Lunar nodal tide in the Baltic Sea

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Abstract

The nodal tide in the Baltic Sea was studied on the basis of the Stockholm tide-gauge readings for 1825–1984; data from the tide gauge at Świnoujście for the same period provided comparative material. The Stockholm readings are highly accurate and are considered representative of sea levels in the whole Baltic; hence, the final computations were performed for the readings from this particular tide gauge for the period 1888–1980. The tidal amplitude obtained from measurements uncorrected for atmospheric pressure or wind field was compared with that forced only by atmospheric effects. The amplitude of the recorded nodal tide was the same as the equilibrium tide amplitude calculated for Stockholm. Calculations for equilibrium tide amplitudes were also performed for the extreme latitudes of the Baltic basin.

1. Introduction

The nodal tide is caused by variations in the Moon's declination, which, over a mean period of 18.6 years, range from $28°35'$ to $18°19'$. The tide-producing force resulting from such variations brings about sea level changes detectable in the atmosphere and in the temperature and circulation of oceanic waters. The tidal maximum is detectable at the poles; the nodal line occurs at around latitudes 35°N and 35°S.

The tide-producing forces have been presented in general by Pugh (1987). Proudman (1960) demonstrated analytically that the nodal tide in the open seas should correspond to the equilibrium tide. However, actually obtaining empirical evidence of this regularity has often proved difficult, because the low amplitude of this tide is concealed among random oscillations of diverse origin, which include sea-level changes caused by atmospheric forcing. Apart from this, one has to take account of the non-linear generation of low-frequency tides when diurnal and semi-diurnal tides are modulated by the nodal tide. A semi-diurnal tide reaches its maximum amplitude when that of the nodal tide falls to a minimum.

The difficulties involved in isolating the nodal tide from low-amplitude and low-frequency changes in sea level lie in the fact that not all analyses of long-term tide-gauge readings provide evidence of the relevant oscillations. The literature covering this area of research is very much slimmer than that covering investigations of the pole tide. One of the reasons for this is that long series of readings are essential for a reliable data analysis. Research into the nodal tide has usually been reported in publications whose titles give no clue to this fact, e.g. Rossiter (1967), Maximov (1970), Currie (1981).

Substantial low-frequency oscillations, probably forced by the atmosphere, are typical of long series of tide-gauge readings. Thus, in order to define the nodal tide, several tide-gauge series have recently been linked by the least squares method to the spatial distribution of the equilibrium tide. This has made for amplification of the spatial measurement signal and attenuation of the correlation with wind variability (Trupin & Wahr 1990). In the present paper, the nodal tide has been analysed on the basis of the Stockholm tide gauge readings, a meticulous compilation regarded as representative of the long-term oscillations of the Baltic Sea level. The effect of atmospheric pressure and wind on the sea level is statistically determined and the sea-level forced by the atmosphere is subsequently calculated. The sea level series thus obtained are then subjected to Fourier analysis. The resulting amplitudes differ only slightly from the equilibrium tide or are the same. The range of variability of the equilibrium tide amplitude in the Baltic basin has also been determined.

2. Measurement data

The sea levels at Stockholm in the period 1825–1984 were analysed on the basis of the mean annual measurement data published by Ekman (1988). The corresponding data for the same period were collected for Świnoujście (Dziadziuszko 1995). Comparison of the two sets of readings showed, however, that their correlation coefficient was 0.77, a low value in the context of this study of the nodal tide in the Baltic. In view of the considerable gap in readings caused by the Second World War, and the hydrodynamic conditions of the Świnoujście tide gauge, it was finally decided to concentrate on the Stockholm data for this analysis. Situated close to the nodal line of seiches in the Gulf of Bothnia – Danish Straits and Gulf of Finland – Danish Straits systems, the oscillations at the Stockholm tide gauge are relatively small and, with respect to long-term oscillations, have been proved representative for the Baltic (Wróblewski 1998a). The nodal tide was calculated for various measurement periods, the results showing that a meaningful analysis was possible for the final decades of the 19th century and the years thereafter: the period 1888–1980 was thus selected for the computations. The upper limit of this time interval was set by the range of meteorological data available, while the quantity of data in the Fourier analysis allowed for determination of the frequency corresponding to the nodal tide oscillations.

