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OPTIMIZATION OF A TRANSPORT APPLYING GRAPH-MATRIX METHOD

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Summary. The paper presents the procedure for solving a transportation task. The optimization was carried out in two phases. In the first one, a preliminary flow matrix was achieved on a base of the information on the demands and supplies values as well as transport costs, whereas some shifts within the preliminary flow matrix were made. All changes resulted in an optimum matrix, for which Z_x function held the lowest possible value. Presented method appeared to be efficient for solving the transportation tasks of particular type.

Key words: transport, optimization, Vogel's Approximation Method, Graph-Matrix Method.

INTRODUCTION

Transport is a very important element of any economy, because it makes possible to flow goods and services among its particular branches. It assures supplying raw materials, materials, semi-products, and final products, and becomes one of intermediate stages during production process, which makes various institutions closer to one another. Connections between transport and economy branches is not of a single-directional character; the transport uses economy's achievements such as road system, fuels, etc. [Basiewicz, Gołaszewski, Rudziński, 2007, Burski, Mijalska-Szewczak 2008, Towpik, Gołaszewski, Kukulski 2006, Zielińska, Lejda 2010]. Close inter-relations between transport and economy branches make the development level of the latter depends on development level of the former and vice versa [Pang 2004]. However, notion *development* should be understood not only as modernization of transport means, but also optimization of transporting tasks, the means take part in. The development determines not only the increase of weight of goods possible to transport during a time unit, but also the decrease of expended costs, which can be modified by means of route optimization, for instance [Jacyna 2009, Marczuk 2009].

The study aimed at presenting the procedure for solving the transportation tasks using selected optimization methods. The optimization was carried out in two phases. The objective of the first was to formulate a preliminary flow matrix $X = [x_{ij}]$, on a base of information related to the demand and supply data along with cost values for transporting goods between suppliers and receivers. The second phase focused on shifting within such achieved flow matrix in such a way to make matrices held lower or equal (not higher) values of function Z_x, which is the costs expended to all transportation operations. Many procedures were prepared both for the first and second phase

[Tiwari, Shandilya 2006, Kasana, Kumar 2004, Bernard 1999]. Among methods related to the first phase, such ones as The Northwest Corner Method, Vogel's Approximation Method, The Minimum Cost Method, etc. The Graph-Matrix Analysis with its modifications is counted to the second phase. Particular methods differ relating both to actions, and conditions, as well as complexity, difficulty in implementing, and precision of achieved results [Srinivasan 2008, Jain, Aggarwal 2009, Sen 2010].

VOGEL'S APPROXIMATION METHOD

The method is a specific procedure leading to solve a transportation task; it allows for achieving a final solution. However, the result is not always optimum, because sometimes it is only some approximation of the optimum. Inaccuracy of the method makes that it can be efficiently applied only for defining the preliminary solution (achievement of preliminary flow matrix). Applying the Vogel's Approximation Method in the first phase of procedure contributes to time saving when the whole optimization action is performed. The method is usually much more accurate than other ones used in that phase, which results in lower number of iterations necessary to be made in subsequent phase.

The starting point for Vogel's Approximation Method – as similar as for other first-phase methods – is made of the system C, M consisted of transport cost matrix $C = [c_{ij}]$ and vector $M = [a_1, a_2, \ldots, a_m; b_1, b_2, \ldots, b_n]$ representing receivers' demands and suppliers' supplies (where: m – number of suppliers, n – number of receivers). It should be on mind that optimization is possible when C, M system meets the equation:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \tag{1}$$

where: $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Calculation methodology

1) Firstly, the Vogel's number should be calculated for particular rows and columns within transport cost matrix. For rows, the number holds value equal to the difference between two the lowest (representing the lowest costs) elements present in a given row. In the case of columns, the situation is analogous – Vogel's number is a difference between two the lowest values in a given column.
2) In subsequent step aiming at assigning the transport size to particular routes, four actions should be performed:
   – selection of a row or a column, for which calculated Vogel's number holds maximum value. Various cases can arise at that step, which may affect further procedure; (i) it may be a situation when the highest Vogel's number is achieved both for a row and a column, and matrix element present at the intersection represents the lowest costs; (ii) another situation consists in that the element present at the intersection of the row and the column does not represent the lowest costs; (iii) the case if the highest Vogel's number is achieved for several rows or columns at the same time; (iv) another situation differing from the previous one in that the element present at the intersection of the row and the column does not represent the lowest costs; such case is met when the highest Vogel's number is achieved for several rows and columns at the same time.

