Annals of Warsaw University of Life Sciences – SGGW
Agriculture No 66 (Agricultural and Forest Engineering) 2015: 79–88
(Ann. Warsaw Univ. of Life Sci. – SGGW, Agricult. 66, 2015)

Changes in the parameters of the transfer function of a twin screw extruder

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Abstract: *Changes in the parameters of the transfer function of a twin screw extruder.* The objective of this work was to determine the static characteristics and then the transfer function of the process of extrusion for selected process variables, such as screw rotational speed (*n*) and raw material flow rate (*Q*). In the study, the process of extrusion of a mixture of raw brewers' grains and maize cereal of humidity of 18%. Tests were conducted using the Clextral EV25 twin screw extruder. It was found that the extrusion process was an object that reflected well the areas of variability of parameters, such as object gain, time constant and transfer function.

Key words: extrusion, extruder, transfer function, transient state

INTRODUCTION

Extrusion of cereal products is a "continuous" process that allows for performance of several unit operations in a single device, such as: mixing, kneading, cooking and forming. The proper setting of parameters and stable course of the extrusion process is thus an essential factor that allows for keeping of constant quality of the product. Changes in input values, which are even of short duration, may exert significant impact on maintenance of stability of the process, and thus on the quality of end products [Ekielski 2006, Żelaziński 2014, Siwek et al. 2014]. It is thus significant to get familiar with the features of the extrusion process as the object of adjustment [Erikainen and Linko 1987].

Analysis of such processes as extrusion is complicated, since any change in the conditions of the process usually leads to their stabilization on a different level [Strejc 1981, Li et al. 2005]. Therefore, it is difficult to apply a model developed under the conditions established to foresee the course of the process while going from one level to another [Janaszek et al. 2007, Golisz 2009, Zawistowski et al. 2010]. This often leads to excessive energy and raw material losses [Wójtowicz and Mościcki 2008]. It is therefore important to develop the model of the process as an object of adjustment, taking into account its dynamics. Such model would be a valuable tool for: analysis of energy efficiency and performance, selection of the appropriate settings of the adjustment systems, defining of general laws, describing behavior of the extrusion process as a result of the input parameters.

Dynamic behavior of the extrusion process has been discussed broadly in the context of plastic extruders, which served as a basis for early models describing the processes of extrusion of vegetable raw materials. The models of extrusion of vegetable raw materials, however, are more complex in comparison with plastics [Pabedinskas and Cluett 1994]. This is associated with the specific characteristics of vegetable materials, which, in the case of extrusion, consist of mixes of various components. Moreover, various components in mixtures usually interact strongly with one another [Kokini et al. 1992], which results in changes taking place in the products during the extrusion process (such as gelatinization of starch, denaturation of proteins etc.), which makes the entire model even more complicated.

One of the methods of development of the extrusion process is the experimental approach. In this method, the transfer function is used, expressed as a proportion between the process outputs, described using Laplace's equations.

The objective of this study was to define the static characteristics of the extrusion process for the selected process variables (N, Q), and then to establish the area of variability of parameters k_{ob}, T and τ, of the transfer function $G_e(s)$ described by equation:

$$G_e(s) = \frac{k_{ob}}{(Ts+1)^n} e^{-\tau s} \quad (1)$$

where:
$G_e(s)$ – transfer function;
k_{ob} – object gain;
n – object order (number of inertial units);
T – time constant.

METHODOLOGY

Overview

The study was conducted using a mixture of maize grits with 10% addition of raw brewers' grains. The mixture was obtained by mixing the two ingredients immediately before processing. The humidity level of the mixture used for tests amounted to 18%. The DOE module of Statistica 10 software was used to prepare the experiment and to process the statistical results.

Examination of static characteristics of the process and its transfer function was conducted during extrusion of the mix in the laboratory twin screw extruder Clextral EV25. The proportion of the screw length to the diameter was $L : D = 27 : 1$. The screw configuration was a standard factory setting.

The extruder consisted of six-cylinder sections, equipped with a temperature adjustment system with two setpoint values each. During the tests, a constant temperature profile was maintained, amounting to: 30, 50, 80, 100, 110, 120°C, for each of the sections from 1 to 6, respectively (Fig. 1). The material was transferred by a single nozzle die of circular cross section and diameter of 2.5 mm.