The mean annual atmospheric pressures at the grid nodes according to the data published by Vose et al. (1992) were used to determine the influence of the atmosphere on sea levels at Stockholm. The grid was restricted to the area directly affecting Baltic Sea levels. The atmospheric pressure

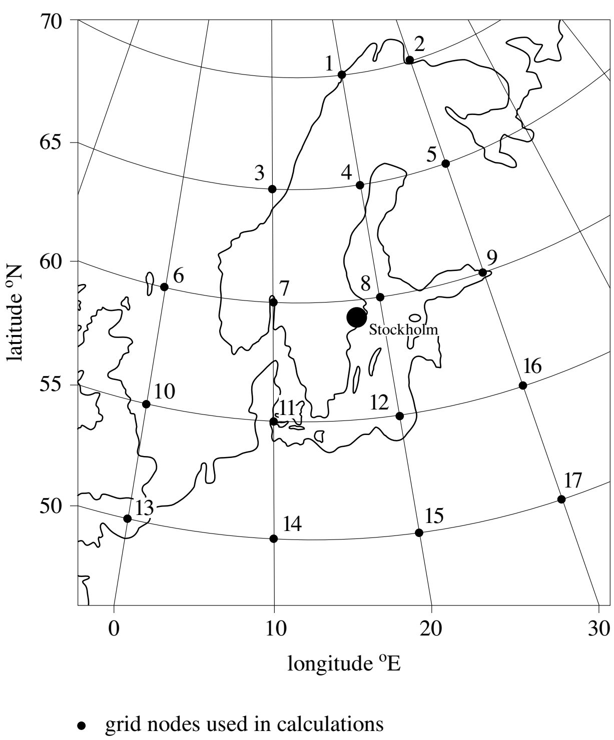

Fig. 1. The geographical location of the atmospheric pressure grid

field between the coordinates 0°–30°E and 50°N–70°N covered central and northern Europe, including the North Sea and Scandinavia. The grid nodes were spaced every 5° of latitude and every 10° of longitude. The geographical location of the atmospheric data is given in Fig. 1.

3. The effect of the atmosphere on sea levels

In order to calculate the nodal tide from tide-gauge readings, all the factors that could alter the tidal amplitude have to be analysed. Conventionally, the computations are based on the mean annual readings, so $\mathrm{S_a}$ and $\mathrm{S_{sa}}$ oscillations together with the pole tide are practically eliminated. Subtraction of the linear trend determined by the least squares method eliminates the long-term sea-level rise. During the computation period, the trend line was lowered by –1.2 cm. After subtraction of the trend, the standard deviation of the series $\xi_0'(\mathrm{t})$ was $\sigma = 5.4$ cm. There remains, however, the fundamental problem of analysing the atmospheric effect. As regards the static effect of the atmospheric pressure, global research depends primarily on the verification of the reverse barometer law. But in the case of the Baltic, where the water level is governed mainly by the barotropic exchange of water with the North Sea via the Danish Straits, the effect of zonal circulation winds is the crucial factor. Long-term changes in sea level at Stockholm can be given by the general equations (1) and (1a)

$$\begin{aligned} \xi_0(t) \;&=\; \overline{\xi} + \mathrm{at} + \xi_N(t) + P(t) + \mathrm{S_a}(t) + \mathrm{S_{sa}}(t) + \int_{-L}^{t} \times \\ &\times \left[Q_1(\tau) P_A(t-\tau) + Q_2(\tau) \frac{\partial P_A}{\partial \overline{\omega}}(t-\tau) \right] d\tau + \varepsilon(t), \end{aligned} \tag{1}$$

$$\begin{aligned} \xi_0'(t) \;&=\; \overline{\xi} + \xi_N(t) + \int_{-L}^{t} \times \\ &\times \left[Q_1(\tau) P_A(t-\tau) + Q_2(\tau) \frac{\partial P_A}{\partial \overline{\omega}}(t-\tau) \right] d\tau + \varepsilon(t), \end{aligned} \tag{1a}$$

where

$\xi_0(t)$ – changes in the Stockholm sea levels (long-term oscillations),
$\xi_0'(t)$ – $\xi_0(t)$ mean annuals after elimination of the trend, pole tide and seasonal changes,
$\overline{\xi}$ – mean level,
at – linear trend,
$\xi_N(t)$ – nodal tide,
$P(t)$ – pole tide,
$\mathrm{S_a}(t)$ – annual tide,
$\mathrm{S_{sa}}(t)$ – semi-annual tide,

$P_A(t)$ – mean annual atmospheric pressure field over the Baltic region,

$\frac{\partial P_A}{\partial \overline{\omega}}(t)$ – mean annual gradient of the atmospheric pressure field along the zonal circulation horizontal axis $\overline{\omega}$,

$Q_1(\tau)$ – Green's function defining sea level changes due to atmospheric pressure,

$Q_2(\tau)$ – Green's function defining sea level changes due to the tangential friction of the wind,

$\varepsilon(t)$ – oscillations not caused by the previous components, including influence of the atmosphere at other locations (e.g. oscillations of the North Sea and Atlantic).

