INFILTRATION INTO LAYERED SOIL COVERED WITH A DEPOSITIONAL SEAL: A GREEN-AMPT APPROACH

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A b s t r a c t. Our objective was to develop a method of infiltration simulation that could be applied in crop models which use simplified soil water transport models. Infiltration into layered soil profiles topped with a depositional crust is considered in this paper. The proposed method, based on the Green-Ampt equation, uses harmonic mean hydraulic conductivity in the wetting zone, assumes the pressure head in the wetting zone to be close to pressure head under the seal, and treats the wetting front pressure head as a value which provides for conservation of the integral mean hydraulic conductivity of the wetting front. The method was validated against results of numerical solution of Richards' equation for infiltration into a three layered soil profile and two homogeneous soil profiles with constant and transient seal conductance. Differences of 2-5 % were found between cumulative infiltration and infiltration rate values calculated by the proposed method and by the finite difference solution of the Richards' equation. Largest differences arose during the period when the wetting front passed the interface between layers. When the conductivity of the lower layer was greater than the conductivity of the upper layer the differences between the finite difference solution and the proposed method was 10 to 15 % after 360 min.

K e y w o r d s: infiltration, Gren-Ampt aproach, hydraulic conductivity

INTRODUCTION

Solutions of the infiltration problem under a range of conditions are particularly important for crop modeling. These conditions include layered soil profiles and soil sealing. Sealing of the soil surface was gradually recognised as an important phenomena that can influence both infiltration and surface runoff [9,16]. Different management practices may result in soil sealing, such as (a) precipitation and sprinkler irrigation, (b) formation of depositional seals due to surface irrigation, and (c) artificial soil sealing resulting from application of polymeric substances.

A well known equation to predict infiltration into homogeneous soils without a seal is the Green-Ampt equation [12]. The parameters of the Green-Ampt equation were originally claimed to be purely empirical. However Philip [22], Morel-Setoux and Khanji [19], and Neuman [21] have shown that these parameters, in fact, have a clear physical meaning.

Several approximate methods have been developed to extend the Green-Ampt equation to cases of infiltration into a homogeneous soil layer covered by a crust. These methods include a derivation and analytical solution of the modified Green-Ampt equation [11,13] a piece-wise approximate solution of the modified Green-Ampt equation [12,18], and the use of the original Green-Ampt equation with parameter values altered to take into account the effect of the seal [5,23,24]. The use of latter approximations was limited to homogeneous soil topped with a crust. Later Chu *et al.* [8] and Wolfe *et al.* [30] expanded this technique to the three-layer systems 'crust-soil-subsoil'. Their methods also required estimation of 'effective' [8] or 'wetted' [30] hydraulic conductivities of the soil layers under crust directly from infiltration experiments.

When Richards' eqation is used to simulate infiltration, one can use conventional data on hydraulic conductivity and water retention dependencies on soil matric potential. No effective conductivities are required. Numerical solutions of the Richards' equation have been used extensively to study the effects of sealing on infiltration and runoff in layered soils. Finite difference and later finite element approximations of Richards' equation have been successfully applied to analyze infiltration into sealed soil [1,28,29]. The development of this methodology continues today [3,25,27].

Numerically intensive solutions for infiltration, however, are not always needed in crop models. Crop models generally require short execution times to allow multiple runs with varying scenarios. As a result, most crop models contain rather simplistic representations of infiltration. The IBSNAT [14] models, which include CERES-MAIZE [15], for example, use a capacity based infiltration model where each layer of soil can hold a set amount of water (defined by field capacity) and any excess water is instantly moved to the layer below. If crop models are to calculate more realistic nutrient budgets and water availability to root systems, however, better infiltration subroutines need to be implemented. A reasonable compromise between the capacity based models and numerically intensive solutions of the Richards' equation would be an approximate method based on the Green-Ampt equation that can be used in layered soils with and without a crust.

The aim of this work was to propose and test the extention of the Green-Ampt methodology to quantify infiltration into layered soil with a depositional crust using dependencies of water retention and hydraulic conductivity on soil matric potential for each layer. We considered a depositional crust which results from surface irrigation and has both variable and constant hydraulic conductance. We also considered both constant and variable pressure head on the soil surface above crust.

THEOR Y

We consider the physical system shown in Fig. 1 which consists of a layered soil profile with a surface seal. A layer of water present above the seal infiltrates into the soil. The air is assumed to flow freely in the profile, concentrations of solutes in the soil water are assumed to be negligible, and therefore pressure head of the soil water is equal to capillary pressure head. The air entry pressure of the seal is assumed to be a large negative number so that the seal remains saturated.