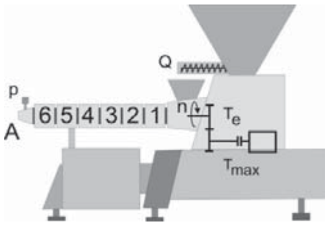

FIGURE 1. Diagram of the extruder with the temperature stabilization sections: A – extruder die, 1–6 – subsequent heating sections, *Q* – mass flow rate, *p* – pressure in the extruder head

The load of the extrusion unit (*N*) was measured as the ratio of torque (T_e) needed to drive the extruder screws to the maximum value of torque (T_{max}): a maximum load, at which the drive transmission was disconnected by the overload clutch. A measurement of the torque was performed in the gearbox of the extruder and described by formula (1):

$$N = \frac{T_e}{T_{max}} 100\% \tag{2}$$

For pressure measurement, a pressure transducer was used as a standard, located in the extruder head, with the measurement range of 0–25 MPa. The material was fed to the extruder by a feeding screw that had been calibrated for measurement of the mixture mass flow rate (*Q*).

The extruder was equipped with a system for recording of measurement data. Measurement data was saved on the hard disk with the frequency $f = 1$ Hz. Filtration of measurement signals was conducted after the measurements by the algorithm using the adaptation filters. Algorithms written in C++ were implemented to Matlab v12. Determining of the transfer function parameters was conducted using the standard Matlab procedures.

The process input values were the extruder screw rotational speed (*n*) and mass flow rate of the material fed into the process (*Q*) – Figure 2.

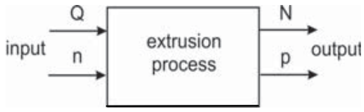

FIGURE 2. The input and output process parameters measured

Specification of the transfer function model using a response of the object to step forcing

The transfer function allows for comparison of the impact of input factors for various inputs. In reality, it is the ratio of Laplace transform of the input signal to the input signal. Thus, it allows for presentation of the manner, in which the object will process the input signal and how the input signal will determine the output of the object examined. Experimental determination of the transfer function consists of emitting of a forcing signal – $x(t)$ – to the input and measuring of changes in the response value – $y(t)$ – at the output. Each of the functions examined should be transformed to the following format Laplace transform:

$$Y(s) = \alpha\{y(t)\} \tag{3}$$

$$X(s) = \alpha\{x(t)\} \tag{4}$$

By dividing equations by one another, we receive the $G(s)$ transfer function equation:

$$G(s) = \frac{Y(s)}{X(s)} = \frac{\alpha\{y(t)\}}{\alpha\{x(t)\}} \quad (5)$$

Determination of the transfer function of an object with unknown characteristics is performed by examining changes in the input signal of the system upon emission of the forcing signal on the output. In one of the most frequently used methods, this is a step forcing, consisting of a change of the stable initial value of the input signal into the invariable end value (standard forcing). The value of forcing at the object input is described by the following function:

$$x(t) = k \cdot 1(t) \quad (6)$$

The response of the stable system approaches the steady state:

$$y(t) = k \cdot k_{ob} \quad (7)$$

where:
k_{ob} – examined object gain.

The extrusion process is usually presented as the 2nd order object. Such object is described by a general linear differential equation:

$$a_2 \cdot \frac{\delta^2 y}{\delta t^2} + a_1 \cdot \frac{\delta y}{\delta t} + a_o \cdot y = b_o \cdot x \quad (8)$$

For static conditions, differential elements assume zero values. Thus, the differential equation format is as follows:

$$a_o \cdot y = b_o \cdot x \quad (9)$$

and further:

$$y = \frac{b_o}{a_o} \cdot x \quad (10)$$

Using the shape of the step response of the object, it is possible to estimate the order of the object. If the response of the object to the step interference is not periodic, like presented in Figure 3, the object transfer can be modelled using the transfer function of the n-th order described by the equation (1).

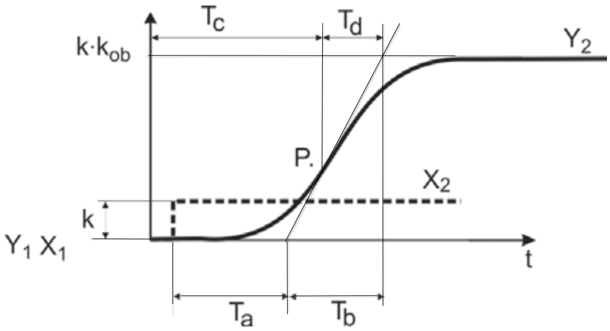

FIGURE 3. Response of the static object – $y(t)$, as a response to step forcing – $x(t)$. Determination of constant proportional and inertial units