The influence of the atmosphere presented generally in eqs. (1) and (1a) can be determined statistically by the stochastic two-input/one-output dynamic system (Bendat & Piersol 1986) and has already been applied in the single-input/one-output range for analysing the effect of atmospheric pressure on long-term sea level changes (Trupin & Wahr 1990). It is obvious that long-term atmospheric pressure oscillations are correlated with the atmospheric circulation. However, since this impairs the statistical computations, the stochastic system has to be applied to correlated inputs.

It is assumed in eq. (1) that the sea level is forced by atmospheric pressure oscillations and the wind. The latter is taken to be the geostrophic wind. In principle, this representation of forcing reduces the influence of sea level changes elsewhere in the basin, provided that this influence is correlated with the introduced atmospheric data. In this equation, one makes the assumption that the cause-effect relationships restrict the integration to the interval from values of τ greater than the past time $-L$ and at most equal to zero for the present time. This is due to the lack of the basin memory for the excitation $< -L$. The weighting functions $Q_i(\tau)$ can be interpreted physically as Green's functions representing dynamic sea level forcing by atmospheric factors.

The analysis of atmospheric effects begins with the calculation of the correlation coefficients between the atmospheric pressure at the grid nodes and the sea level at Stockholm; the isolines of these coefficients are shown in Fig. 2. The high correlation had been expected to include a larger number of nodes, but the –0.8 isoline includes the northern and central areas of the Gulf of Bothnia to the north of the tide gauge. This is very likely due not so much to the dominant effect of the Gulf of Bothnia sea level, forced by the atmospheric pressure, on the sea level at Stockholm or to the static effect of this pressure, as to the correlation with the horizontal components of the atmospheric pressure gradients, to be discussed shortly. Computations of the amplitude functions of the empirical orthogonal functions (EOF) of the atmospheric pressure field were unsuccessful in correlating their links with the sea level. In the further analyses, the atmospheric pressure was accounted for by the introduction of the measurement series $P_A^\circ(t)$ for the grid node at 65°N, 20°E. This node has the highest absolute value of the correlation coefficient with sea level: r = –0.80.

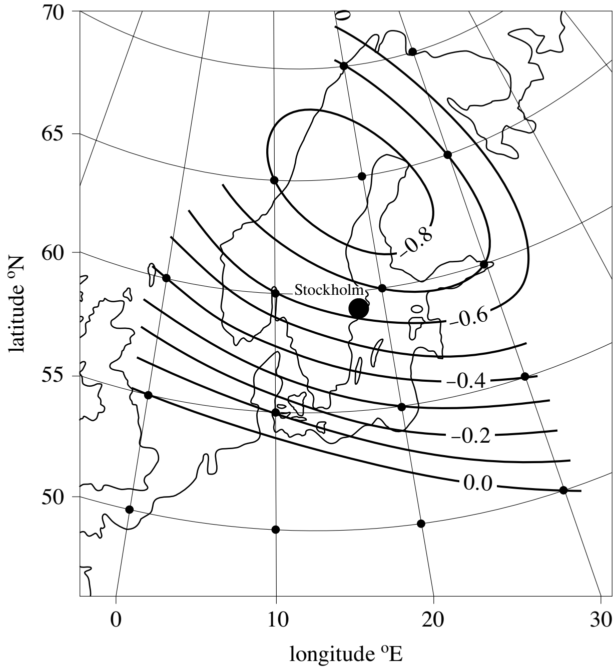

Fig. 2. Isolines of correlation coefficients of the mean annual sea levels at Stockholm and the mean annual atmospheric pressures at the grid nodes. Measurements 1888–1980