- selection of the element that contains the lowest costs and belongs to a row or a column selected in previous step. Existence of any of above described case, requires special procedure: (i) common assignment both for a row and a column should be introduced in place where element representing the lowest costs is present; (ii) there is a discretion in introducing the assignment to the row or the column where element representing the lowest costs is present; (iii) the case allows for introducing the assignment to any of matrix element, regardless of where the element representing the lowest costs is present;
- assignment of the highest possible number of transport quantity to the transport costs matrix element selected in previous action;
- cancelling the column (columns) or row (rows), for which the assignment made the receiver's (receivers') demands or supplier's (suppliers') supplies are met.

3) Calculating the Vogel's number for particular rows and columns (without cancelled ones).
4) Repeating second and third steps until all assignments are made, i.e. achieving preliminary flow matrix $X = [x_{ij}]$.
5) Calculating value of Z_x function [Bocchino 1975, Krawczyk 2001, Shenoy 1998, Khanna 2009, Bandopadhyaya 2007]:

$$Z_x = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}. \tag{2}$$

GRAPH-MATRIX METHOD

The Graph-Matrix Method is one of the second-phase method during the optimization procedure.

Verifying if preliminary flow matrix achieved in previous actions has been properly created should be carried out prior to this method applying. The verification can be performed applying following criteria:

$$\sum_{j=1}^{n} x_{ij} = a_i, \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \tag{4}$$

where: i = 1, 2, …, m; j = 1 , 2, …, n.

Calculation methodology

1) Creating the matrix graph by means of connecting all nodes; nodes can be connected only in vertical or horizontal directions.
2) Evaluating the type of basis solution represented by preliminary flow matrix achieved in the first step. It is extremely important, because the necessary and sufficient condition for the method to be applied states that any preliminary solution has to be the basic of the first or the second type. In order to set the type of the basic solution, it should be checked whether: (i) number of nodes of the graph constructed using the flow matrix equals to m + n – 1; (ii) graph is consistent; (iii) graph does not contain any cycle; (iv) graph's nodes

correspond to positive node elements of the flow matrix; (v) all –no-node elements of the matrix are null. If all above conditions are met, the flow matrix X is a basic solution of the first type; otherwise, the matrix is no basic solution. Then, it is necessary to make transformation aiming at achieving the basic solution of the second type, which consists in distinguishing the null elements of discussed matrix as node elements to get their m + n – 1 number. The distinguishing should be made in such a way to make graph stretched on those nodes consistent and not containing any cycle.

3) Making some actions aiming at verifying the optimization criterion for a given flow matrix; the process consists of the following steps:
   - creating the equivalent null matrix of costs C^1 by means of finding some constants u_i and v_j, which added to particular rows (u_i) and columns (v_j) would bring all node elements of the matrix to zero. In order to make procedure easier, action should be begun from the separate node element in a given row or a column. If the node element is separate in a column, number equal to that element should be subtracted from the row, which results in finding the first u_i. If the node element is separate in a row, number equal to that element should be subtracted from the column, which results in finding the first v_j.
   - creating the equivalent null matrix C^1 by means of adding previously found values u_i and v_j to particular rows and columns of the cost matrix C.
   - verifying the optimization criterion for the flow matrix X. According to a theorem, the flow matrix X is optimum for transportation problem, if positive values of its elements correspond to elements from equivalent null matrix C^1, value of which is zero, while other elements are not negative.
4) If a given matrix appears not to be optimum flow matrix for transportation problem, it is necessary to execute step 4 consisting in making some corrections in flow matrix aiming at reducing the value of Z_x function. First, the lowest element among all negative ones in equivalent null matrix of costs C^1, should be selected. If there are several such elements, any of them can be selected. Then, a node with corresponding selected element, should be incorporated into the graph, which results in creating the graph spanned on m + n nodes and containing a simple cycle. Next, a cycle should be found and a set of nodes that belongs to it should be divided into two subsets $A_1(+)$ and $A_2(-)$: node that was incorporated to the previous action has to be assigned to subset $A_1(+)$, while every two adjacent nodes are assigned to other subsets. Then, iteration should be performed, which is based on making some changes in transport size values. These changes consist in subtracting the assignment corresponding to the lowest of elements x_{ij} at nodes of subset $A_2(-)$ and assigning to nodes of subset $A_1(+)$, which in consequence cancels the cycle.
5) Calculating the value of Z_x function using the same formula that was applied in Vogel's Approximation Method.
6) Repeating the actions of steps 3 to 5 for newly created flow matrices (X_1, X_2, …, X_k) until the optimum matrix is achieved [Całczyński 1992, Natarajan, Balasubramani, Tamilarasi 2005].

