Modeling of kinematics of movement on turn of a wheeled tractor with a hinge-operated drawbar on semitrailer

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Summary. In the article the mathematical description and results of modeling of kinematics of turn of an agricultural tractor train with a hinge-operated drawbar on semitrailer is presented at various modes of turn. The developed technique allows for the solution of many practical research problems of shunting property of tractor trains with trailers of the new constructive scheme, for the purpose of a choice at a stage of working out of the trailer of the best parameters both of the trailer, of its connection with the tractor.

Key words: tractor train, semitrailer with hinge-operated drawbar, mathematical modeling of kinematics of unsteady turn.

INTRODUCTION

Development of tractor trains goes by the way of their specific increase, on units of constructive weight, loadcarrying capacity, decrease in a body weight, increase of manoeuvrability and admissible speeds of movement with a tractor, and also indicators of other major properties [2, 5, 14, 17, 21]. In realization of the listed ways advantages of the semi-hook scheme of execution of a link of a tractor train bearing cargo before the scheme of the trailer with free hitch and the scheme of the semi-hinged trailer [3, 8, 10, 12, 16, 22] are accurately traced. Thus, the growth of load-carrying capacity of semitrailers is reached by an increase in the number of tandem-type wheel axes of the wheel cart, and increase of manoeuvrability of trailers with long-base ("breaking") operating hitch. However, many questions of dynamics of movement of such tractor trains demand further consideration [1, 5, 11, 18], for the purpose of the proved choice of rational design data of created new semitrailers, a control system of a forward rotary part drawbar and, if necessary, the turn of wheels or of the wheel cart as a whole with the semitrailer in the plane.

In the article methodical positions of modeling and an estimation of one of the major properties of tractor trains with a hinge-operated drawbar on semitrailer – the shunting property - are considered.

Growth of capacity and tonnage of agricultural tractor trains is accompanied by the increase of inertion and lengths. Thus they aim at an increase of speeds of movement at the expense of use of tractors with a gear change without rupture of a stream of capacity or transmissions with stepless change of the transfer relation, for example, the two-line. All it demands is the careful analysis of dynamics of movement of tractor trains with new constructive scheme and, in particular, their shunting property.

The purpose of the given article consists in working out of the mathematical description and reception of results of modeling of kinematics of turn of a tractor train with a hinge-operated drawbar on semitrailer at various modes of turn.

METHODICAL POSITIONS OF MODELING OF SHUNTING PROPERTY

At creation of the tractor transport technological unit on the basis of a wheel tractor obligatory section of design researches is the estimation of its shunting property. At such estimation for a case of movement of the machine tractor unit on a circular trajectory with the established speed radiuses of turn of characteristic points and corners of a relative positioning of links of the unit are defined. As the generalising estimated parametre of shunting property, the factor of manoeuvrability equal to the relation of dimensional width of the unit to dimensional width of its circular rotary strip is used, for example.

It is considered that the value of the factor of manoeuvrability is closer to unit, the best shunting properties the unit possesses. For an estimation of fitness of the tractor the unit to passage of turns of a surface of movement the size of width of a dimensional lane is used at turn, and for an estimation of manoeuvrability of the unit at the maximum corners of turn of operated wheels of a tractor - the minimum dimensional radius of turn of the tractor unit [4, 6, 7, 9, 14, 19, 20].

To manoeuvrability estimations at static turn estimations of phases of trajectories of unsteady movement on turn are in case of need added: an input in turn and an exit from it, that is a phase which precedes the period of circular turn and which follows it [3, 8, 10].

Reception estimations of curvilinear movement of a tractor train is made on the basis of the general expressions for characteristics of curvilinear movement of wheel tractor units as a corner of turn of operated wheels at various values of regime parameter of turn which represent the system of the differential equations and allow to make a comparative estimation of units with one tractor and the same semitrailer, but with various laws of management of turn "breaking" on hinges operated trailer hitch the semitrailer [11, 13, 15, 20].

In the figures characteristic conditions of a relative positioning of links of the considered tractor unit are presented at circular turn.





Fig. 1. The scheme of the tractor unit on turn: a) corner $\gamma_2 = 0$; b) corner $\gamma_2 > 0$

Figure 1 presents the size e - displacement of trajectories of the middle of the back bridge of a tractor (point *C*) and the middle of the wheel dual cart of the semitrailer (point *K*). The corner size θ_o is equal to a half-sum of corresponding corners of turn of the right and left operated wheels of a tractor-tractor, other designations are clear from the figure.