Fig. 1. The physical system under consideration.

We consider movement of soil water as obeying the Richards' equation:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} \tag{1}$$

where coordinate z increases downwards, q is the downward flux of water expressed as:

$$q = -k \frac{\partial h}{\partial z} + k \tag{2}$$

h is capillary pressure head, *k* is soil hydraulic conductivity, and θ is volumetric soil water content.

Integration of Eq. (1) over z from the soilseal interface z=0 to the depth z_i below the wetting front gives:

$$\frac{dV}{dt} = q_0 - q_i \tag{3}$$

where:

$$V = \int_{0}^{z_{f}} (\theta - \theta_{i}) dz;$$

$$q_{0} = \left[-k \frac{\partial h}{\partial z} + k\right]_{z=0}; \qquad (4)$$

$$q_i = \left[-k \frac{\partial h}{\partial z} + k \right]_{z = z_i}$$

Here V is the total volume of water infiltrated between the interface and the wetting front, q_0 is the flux through the interface between seal and bulk soil, q_i is a flux below the wetting front, and z_f is a vertical coordinate of the wetting front.

We assume that:

a) q depends only on time and is equal to $q_0(t)$ everywhere between the interface 'seal-bulk soil' and the wetting front; below the wetting front, q_i is constant over the each layer since the initial pressure head is constant over layers but q_i may vary for different layers;

b) differences between pressure head values at opposite borders of any layer within the wetted zone are small enough to allow the use of the same pressure head value h_0 in calculations of soil hydraulic conductivity in each layer, where h_0 is a pressure head at the interface 'seal-bulk soil'.

We now derive expressions that define q_0 and q_i . If the wetting front is in the nth layer, then after integrating q expressed by Eq. (2) over each layer above the wetting front, we have:

$$\frac{\Delta z_1}{k_1 (h_0)} q_0 = h_0 - h_1 + \Delta z_1$$
$$\frac{\Delta z_2}{k_2 (h_0)} q_0 = h_1 - h_2 + \Delta z_2$$

$$\frac{\Delta z_n - 1}{k (h_0)} q_0 = h_{n-2} - h_{n-1} + \Delta z_{n-1}$$

$$\frac{\Delta z_f}{k_n(h_0)} q_0 = h_{n-1} - h_f + \Delta z_f$$
(5)

Here the subscripted h's are pressure heads at the interfaces between the layers beginning from beneath the seal (h_0) to the layer just above the wetting front (h_{n-1}) ; h_f is a pressure head at the wetting front; Δz_j is a thickness of the layer j, j=1,2,..., n-1, Δz_f is a distance between wetting front and upper border of the *n*th layer (the layer that contains the wetting front); k_j (h_0) is a soil hydraulic conductivity in the *j*th layer at pressure head h_0 , j=1,2,...,n.

Following Bouwer [4], we define harmonic mean hydraulic conductivity as:

$$k_{H}(h_{0}) = \frac{\sum_{j=1}^{n-1} \Delta z_{j} + \Delta z_{f}}{\sum_{j=1}^{n-1} \frac{\Delta z_{j}}{k_{j}(h_{0})} + \frac{\Delta z_{f}}{k_{n}(h_{0})}}$$
(6)

and make a summation of all equations in Eq. (5). As a result, we obtain the expression for the flux q_0 :

$$q_0 = k_H \; \frac{h_0 - h_f + z_f}{z_f} \tag{7}$$

which is essentially the same as in the Green Ampt equation with wetting front depth z_f equal to $\sum z_j + \Delta z_f$.

Finally, q_i is equal to:

$$q_i = k_n \left(h_{n,i} \right) \tag{8}$$

where $h_{n,i}$ is the initial pressure head value in the *n*th layer.

To define the wetting front pressure head h_f , we use the fact that, in this model, the continuous hydraulic conductivity dependence on pressure head $k_n(h)$ is replaced by piece-wise constant dependence near the wetting front:

$$\hat{k}(h) = \begin{cases} k_n(h_0), \ h > h_f \\ k_n(h_{n,i}), \ h < h_f \end{cases}$$
(9)

We select the value of h_f which allows us to preserve the average capillary conductivity of the soil at the wetting front after replacement of the continuous conductivity by the piece wise one. Such a value of h_f requires:

$$(h_0 - h_f) k_n (h_0) + (h_f - h_{n,i}) k_n (h_{n,i}) = \int_{h_{n,i}}^{h_0} k dh.$$
(10)

In the right hand side of the equality in Eq. (10) we have the integral average conductivity of the wetting front calculated for the continuous hydraulic conductivity function, and in the left hand side of Eq. (10) we have the integral average hydraulic conductivity calculated for the piece-wise hydraulic conductivity function. The explicit expression of h_f follows from Eq. (10):

$$h_{f} = \frac{1}{k_{n}(h_{0}) - k_{n}(h_{i})} \int_{k_{n}(h_{i})}^{k_{n}(h_{0})} hdk \qquad (11)$$

This method of calculating the wetting front capillary pressure head was proposed by Mein and Larson [17] and later used by Morin *et al.* [20].