It was assumed that the winds forcing sea level changes were represented by the geostrophic winds introduced into the analysis as the horizontal components of the atmospheric pressure gradients. Many a time, this approximation was introduced into the computations for lack of a suitably dense grid for acquiring wind data and because of the very approximate nature of the wind characteristics calculated from the gradients. In order to determine the gradients with the greatest predictive power, all the gradients between the 17 nodes of the grid were computed, after which their correlation coefficients with sea levels were calculated. The maximum correlation coefficients in the 0.70–0.85 interval are thus associated with the gradient vectors generally directed from south to north. Since the Coriolis force affects the direction of these vectors, real winds take up zonal circulation directions forcing water exchange through the Danish Straits, that is, they bring about water level changes throughout the

Table 1. The geographical coordinates of the vectors of atmospheric pressure gradients with a correlation coefficient with the Stockholm sea level of > 0.80

Correlation coefficient	r = 0.85	r = 0.81	r = 0.85	r = 0.85	r = 0.85
beginning of vector	50°N, 0°E	50°N, 0°E	50°N, 10°E	50°N, 20°E	50°N, 20°E
end of vector	60°N, 10°E	60°N, 20°E	60°N, 10°E	60°N, 20°E	60°N, 20°E

Table 2. Expansion according to the EOF scheme of the vectors given in Table 1

Amplitude functions	$\beta_1(t)$	$\beta_2(t)$	$\beta_3(t)$	$\beta_4(t)$	$\beta_5(t)$
percentage of the total variance	0.935	0.039	0.021	0.005	0.0
total percentage	0.935	0.974	0.995	1.000	0.0

Baltic Sea basin. The vectors with the highest coefficients of correlation were calculated for the grid nodes lying between 55°N–60°N, 50°N–60°N and 50°N–65°N respectively. The five vectors coplanar to sea level with the highest correlation coefficients are given in Table 1. These lay between 50°N–60°N (see Fig. 3) and were used in the subsequent calculations. The expansion of these vectors according to the EOF scheme is given in Table 2. The direction of the first eigenvector of the expansion, together with the influence of the Coriolis force, corresponds to the axis $\overline{\omega}$ in eq. (1), along which the vectors jointly exhibit the greatest variance of the geostrophic wind field components correlated with the sea level analysed. The correlation coefficient of $\beta_1(t)$ with sea level is 0.80 and this function is used in further calculations as the component $\frac{\partial P_A}{\partial \overline{\omega}}(t)$ in eq. (1). Calculation of the weighting function $Q_1(\tau)$ for the previously assumed atmospheric pressure $P_A^\circ(t)$ at the grid node and sea level yielded a maximum absolute value for $\tau = 0$; the function's other values for $\tau \neq 0$ do not improve the dependence of the predictand on the pressure. Similar results were obtained for the weighting function $Q_2(\tau)$ calculated for $\beta_1(\tau)$.

Following analysis of the weighting functions, the influence of the atmosphere in equation (1) is simplified to the single equation (2), where the coefficients replacing the weighting functions $Q_1(\tau)$ and $Q_2(\tau)$ for $\tau = 0$ can be determined for correlated predictors by the most efficient method, i.e. by the least squares method.

$$\xi_A(t) = 1.07\beta_1(t) - 1.78P_A^\circ(t), \tag{2}$$

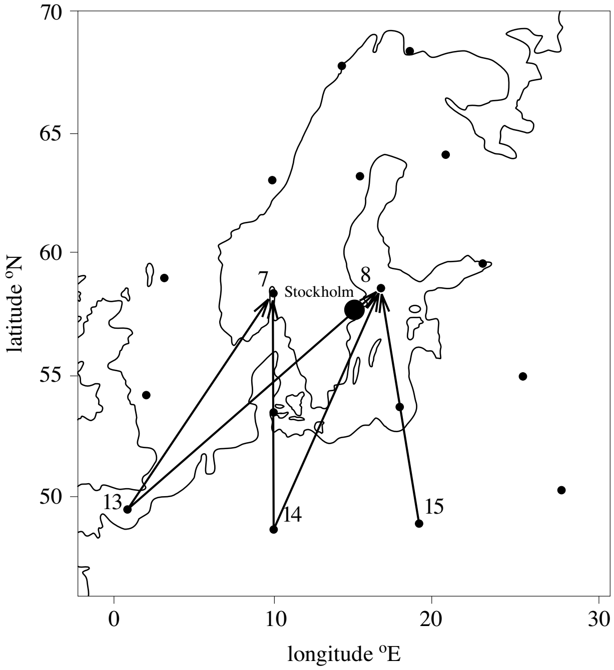

Fig. 3. Vectors of the mean annual horizontal components of atmospheric pressure gradients maximally correlated with the mean annual sea level at Stockholm. Measurements 1888–1980

where

$\xi_A(t)$ – the long-term sea level changes at Stockholm forced by atmospheric pressure and zonal circulation winds.