At the analysis of the investigated tractor unit represents practical interest an estimation of influence on indicators of shunting property:

- base distance of the semitrailer;
- parities of base distances of a tractor and the semitrailer;
- relative length of a breaking-rotary part compound on hinges operated trailer hitch the semitrailer;
- a corner of turn of a rotary part compound on hinges operated trailer hitch the semitrailer;
- a start of a point of connection of links of the unit;
- a semitrailer track.

For definition of numerical values of indicators of manoeuvrability their analytical expressions have been received, lateral withdrawal of wheels was not considered. From geometrical parities on the basis of Figure 1a and Figure 1b, at introduction of a designation of radiuses of trajectories of characteristic *i*-th points of the settlement scheme of the tractor unit - R_i , it is received:

$$R_A = \overline{AO_c} = \sqrt{l_0^2 \cdot \operatorname{ctg}^2 \theta_0 + c_0^2}, \qquad (1)$$

$$\Delta \alpha_0 = \operatorname{arctg} \frac{c_0}{l_0 \cdot \operatorname{ctg} \theta_0},\tag{2}$$

$$R_L = \sqrt{l_2^2 + R_K^2},$$
 (3)

$$R_{K} = l_{0} \cdot \operatorname{ctg} \theta_{0} \cdot \cos(\gamma_{1} + \gamma_{2}) + c_{0}$$

$$\cdot \sin(\gamma_{1} + \gamma_{2}) + l_{1} \cdot \sin\gamma_{2} \qquad , \qquad (4)$$

$$\overline{TK} = l_0 \cdot \operatorname{ctg} \theta_0 \cdot \sin(\gamma_1 + \gamma_2) = c_0$$

$$\cdot \cos(\gamma_1 + \gamma_2) + l_1 \cdot \cos\gamma_2 + l_2 \quad , \tag{5}$$

whence we will find the unknown corner γ_i . It is known, that:

$$\sin(\gamma_1 + \gamma_2) = \sin \gamma_1 \cdot \cos \gamma_2 + \cos \gamma_1 \cdot \sin \gamma_2, (6)$$

$$\cos(\gamma_1 + \gamma_2) = \cos\gamma_1 \cdot \cos\gamma_2 - \sin\gamma_1 \cdot \sin\gamma_2, \quad (7)$$

that we will transform last equality, having designated $l_0 \cdot \operatorname{ctg} \theta_0 = A_0$:

$$A_{0} \cdot \sin \gamma_{1} \cdot \cos \gamma_{2} + A_{0} \cdot \cos \gamma_{1} \cdot \sin \gamma_{2}$$

= $c_{0} \cdot \cos \gamma_{1} \cdot \cos \gamma_{2} - c_{0}$
 $\cdot \sin \gamma_{1} \cdot \sin \gamma_{2} + l_{1} \cdot \cos \gamma_{2} + l_{2}$, (8)

Let us group the similar:

$$\underbrace{\left(\underline{l_0 \cdot \operatorname{ctg}} \theta_0 \cdot \operatorname{cos} \gamma_2 + c_0 \cdot \operatorname{sin} \gamma_2\right)}_{C_1}_{C_2} \\ \cdot \operatorname{sin} \gamma_1 + \underbrace{\left(\underline{l_0 \cdot \operatorname{ctg}} \theta_0 \cdot \operatorname{sin} \gamma_2 - c_0 \cdot \operatorname{cos} \gamma_2\right)}_{C_2}_{C_2}}_{C_2} \\ \cdot \operatorname{cos} \gamma_1 = \underbrace{\underline{l_1 \cdot \operatorname{cos}} \gamma_2 + l_2}_{C_3}.$$
(9)

Taking into account the entered designations we will receive:

$$C_1 \cdot \sin \gamma_1 + C_2 \cdot \cos \gamma_1 = C_3. \tag{10}$$

Having solved last equation, we will receive size of a required corner γ_1 .

The scheme a) (fig.1a):

$$\overline{O_c C} = R_c = l_0 \cdot \operatorname{ctg} \theta_0 \quad \overline{O_c A} = R_A = \sqrt{R_c^2 + c_0^2}, \quad (11)$$
$$\overline{O_c K} = R_K = \sqrt{R_A^2 + (l_1 + l_2)^2} =$$

$$=\sqrt{l_0^2 \cdot \operatorname{ctg}^2 \theta_0 + c_0^2 + (l_1 + l_2)^2} , \qquad (12)$$

$$\angle \alpha_0 = \operatorname{arctg} \frac{c_0}{l_0 \cdot \operatorname{ctg} \theta_0},\tag{13}$$

$$\angle (\gamma_{1} - \alpha_{0}) =$$

$$= \operatorname{arctg} \frac{l_{1} + l_{2}}{\sqrt{l_{0}^{2} \cdot \operatorname{ctg}^{2} \theta_{0} + c_{0}^{2} + (l_{1} + l_{2})^{2}}},$$
(14)