Earlier, we made the assumption that the

soil hydraulic conductivity above the wetting front is equal to the soil hydraulic conductivity at the pressure head $h=h_0$ in any layer. Now we add the assumption that soil water contents above the wetting front are equal to the soil water content at pressure head h_0 in any layer. In this case, the value of the total mass of the infiltrated water above the wetting front will be equal to:

$$V(z_{f}) = \sum_{j=1}^{n-1} \left[\theta_{j} (h_{0}) - \theta_{j} (h_{j,i}) \right] \Delta z_{j} + \left[\theta_{n} (h_{0}) - \theta_{n} (h_{n,i}) \right] \Delta z_{f}$$
(12)

where Δz_j are the same as in Eq. (5), the wetting front is assumed to be in the *n*th layer, and $\theta_j(h)$ is the dependence of soil water content on pressure head in the *j*th layer, index *i* means initial value.

In addition to the equations of soil water movement, we have the equation of infiltration through the seal:

$$q_0 = \frac{h_s - h_0}{S} \tag{13}$$

where h_s is the pressure head on the surface of the seal, and S is the hydraulic conductance of the seal. We assume that the value of S may explicitly depend on time but not on the pressure head under the seal, h_0 (because the seal is always saturated). The value of h_s also may depend on time. Eliminating q_0 from Eq. (13) and Eq. (7) results in the following relationship between h_0 and z_f :

$$\frac{h_s - h_0}{S} = k_H \frac{h_0 - h_f + z_f}{z_f}$$
(14)

where k_H depends on h_0 according to Eq. (6) and h_f depends on h_0 according to Eq. (11).

Now we have all the equations necessary to calculate infiltration through the seal. The calculations for the case of constant h_s and S are presented here. Modifications can be readily done for the case of h_s decreasing during infiltration.

We consider a set of sequential depths of the wetting front: $z_{f,0}=0$, $z_{f,1}$, $z_{f,2}$,..., $z_{f,L}$. Assume that we know the values of V_{k} , $h_{0,k}$, $q_{0,k}$, $q_{i,k}$, and time t_k for the moment when wetting front has reached the depth $z_{f,k}$. We calculate new values V_{k+1} , $h_{0,k+1}$, $q_{0,k+1}$, $q_{i,k+1}$, and time t_{k+1} for the wetting front arrival to the next depth z_{fk+l} . First, we find the layer n corresponding to depth $z_{f,k+l}$. Second, we compute $h_{0,k+1}$ from Eq. (14) solving this nonlinear equation with known n and $z_f = z_{k+1}$. Third, we find $q_{0,k+1}$ from Eq. (13) with known $h_0 = h_{0,k+1}$. Fourth, we calculate $q_{i,k+1}$ from Eq. (8). Fifth, we obtain V_{k+1} from Eq. (12). Finally, to obtain t_{k+1} , we will use an approximation of Eq. (3):

$$\frac{V_{k+1} - V_k}{t_{k+1} - t_k} = \frac{1}{2} \left(q_{0, k+1} + q_{0, k} - q_{f, k+1} - q_{f, k} \right)$$
(15)

Up to this time, all values in Eq. (15) are known except t_{k+1} . Finding t_{k+1} completes a step of calculations. Now all values with subscript k+1 are known, and we can find the values for the wetting front depth z_{k+2} . To begin calculations, we put $V_0=0$, $t_0=0$, $h_0=h_i$, and determine $q_{0,0}$ from Eq. (13) and $q_{i,0}$ from Eq. (8).