The calculations of the oscillations presented in Fig. 4 show the spectrum of amplitudes of measured sea levels $\xi_0'(t)$ determined by Fourier transforms, while Fig. 5 illustrates the same characteristic of the series $\xi_A(t)$. Analysis of the measurement series $\xi_0'(t)$ yielded a nodal tide amplitude of 7.2 mm, a period 18.6 years and a phase of 144°. This tide determines 1% of the variance of periods $\geq$ 2 months. On the plot this tide is not visible, concealed as it is behind a neighbouring oscillation with a period of 23.3 years and amplitude 14 mm. After calculation of the component $\xi_A(t)$ the tidal amplitude in series $\xi_A(t)$ rose to 7.8 mm, which made up 4% of the variance; the phase was –26° (–16 months). This difference is, however, extremely small – only 8% of the amplitude of the fundamental series $\xi_0'(t)$ – and does not allow unequivocal determination of the real effect of the atmosphere in the presence of the computation noise. With the aid of the non-parametric bootstrap method the significance of the tidal amplitude was assessed nearly at 10% (Efron 1981, Efron & Gong 1983). In the

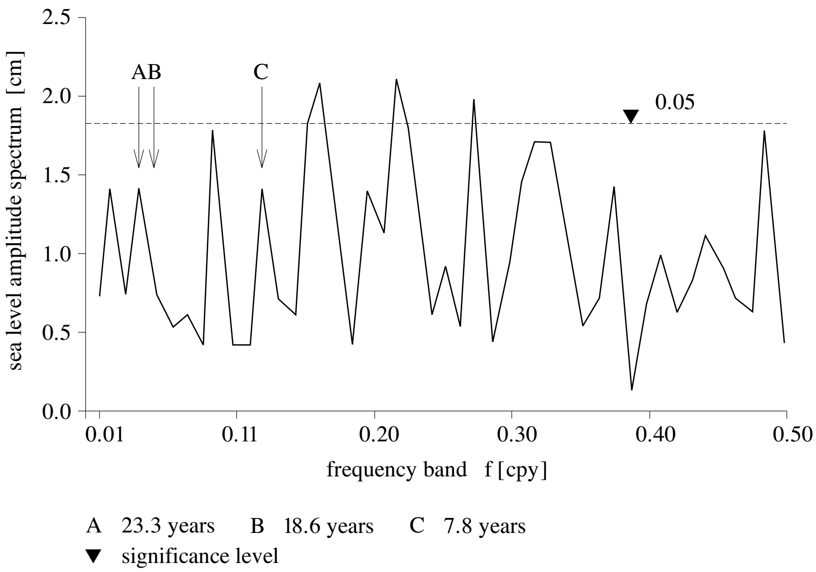

Fig. 4. The amplitude spectrum of mean annual sea levels at Stockholm 1888–1980; the linear trend has been eliminated