$$\angle \gamma_1 = \operatorname{arctg} \frac{c_0}{l_0 \cdot \operatorname{ctg} \theta_0} + \operatorname{arctg} \frac{l_1 + l_2}{\sqrt{l_0^2 \cdot \operatorname{ctg}^2 \theta_0 + c_0^2 + (l_1 + l_2)^2}},$$
(15)

$$e = R_C - R_K = l_0 \cdot \operatorname{ctg} \theta_0 - \frac{1}{\sqrt{l_0^2 \cdot \operatorname{ctg}^2 \theta_0 + c_0^2 + (l_1 + l_2)^2}}.$$
(16)

The scheme b) (fig. 1b):

$$R_{C} = l_{0} \cdot \operatorname{ctg} \theta_{0},$$

$$R_{A} = \sqrt{R_{C}^{2} + c_{0}^{2}} = \sqrt{l_{0}^{2} \cdot \operatorname{ctg}^{2} \theta_{0} + c_{0}^{2}}$$

$$\angle CAO_{c} = \operatorname{arctg} \frac{R_{C}}{c_{0}} = \frac{l_{0} \cdot \operatorname{ctg} \theta_{0}}{c_{0}},$$

$$\langle \beta_{1} = 180^{\circ} - \gamma_{1} - \angle CAO_{c}$$

$$\angle \alpha_{0} = \operatorname{arctg} \frac{c_{0}}{l_{0} \cdot \operatorname{ctg} \theta_{0}}.$$
(17)

The resulted expressions of the established turn allow for the investigation of influence on indicators of manoeuvrability of design data of a tractor and the semitrailer with a rotary part compound on hinge-operated trailer hitch. For studying of phases of unsteady turn of the tractor unit (Fig. 2) the differential equation concerning unknown size – a corner of folding γ_1 is received at change of a corner of turn of operated wheels $\theta \sigma$ with constant speed for cases of a constant corner of installation γ_2 rotary parts compound on hinges operated trailer hitch and its operated change by means of the crosswise communications shown in figure 2. The equation conclusion is not resulted because of bulkiness.



Fig. 2. Scheme of unsteady turn of a tractor with the semitrailer

Qualitative character of change estimations of shunting property at an input in turn and an exit from turn depending on a corner of turn of operated wheels of a tractor is shown in Figure 3.



Fig. 3. Character of change of sizes of corners and angular speeds at turn of a tractor with the semitrailer

So, for example, the mentioned differential equation $\gamma_2 = 0$ looks like:

$$(\gamma_{1})_{\theta_{o}}^{\prime} = \frac{d\gamma_{1}}{d\theta_{0}} = \frac{1}{k_{p} \cdot R_{o}(\theta_{o})} \cdot \left[1 - \frac{C_{o}}{\ell_{1}} \cdot \frac{\sin\left[\gamma_{1} - \alpha_{o}(\theta_{o})\right]}{\sin\left[\alpha_{o}(\theta_{o})\right]}\right],$$
(18)

which allows for the choice of one of the known methods, for example, Euler's method to receive communication of corners θ_{a} and γ_{1} during curvilinear movement of tractor units of the considered schemes. In equation

 $k_p = \frac{\theta_o}{v_o}$ - regime parameter of turn,

where: v_{a} - forward speed of the centre of a back axis of the tractor. For tractor units at small radiuses of turn in constrained conditions k_p lays in a range 0,05 ... 0,15 it is rad·m⁻¹.

Initial values

$$\frac{d\gamma_1}{dt} = \frac{d\gamma_1}{d\theta_o} = 0; \quad \theta_o(t=0) = 0.$$
(19)

 $\theta_{i(\textit{the}_\textit{final})}$ - its function from the accepted regime parameter and for range k_p , characteristic for universal wheeled tractors, it is possible to accept:

 $\theta_{i(the_final)} \leq 0,58$ it is radian,

as limiting values of corners of turn of operated wheels or folding corners. As a rule, they do not exceed this value [3]. The step of integration of last equation can be accepted as equal to:

 $\Delta \theta_{a} \approx 0.02$ radian or $\Delta \theta_{a} = 1$ degree.

As at turn of a tractor train, for example, on 90° the driver turns in the beginning wheels to some corner $\theta_{i_{max}}$, and then turns them to zero in the opposite direction it is necessary for considering at integration of the differential equation.

The resulted expressions for a case of unsteady turn allow to receive a picture of change of characteristics of the maneuver made by the unit:

 $\dot{\gamma}_1, \omega_a, \omega_1, \overline{O_aO_1, \alpha_a}, R_a, R_C, R_D, \overline{O_aO_1}, R_F, \theta_a$

as time from the beginning of its fulfilment which have the following appearance (see fig. 3).