Model testing

We tested the model using the results of numerical simulations of infiltration into sealed soils published by Eisenhauer *et al.* [10]. In their paper, a broad variety of infiltration events was simulated using a finite difference numerical solution of the Richards' equation. We selected data on infiltration into five soil profiles; three had two layers and two were homogeneous. Both time-dependent and constant seal conductance was simulated. Data on soil properties are in the Table 1. This table also contains parameters of the equations used to describe soil hydraulic properties:

$$\theta = \begin{cases} (\theta_w - \theta_r)(\frac{h_w}{h}) \ \lambda, & h < h_w, \\ \theta = \theta_w, & h \ge h_w \end{cases}$$

$$k = \begin{cases} k_w \left(\frac{h_w}{h}\right)^2 + 3\lambda, h, h_w \\ k_w, h \ge h_w \end{cases}$$
(16)

Here θ is a volumetric water content, θ_w is a saturated water content, θ_r is the curve-fitting parameter, *h* is a capillary pressure head, h_w is the water entry capillary pressure head, λ is the Brooks-Corey's pore size distribution index, *k* is a hydraulic conductivity, k_w is the saturated soil hydraulic conductivity.

The soils, which were layered, were sandy loam, loam and silty clay loam textures and had marked differences in clay and silt content (Table 1). The properties in the tillage layer were assumed to represent uniform sandy loam and silty clay loam profiles. Initial pressure head was -4000 mm, -7000 mm, and -10000 mm in sandy loam, loam, and clay loam, respectively, in all layers. The pressure head at the seal surface was 30 mm.

To simulate seal conductance dependence on time, Eisenhauer *et al.* [10] fitted their experimental data on crust resistances with nonlinear equations. In our notations, these equations are:

 $S=3.45[1+(t/82)^{2.33}]$, S=15.6/(1-t/420.), and $S=3.73 \exp(t/103)$ for sandy loam, loam, and silty clay loam, respectively; both S and t are expressed in minutes here.

We performed calculations using the increment $z_{k+1} - z_k = 2$ mm. The nonlinear equation, Eq. (14), was solved by the van Wijngaarden-Dekker-Brent method [6].

RESULTS AND DISCUSSION

In Figures 2 and 3, and in Table 2 we present results of the comparison of two the methods of infiltration simulation: numerical solution of Richards' equation and the approximate Green-Ampt method of this paper. After 360 min simulated time, the differences between the results of the two methods do not exceed 2 %, 3 %, and 5 % for sandy loam, loam, and silty clay loam, respectively.

The difference in performance of the two

Sand	Silt	Clay	Organic matter	Aggregate stability	Bulk density	θ	θ,	h _w ,	λ	k_{w}^{\prime}
		(%)			$(g \text{ cm}^{-3})$		(mm)		-	((())))
	Tri	ipp sandy l	loam							
66	22	12	0.9	14	<u>1.29</u> * 1.54**	$\frac{0.42}{0.38}$	$\frac{0.13}{0.18}$	<u>-429</u> -641	<u>1.16</u> 1.42	$\frac{1.23}{0.21}$
		Ortello loa	m							
13	45	19	3.2	54	$\frac{1.09}{1.29}$	$\frac{0.48}{0.47}$	$\frac{0.00}{0.17}$	<u>-106</u> -383	$\frac{0.155}{0.656}$	$\frac{0.25}{0.09}$
	Sharpst	ourg silty c	lay loam							
16	54	• 30	2.7	29	$\frac{1.09}{1.29}$	$\frac{0.48}{0.45}$	$\frac{0.18}{0.24}$	$\frac{-23}{-525}$	$\frac{0.228}{0.720}$	$\frac{0.82}{0.19}$

Table 1. Properties of soils from Eisenhauer et al. [10]

*tillage layer, **subsoil.



Fig. 2. Calculated dependencies of the cumulative infiltration on time using the modified Green-Ampt method and numerical simulations of the Richards' equation.

methods may be clearly seen in Fig. 4, where the infiltration rate dependence on time is shown. There is little difference while the wetting front is in the upper layer or after the wetting front passes the interface between layers. The largest differences occur during the passage of the interface between layers. The numerical solution of the Richards' equation provides smooth gradual changes of the infiltration rate. The Green-Ampt solution has a sharp increase of the infiltration rate at the moment of passage through this interface. The increase shown in Fig. 4 can be understood from data in Fig. 5 which shows the simulated dynamics of pressure heads under the seal and at the wetting front. The seal conductance (S) is kept constant. Both the pressure heads undergo a sharp decrease immediately after the wetting front passes the interface. This decrease is primarily the consequence of differences in air entry pressure head values. Eq. (16) shows that the soil



Fig. 3. Simulated cumulative infiltration in layered soil with transient seal using the modified Green-Ampt method and numerical simulations of the Richards' equation.

hydraulic conductivity stays the same for any pressure head value exceeding the air entry value. Therefore, according to Eq. (11), the wetting front pressure head will also be the same for any pressure head h_0 greater than h_{w} .