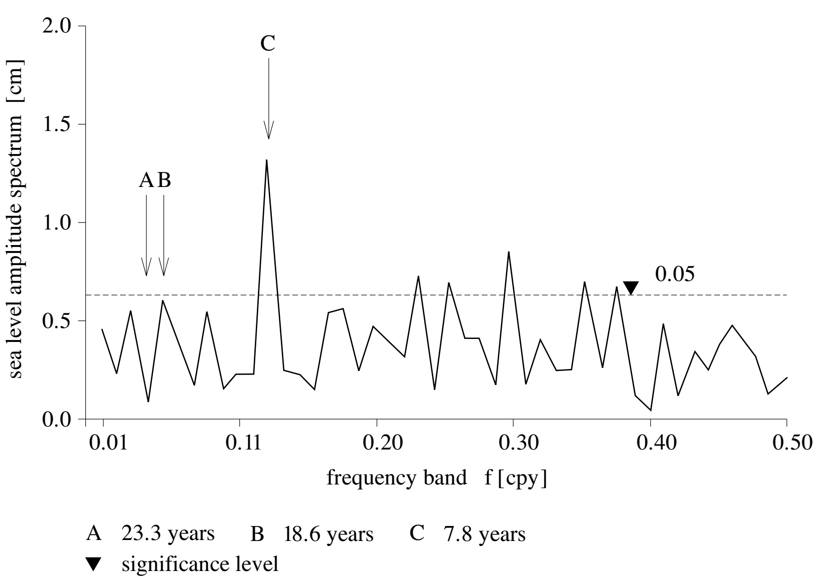

Fig. 5. The amplitude spectrum of mean annual sea levels at Stockholm 1888–1980 forced by the local atmosphere of the Baltic basin

calculations performed for the recorded series, this significance was not at a level regarded as critical in geophysical analyses. The bootstrap method is based on calculation of artificial data sets obtained by resampling the measurement data, after which the significance level for a given parameter can be defined. The method seems more reliable than estimating the noise level on the basis of single recorded series. The value of the correlation coefficient calculated for both series of amplitudes illustrated in Figs. 4 and 5 is very low i.e. –0.15. The atmospherically forced sea level variance was lowered from 28.9 cm^2 to 7.7 cm^2, i.e. to 27% of the variance of the recorded series. These results can demonstrate the strength of the component $\xi_A(t)$ in the mean annual variability of the Baltic Sea level as well as the efficacy of the simplified computation methods used here. According to the calculations, the fundamental long-term oscillation of series $\xi_A(t)$ has a period of 7.8 years, an amplitude of 17 mm and is significant at the 1% level. The physical explanation for this oscillation is that the annual and pole tides overlap (Wiśniewski 1978). Of fundamental importance in the isolation of the nodal tide in the series $\xi_A(t)$ is the elimination of the 23.3-year oscillations. Not generated by the local atmosphere, as calculations have shown, those oscillations are regarded as long-term noise that have masked the nodal tide in the computations performed so far.

The Baltic is a region where the noisy stack data characteristics of the nodal tide deviate considerably from the calculations done on global data; as a result, the Baltic was removed from them (Trupin & Wahr 1990). It must be stressed that the calculated influence of the atmospheric fields defines local atmospheric forcing in the region shown in Figs. 2 and 3. The differences between the two calculated series are meaningful, and the series do not allow unequivocal determination of the real effect of the atmosphere on the weak nodal tide in the presence of the computation noise. Nevertheless, this particular case analysis has shown that, once the effect of the atmosphere on sea levels has been calculated, it is possible to determine the tidal amplitude from readings at the 10% level of significance. Moreover, while calculating the effect of the atmosphere on the sea level does make it easier to define the significance of the tidal amplitude in the amplitude spectrum, it is of no great importance for determining the height of the amplitude, which differs only slightly in the two calculations.

The effect of atmospheric pressure fields above the North Atlantic on the levels of this ocean was not included directly, and the interaction of the North Sea levels, correlated with Atlantic levels, was brought about by the influence of the wind on water exchange through the Danish Straits. The previous publication had shown that the exchange of water through the Straits also depends on the mean North Sea level. The flow of water through the Danish Straits corresponds to low-pass filtration, and oscillations with periods longer than 2.8 months in both basins are coherent with a significance of 1%; for periods longer than 9 months the phase differences are practically non-existent (Wróblewski 1998b). It can thus be concluded that the nodal tide wave travels unhindered through the Danish Straits and the basin reacts to the tide-producing force as part of the Atlantic Ocean with a local atmospheric effect.

4. The nodal tide under equilibrium conditions

Proudman (1960) demonstrated theoretically that the nodal tide in seas should correspond to a tide under equilibrium conditions. Such an equilibrium tide was calculated using the method of Doodson (1921) presented by Rossiter (1967). Eqs. (3) and (4) represent the nodal tide oscillation as a function of time in accordance with the classical tide theory:

$$\xi_N(t) = \frac{9}{8}\,\frac{M}{E}\,e\left(\frac{e}{\rho}\right)^3\left(\sin^2\lambda - \frac{1}{3}\right)\cos N'(t) \times 0.06552, \tag{3}$$

where

$\xi_N(t)$ – oscillations of the nodal tide under equilibrium conditions (metres),
M, E – masses of the Earth and the moon, $M/E \approx 1/81.5$,
e/ρ $\approx 1/60.26$ lunar parallax (e is the mean radius of the Earth; ρ is the mean distance between the centre of the moon and the Earth),
λ – latitude $59°19'$,
$N'(t)$ – longitude of the Moon's ascending node.

$$\xi_N(t) = 2.63\gamma(\sin^2\lambda - 0.33)\cos N'(t) \quad [\mathrm{cm}], \tag{4}$$

where

$\gamma = 0.7$ the effect of a yielding Earth.