RESULTS

Execution of optimization methods is going to be presented in practice using the example of goods distribution between warehouses "H" (localized in: Lublin, Chełm, Ostrów Lubelski, and Ryki) and supplied shops "S" (situated in: Łęczna, Fajsławice, Łuków, Bychawa, and Kazimierz

Dolny). The transportation costs on particular routes have been set as the product of the transport fee rate (3.80 PLZ/km) and doubled distance between given enterprises (km). Levels of receivers' demand (a_i) and suppliers' supplies (b_j) have been randomly selected.

When Vogel's Approximation Method was applied, the calculation procedure is following:

Table 1. The C, M system (highlighted fragment is matrix of costs C)

	S_1	**S_2**	**S_3**	**S_4**	**S_5**	**a_i**
H_1	190	273,6	722	243,2	433,2	200
H_2	425,6	311,6	1238,8	570	942,4	90
H_3	190	418	547,2	585,2	638,4	30
H_4	668,8	752,4	615,6	722	334,4	90
b_j	100	50	80	60	120	410

Table 2. The C, M system improved with calculated values of Vogel's numbers (first step of the procedure)

		0	38	68,4	326,8	98,8	
		S_1	S_2	S_3	S_4	**S_5**	**a_i**
53,2	**H_1**	190	273,6	722	243,2	433,2	200
114	**H_2**	425,6	311,6	1238,8	570	942,4	90
228	**H_3**	190	418	547,2	585,2	638,4	30
281,2	**H_4**	668,8	752,4	615,6	722	334,4	90
	b_j	100	50	80	60	120	410

Table 3 presents bold characters representing the route, where transport is going to be realized; number in the superscript stands for the transport size through a given route.

Table 3. The C, M system after the second-step procedure

		0	38	68,4	326,8	98,8	
		S_1	**S_2**	**S_3**	**S_4**	**S_5**	**a_i**
53,2	**H_1**	190	273,6	722	**243,2**^{60}	433,2	200
114	**H_2**	425,6	311,6	1238,8	~~570~~	942,4	90
228	**H_3**	190	418	547,2	~~585,2~~	638,4	30
281,2	**H_4**	668,8	752,4	615,6	~~722~~	334,4	90
	b_j	100	50	80	~~60~~	120	410

Due to a fact that further procedure aiming at making the assignment, proceeds the same way, it was omitted.

Table 4. Preliminary flow matrix X (achieved by making all assignments)

	S_1	S_2	S_3	S_4	S_5	a_i
H_1	**190^{30}**	273,6	**722^{80}**	**$243,2^{60}$**	**$433,2^{30}$**	200
H_2	**$425,6^{40}$**	**$311,6^{50}$**	1238,8	570	942,4	90
H_3	**190^{30}**	418	547,2	585,2	638,4	30
H_4	668,8	752,4	615,6	722	**$334,4^{90}$**	90
b_j	100	50	80	60	120	410

Values of Z_x functions according to dependence (2) amounted to 159 448 PLZ.

Applying the Graph-Matrix Method, calculations start from creating the preliminary flow matrix.

Table 5. Preliminary flow matrix X with spanned matrix graph

	S_1	S_2	S_3	S_4	S_5
H_1	**190^{30}**	273,6	**722^{80}**	**$243,2^{60}$**	**$433,2^{30}$**
H_2	**$425,6^{40}$**	**$311,6^{50}$**	1238,8	570	942,4
H_3	**190^{30}**	418	547,2	585,2	638,4
H_4	668,8	752,4	615,6	722	**$334,4^{90}$**

Matrix C^1 included in Table 6 contains negative elements, hence the preliminary flow matrix is not the optimum.