RESULTS OF MODELLING OF MANOEUVRABILITY OF THE UNIT

The analysis of the spent variants of calculations shows, that changes of the sizes characterising phases of unsteady turn, it is possible to present, as is shown in figure 3. On settlement data for the considered unit with the set sizes $(l_0 = 2,75 \text{ m}; l_1 = 1,2 \text{ m}; l_2 = 2,5)$ the schedules resulted in figures 4, 5 are constructed. From schedules follows, that to a corner of turn of operated wheels of a tractor in 11 degrees growth of a corner of folding of the semitrailer concerning a tractor lags behind, and then advances corner growth θ_0 , reaching value in 71,5 degrees at maximum on a technical characteristics of a tractor-tractor coal $\theta_0 = 37$ degrees. And after return of operated wheels to a starting position the corner γ_1 is yet equal to zero and the exit on rectilinear movement occurs later in time, which confirms the generalized picture of process of turn of a tractor with the semitrailer (Fig. 3). However, at the maximum corners γ_1 and $\gamma_2=0$ the metal design of the semitrailer can adjoin to a tractor.

With increase in the corner γ_2 , at installation of a rotary part under a corner to a longitudinal-vertical plane, possibility of not operational contact of metal designs of links of the unit will exist at smaller corners of turn of operated wheels θ_{a} .

In Figure 5 and 6 results of an estimation of influence of parameters of the unit on indicators of its manoeuvrability are resulted at circular turn. From the schedule in Figure 5 the corner of folding of the semitrailer concerning the tractor it follows that to values of a corner of turn of operated wheels of a tractor $\theta o = 26$ hailstones, at $\gamma_2=0$ changes practically linearly, and then there is its progressive growth, in comparison with the corner θ_{i} .

The resulted results of calculations on the received expressions have allowed for an establishment of the



Interrelation of a corner of turn of wheels and corner of turn of links of the unit

Fig. 4. Interrelation of the corner of turn of operated wheeled tractor and the corner of folding of the semitrailer



Fig. 5. Dependence of angular speed of turn of a tractor on the corner of turn of operated wheels



Fig. 6. Dependence $\gamma_1 = f(\theta_o)$, at $\gamma_2 = 0$



Fig. 7. Parameters of circular turn



Fig. 8. Characteristics of turn of the tractor unit



Fig. 9. Influence of the corner of installation of the rotary part of hinge-operated trailer hitch on manoeuvrability indicators



Fig. 10. Characteristics of manoeuvrability MTA at change of parities of lengths of rotary and non-rotary parts of a hitch on hinge -operated drawbar of semitrailer

character of influence of various installations and parameters on hinge-operated trailer hitch of the semitrailer, on shunting properties of a tractor train of the nonconventional construction scheme (Fig. 7, 8, 9, 10).

CONCLUSIONS

Thus, the technique of modeling and definition of indicators of shunting property of an agricultural tractor train with the hinge-operated trailer hitch is developed at various modes of turn.

The developed technique allows for the solution of many practical research problems of shunting property of tractor trains with trailers of the new constructive scheme, for the purpose of the choice at a stage of working-out of the trailer design of the best parameters both of the trailer and of its connection with the tractor. The stated method is applicable also at the decision of the return problem of kinematics of unsteady turn of returning automobile trains of big length, concerning development of operation drawbars for wheeled carts for maintenance of the set dimensional lane. The further development of the stated mathematical model consists in the establishment, at necessity, of corners of withdrawal of wheels and transition from purely kinematic model to dynamic model of the investigated unit.

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MODELOWANIE KINEMATYKI RUCHU CIĄGNIKA KOŁOWEGO Z NACZEPĄ ZE STEROWANIEM ZACZEPEM ZAWIASOWYM PODCZAS SKRĘCANIA

Streszczenie. W artykule przedstawiono matematyczny opis i wyniki modelowania skrętu ciągnikowego agregatu z naczepą ze sterowanym zawiasowym zaczepem. Przedstawiono metodykę badań i określono wskaźniki manewrowości agregatu ciągnikowego. Opracowana metodyka pozwala rozwiązywać zadania badawcze w stadium opracowywania parametrów naczepy i agregatowania z wybranym ciągnikiem. Wykorzystanie wyników analizy pozwoli na przejście od modelu kinematycznego na dynamiczny model badanego agregatu.

Słowa kluczowe: agregat ciągnikowy, naczepa, zawiasowy zaczep, sterowanie zaczepem, matematyczne modelowanie, kinematyka przy skręcaniu agregatu.