Fig. 4. Simulated infiltration rates for layered soils using the modified Green-Ampt method and numerical simulations of the Richards' equation.

Soil	Seal	Cumulative i	nfiltration after 3	Infiltration rate at 360 min (mm min ⁻¹)		
		N*	А	С	N	А
Sandy loam,	Constant	225	225	220	0.365	0.370
layered	Transient	232	229	236	0.363	0.367
Loam, layered	Constant	128	128	126	0.202	0.203
	Transient	133	129	134	0.199	0.202
Silty clay loam, 1ayered	Constant	166	174	170	0.362	0.360
	Transient	206	198	208	0.327	0.336
Sandy loam, uniform	Constant	614	620	560	1.460	1.416
Silty clay loam, uniform	Constant	176	175	170	0.477	0.479

T a ble 2. Comparison of infiltration simulation results for the numerical solution of Richards' equation and the approximate solution of this paper

*N - numerical finite difference solution of the Richards' equation [10], A - Green-Ampt method of this paper, C - method of this paper with wetting front pressure head calculated as capillary drive.

Because air entry pressure head values of the subsoil are much lower than those of the tillage layer, the wetting front pressure heads of the subsoil are also much lower than those of the tillage layer. That is why the wetting front speeds up after passing the interface. This acceleration requires an increase of the water flux through the seal which can be achieved only by decreasing the potential under seal. The secondary effect, which moderates the decrease of the soil-seal interface potential h_0 , is the release of water from the tillage layer to the subsoil due to decrease of the h_0 value. This water moves to the subsoil and partially provides the necessary amount of water to fill the wetting zone.

The harmonic mean of the conductivities may be inaccurate when the hydraulic conductivity increases with depth as when a fine textured layer overlies a coarse textured layer. In this case the upper part of the profile is controlling the infiltration rate but the harmonic mean gives too much weight to the conductivity of the lower part of the profile. We also simulated this system to see how much error results. We kept the thickness of upper layer equal to 10 cm in these simulations. The conductivities are the same as in Table 1 but the sequence of layers has been inverted.



Fig. 5. Dependencies of the pressure head of the wetting front(h_0) and pressure head under a seal(h_0) on time simulated by the modified Green-Ampt method.

The results of the simulations are in Fig 6. The Green-Ampt method predicted greater cumulative infiltration than the numerical simulations did. The differences are larger for the sandy loam soil than for the loam. This is because the difference in conductivities between the surface and subsurface horizons in the sandy loam is greater than for the case of the loam. The overall differences are 12.5 % for the sandy loam and 11.0 % for the loam at 360 min. The differences increase as time increases. Since, during rainfall, most infiltration periods generally do not last longer than one to two hours, the error in these cases will not be large. For longer infiltration periods, as during irrigation, the error will be greater.



Fig. 6. Simulation of a two layer profile where the hydraulic conductivity is increasing with depth.

It has been generally accepted that a capillary drive value h_c is a good approximation of the wetting front pressure head to calculate infiltration using Green and Ampt approach [19,21] for soils without a seal. This value was calculated as:

$$h_{c} = \frac{1}{K_{s}} \int_{hi}^{0} k(h) dh$$
 (17)

where K_s is the saturated hydraulic conductivity. Ahuja [2] generalised the idea of the capillary drive for the case of an unsaturated soil surface. He defined the corresponding value as: h_{0}

$$h_c = \frac{1}{k(h_0)} \int_{h_i}^{h_0} k(h) dh$$
 (18)

When we used the capillary drive value h_c instead of h_f from Eq. (11) we found the correspondence with numerical simulation results to be generally worse (Table 2). For instance, after 360 min of simulated time, we encountered values of 560 mm and 170 mm of the cumulative infiltration into uniform layer of sandy loam and of silty clay loam, respectively. These values are 5-10 % less than the corresponding values resulting from numerical simulation. However, for layered silty clay loam, introduction of capillary drive (h_c) produced better results than application of the Eq. (11). While we feel that Eq. (11) is a better alternative for capillary drive, the results of calculations using h_c from Eq. (17) will be acceptable for most practical cases.

SUMMARY AND CONCLUSIONS

We developed an application of the Green-Ampt infiltration equation for layered soils topped with a crust. For the case where the hydraulic conductivity is decreasing with depth the results compare favorably with numerical simulations using the Richards' equation. When hydraulic conductivity is increasing with depth, the method of this paper overpredicts cumulative infiltration by 10 to 15 % after 360 minutes in a sandy loam soil. The method is an acceptable substitute for infiltration equations in crop models where speed of calculations is important.

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