Table 6. The C^1, X system

	S_1	S_2	S_3	S_4	S_5
H_1	**0^{30}**	197,6	**0^{80}**	**0^{60}**	**0^{30}**
H_2	**0^{40}**	**0^{50}**	281,2	91,2	273,6
H_3	**0^{30}**	342	-174,8	342	205,2
H_4	577,6	775,2	-7,6	577,6	**0^{90}**

Table 7. The C^1, X system with highlighted the lowest negative element

	S_1	S_2	S_3	S_4	S_5
H_1	0^{30}	197,6	0^{80}	0^{60}	0^{30}
H_2	0^{40}	0^{50}	281,2	91,2	273,6
H_3	0^{30}	342	**$-174{,}8^{0}$**	342	205,2
H_4	577,6	775,2	-7,6	577,6	0^{90}

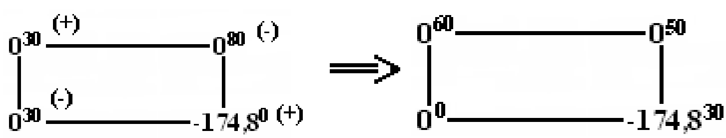

Fig. 1. Changes during iterations

Table 8. The C^1, X system after iterations

	S_1	S_2	S_3	S_4	S_5
H_1	0^{60}	197,6	0^{50}	0^{60}	0^{30}
H_2	0^{40}	0^{50}	281,2	91,2	273,6
H_3	0	342	**$-174{,}8^{30}$**	342	205,2
H_4	577,6	775,2	-7,6	577,6	0^{90}

Like for Vogel's Approximation Method, the procedure has been shortened by repeating actions.

Table 9. Final version of the C^1, X system

	S_1	S_2	S_3	S_4	S_5
H_1	0^{60}	197,6	7,6	0^{60}	0^{80}
H_2	0^{40}	0^{50}	288,8	91,2	273,6
H_3	167,2	509,2	0^{30}	509,2	372,4
H_4	577,6	775,2	0^{50}	577,6	0^{40}

Table 10. Final flow matrix X

	S_1	S_2	S_3	S_4	S_5	a_i
H_1	$\mathbf{190^{60}}$	273,6	722	$\mathbf{243{,}2^{60}}$	$\mathbf{433{,}2^{80}}$	200
H_2	$\mathbf{425{,}6^{40}}$	$\mathbf{311{,}6^{50}}$	1238,8	570	942,4	90
H_3	190	418	$\mathbf{547{,}2^{30}}$	585,2	638,4	30
H_4	668,8	752,4	$\mathbf{615{,}6^{50}}$	722	$\mathbf{334{,}4^{40}}$	90
b_j	100	50	80	60	120	410

Value of Z_x function for calculations made by means of Graph-Matrix Method was 153 824 PLZ.

CONCLUSIONS

The two-step procedure possible to be used at optimization of a transport realized between the demand and supply points, has been presented. In the first step, the preliminary flow matrix X has been achieved, for which value of Z_x function amounted to 159 448 PLZ. Two iterations has been performed in the second step. The first one consisted in incorporation of C^1 matrix element of 174.8 value (the lowest negative element of the matrix) into the graph and making some shifts in the transport quantities of the graph elements included in a cycle. That operation has resulted in assigning the value 30 to previously incorporated C^1 matrix element and non-profitable route with the same transport size assigned has been canceled. The second iteration was identical; however it referred to the lowest negative matrix C^1 element that remained after zeroing previous the lowest negative element of that matrix (according to procedure leading to verification of the optimization criterion). Like for previous iteration, the transport size of 50 value has been assigned to incorporated element and non-profitable route has been eliminated. These changes have led to achieving the final matrix (optimum), for which Z_x function held value of 153 824 PLZ. It means that changes can allow for achieving the savings during the transportation tasks at the level of 5 624 PLZ.

Above method appeared to be efficient for solving the transportation tasks, the example of which was presented in the paper.

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OPTYMALIZACJA TRANSPORTU PRZY WYKORZYSTANIU METODY GRAFÓW MACIERZOWYCH

Streszczenie. Celem pracy było przedstawienie postępowania właściwego dla rozwiązywania zadania transportowego. Działania optymalizacyjne przeprowadzono w dwóch etapach. W pierwszym z nich uzyskano wstępną macierz przepływów, na podstawie informacji o wielkości popytów odbiorców i podaży dostawców oraz wartości kosztów przewozów. W drugim etapie dokonano przesunięć we wstępnej macierzy przepływów, uzyskanej w etapie pierwszym. Zmiany te doprowadziły do uzyskania macierzy optymalnej, dla której funkcja Z_x przyjęła najniższą wartość. Przedstawiona metoda okazała się być skuteczna przy rozwiązywaniu zadań transportowych określonego typu.

Słowa kluczowe: transport, optymalizacja, metoda aproksymacyjna Vogela, metoda grafów macierzowych.