Values of parameters: k_o, T_a, T_b, T_c present in equation (1) can be determined graphically (Fig. 3). For the purpose of the course of change in the output value (Y_2) as a result of forcing (X_2), gain k_{ob} was determined on the basis of the following formula:

$$k_{ob} = \frac{\left|\frac{Y_1 - Y_2}{Y_1}\right|}{\left|\frac{X_1 - X_2}{X_2}\right|} \tag{11}$$

Specification of values of parameters T_a and T_b allows for approximation of the order of magnitude of the object n according to the method developed by Strejc [1959]. The method consists of determination of the point of inflexion of curve of the step response of the object, running of the tangent to this curve at the point of intersection and reading of the values of parameters: T_a and T_b, T_c, and assigning of order n to the estimated model. The estimation values can be found in the conversion table (Table 1).

where:
T_a – substitute delay time;
T_b – substitute time constant.

Indexes d and t refer to values read from the curve (index d) and the closest value read, entered in Table 1 (index t). On the basis of familiarity with the transfer function – $G(s)$ – it is possible to determine the course of change of the signal, which will be obtained at the output of the object for the set change of the input signal, calculating the inverse Laplace transform:

$$y(t) = \alpha^{-1}\{X(s) \cdot G(s)\} \tag{13}$$

Using the shape of the curve expressing the response of the system being examined, it is possible to find the dynamic response, expressed by a change in pressure inside the extruder die and a change in loading of the main drive motor in the time function. The change of the pressure value in the extruder die – $p(t)$ – and load – $N(t)$ – is enforced by

TABLE 1. Assigning of the inertial order and time constant (T_o) depending on the read values of parameters: T_a, T_b and T_c

T_a / T_b	0.000	0.104	0.218	0.319	0.410	0.493	0.570	0.642	0.709	0.773
N	1	2	3	4	5	6	7	8	9	10
T_c / T	0	1	2	3	4	5	6	7	8	9

Value τ is calculated from equation

$$\tau = \left\| \frac{T_a}{T_b} \right|_d - \left| \frac{T_a}{T_b} \right|_t \right| \cdot T_b \tag{12}$$

step change in the rotational speed of the extruder screw – $n(t)$ – or by a change in the quantity of material – $Q(t)$ – fed for the process. These changes can be presented as the course of changes in the value of pressure in the cylinder of the extruder, obtained as a result of change in the input value (*Q*). The values measured have been presented in Table 1.

Due to technical reasons, range *Q* for parameters: K_{ob}, *Tz* and *To*, has been narrowed, and the 3D diagrams provided presented approximated values within the range of 2–9 $kg \cdot h^{-1}$.

Conducting of the experiment

The mathematical model of the extrusion process was developed by measuring the selected output values, depending on the value of the input parameters. On this basis, a model of the course of the process was established. Table 2 presents the input values (*n* – revolutions, *Q* – flow rate of raw material). The output values were recorded only after the process stabilization (about 10 min).

The transfer function around the selected duty points was established by measuring the object response [$N(t)$, $p(t)$] to step forcing of input values (ΔQ, Δn). The size of the change in rotational speed (Δn) and the quantity of material fed (ΔQ) were based on the subsequent measurement points, calculated in the Design of Experiments module of Statistica software. Step forcing was implemented only after stabilization of the process, changing the value of only one variable. The observations were concluded after stabilization of settings at the new level of output values. Table 1 presents absolute forcing values used in research.

RESULTS

Statistic characteristics of the extrusion process

Tables 3 and 4 present the input values set as the parameters of the extrusion process and the output values parameters as the values obtained during the extrusion process.

TABLE 2. Changes in the process input values

<table>
<tr><th>Process input</th><th colspan="5">Value</th></tr>
<tr><td>Rotational speed of N extruder screws [rpm], under set conditions</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td></tr>
<tr><td>Forcing value (Δn) [rpm]</td><td colspan="5">–50</td></tr>
<tr><td>Raw material flow rate (Q) [kg·h^{–1}], under set conditions</td><td>5.04</td><td>10.86</td><td>12.82</td><td>16.00</td><td>×</td></tr>
<tr><td rowspan="2">Forcing value (ΔQ) [kg·h^{–1}]</td><td colspan="2">5.82</td><td colspan="2">3.18</td><td>×</td></tr>
<tr><td>×</td><td colspan="2">1.96</td><td>×</td><td>×</td></tr>
</table>

TABLE 3. Die pressure and extruder screw motor load values obtained under static conditions ($Q = 10.86$ $kg \cdot h^{-1}$)

<table>
<tr><td>Process input</td><td>Extruder screw rotational speed (n) [rpm]</td><td>150</td><td>200</td><td>250</td><td>300</td><td>350</td></tr>
<tr><td rowspan="2">Process output</td><td>Pressure value (p) [bar]</td><td>201</td><td>192</td><td>155</td><td>148</td><td>147</td></tr>
<tr><td>Load (N) [%]</td><td>71</td><td>65</td><td>56</td><td>42</td><td>41</td></tr>
</table>

TABLE 4. Die pressure and extruder screw motor load values obtained under static conditions (n = 250 rpm)

Process input	Raw material flow rate (Q) [$kg \cdot h^{-1}$]	5.04	10.86	12.82	16.00
Process output	Pressure value (p) [bar]	150	155	160	171
	Load (N) [%]	50	56	64	75

The characteristics obtained in Figures 4 and 5 have been presented in form of 3D charts as functions: $N = f(Q, n)$ and $p = f(Q, n)$. It was found that both the percentage load of the motor N and the obtained pressure (p) changed significantly along with the input values: Q and n. Moreover, the tendency of these changes was similar. The trends indicate that N and p values tend to decrease along with reduction in the flow rate of material fed and increasing of the extruder screw rotational speed.

Values of dynamical parameters of the object

The transfer function values were established for two forcing variables: step change in the quantity of raw material fed and change in rotational speed of the extruder screws. Figure 6 presents the response of relative value of motor load

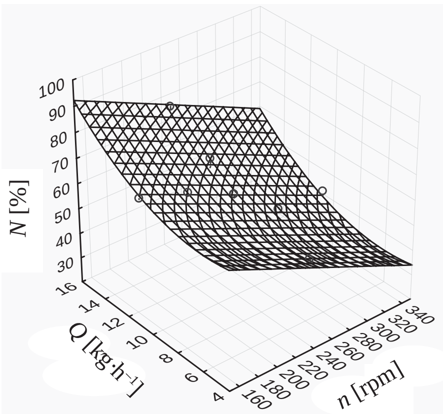

FIGURE 4. Object static characteristics: $N = f(Q, n)$

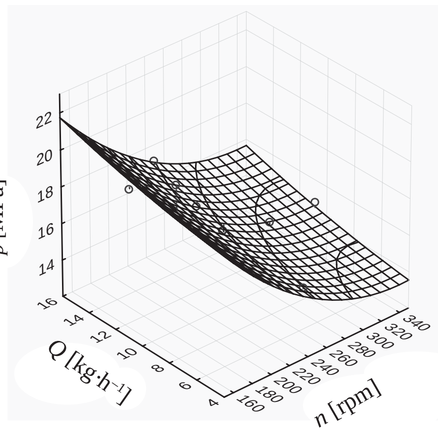

FIGURE 5. Object static characteristics: $p = f(Q, n)$

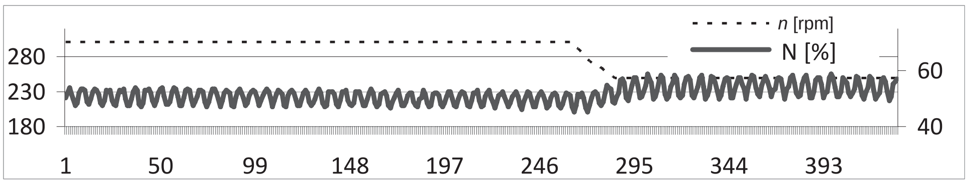

FIGURE 6. A change in the driving system load (N) during change of rotational speed of the screws (n)

to step interference in form of change of the rotational speed of the extruder screws.

The chart presented in Figure 7 indicates that parameter k_{ob} tends to increase along with the flow rate of raw material (*Q*) and reduction of rotational speed of screw (*n*) except for rotational speed range of 140–180 rpm. Under these conditions, the value of parameter k_{ob} decreased slightly and it reached the lowest point at the smallest *n* and *q* values.

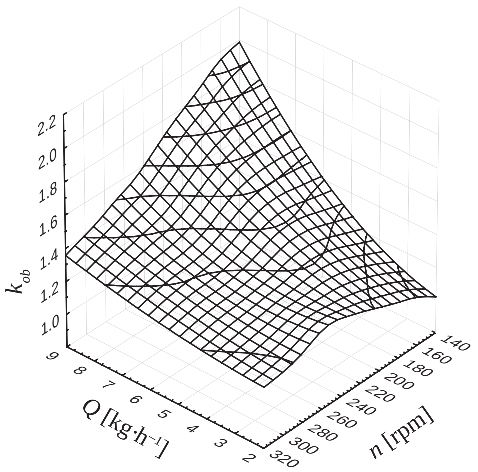

FIGURE 7. The gain of object (k_{ob}) in the function of rotational speed of screws (*n*) and the raw material flow rate (*Q*)

A similar tendency was observed in the chart describing the time constant (*Tz*) in *f*(*n*, *Q*), (Fig. 8). It is also possible to observe a tendency of increase of parameter *Tz* along with reduction of parameter *Q* and rotational speed of extruder screws (*n*).

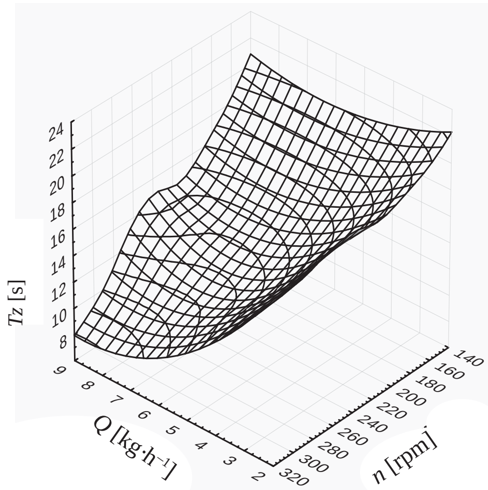

FIGURE 8. Time constant (*Tz*) in the function of rotational speed of screws (*n*) the raw material flow rate (*Q*)

Parameter *To*, like the parameters described earlier, increased along with reduction of *n* and *q* (Fig. 9). Its values, however, changed only within the range of 18.2–25.3%.

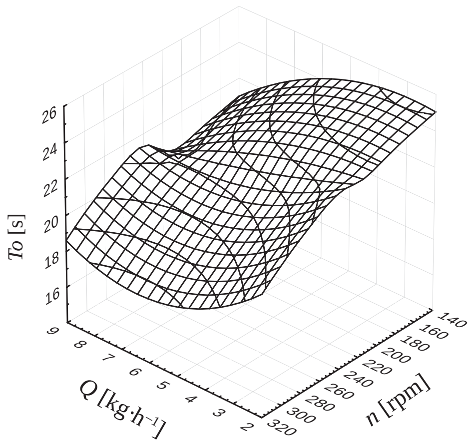

FIGURE 9. Time constant (*To*) in the screw rotational speed function (*n*) and the raw material flow rate function (*Q*)

CONCLUSIONS

1. The extrusion process is an object, which reflects well the parameter changeability areas, such as object gain, time constant and transfer function. The charts allow for identification of positions characterized by sudden increase of k_{ob}, as well as *To*.
2. The process of extrusion of cereal products has visible non-linearity features.
3. The obtained results for static parameters of the object are characterized by patterns similar to those described in literature. The transfer function components change simultaneously to a significant extent, depending on the process parameters.

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Streszczenie: *Zmiany parametrów transmitancji operatorowej ekstrudera dwuślimakowego.* W pracy przedstawiono przykład wyznaczenia charakterystyki statycznej i transmitancji operatorowej procesu ekstruzji. Badania obejmowały analizę danych pomiarowych wybranych zmiennych procesowych takich jak prędkość obrotowa ślimaków (*n*) oraz natężenie przepływu surowca (*Q*). Wyniki pomiarów pozyskano podczas prowadzenia badań nad ekstruzją surowego młóta browarnianego i kaszki kukurydzianej o wilgotności 18%. W badaniach wykorzystywano ekstruder dwuślimakowy Clextral EV25 (oprogramowanie ekstrudera pozwalało na dokładny zapis danych pomiarowych z częstotliwością 1 Hz). Stwierdzono, że proces ekstruzji jest obiektem, który dobrze odzwierciedla obszary zmienności parametrów, takie jak: wzmocnienie obiektu, stała czasowa oraz transmitancja operatorowa. Wyniki badań potwierdziły również doniesienia innych autorów, że ekstruzja to proces o wyraźnych cechach nieliniowości.

Słowa kluczowe: ekstruzja, ekstruder, transmitancja, stan nieustalony

MS received May 2015

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