After the relevant values were substituted in eq. (4) and the elasticity of the Earth's crust taken into account, calculations with these equations yielded a tidal amplitude of 7.2 mm, a value corresponding to those obtained from calculations involving the tide-gauge readings. The reader is reminded that these calculations were based on a meticulously compiled series of sea level readings (Ekman 1988). Fig. 6 illustrates the comparison of the equilibrium tide and sea levels forced by the atmospheric effects. This comparison was presented for the series where the nodal tide has a clear significance.

The global results of a comparison of PSMSL data (Spencer & Woodworth 1993) with theoretical calculations indicated that the nodal tide amplitude of the measurement data corresponds to the equilibrium amplitude multiplied by (1.13 ± 0.22), and that the phase was equal to the equilibrium phase ±30° (Trupin & Wahr 1990). The global data quoted

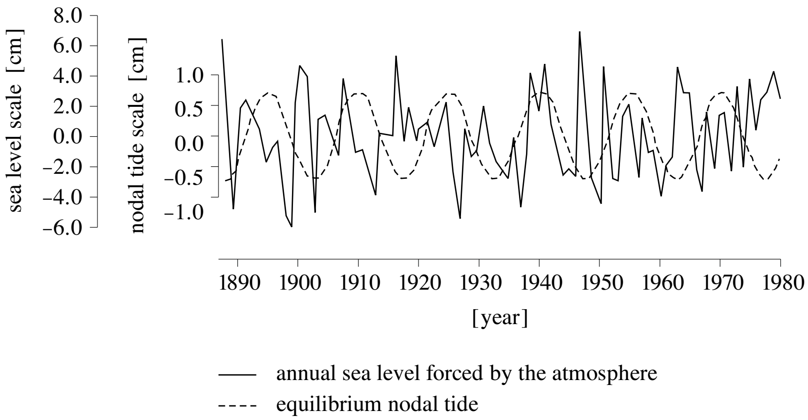

Fig. 6. Time series of mean annual sea levels in 1888–1980 (forced by the local atmosphere of the Baltic basin) and the nodal tide under equilibrium conditions

here were calculated without the Baltic. The range of occurrence of the equilibrium tide in the Baltic can be determined with sufficient accuracy if the latitudes of Świnoujście and Kemi, i.e. 53°55′N and 65°46′N, are used in the calculations. The theoretical interval of occurrence of the tide is 5.7–8.9 mm. Taking into consideration the example of the calculation for Stockholm, one may assume that the real nodal tide in the Baltic Sea lies within a similar interval.

5. Conclusions

The longest series of sea level readings available for the Baltic (1825–1990), measured at the Stockholm and Świnoujście tide gauges, were analysed. The Stockholm series for period 1888–1980 was established as representative of the Baltic.

The geographical location of a time series of atmospheric pressures maximally correlated with sea level data was chosen at the grid node 65°N, 20°E. Zonal circulation winds were assumed to be geostrophic winds represented by the horizontal components of the atmospheric pressure gradients coplanar with sea level. The wind field was represented by the first amplitude function of the empirical orthogonal functions calculated from the set of gradients maximally correlated with the sea level.

The sea level forced by the atmosphere was assumed to be an output of the stochastic dynamic system of constant parameters with two correlated inputs i.e. atmospheric pressure and zonal circulation winds. After the weighting functions had been analysed, this system was simplified for the function argument $\tau = 0$.

The Fourier transform of the recorded series showed a tidal amplitude of 7.2 mm and a phase of 144°. The atmospherically forced series had a tidal amplitude of 7.8 mm and a phase of –26°. The significance of this tidal amplitude was calculated at the 10% level with the bootstrap method. The amplitude of the calculated equilibrium tide was 7.2 mm.

While the tidal force of the nodal tide affects the Baltic and North Seas as part of the Atlantic Ocean, the local influence of atmospheric fields is also discernible. This latter effect of the atmosphere on the generation of the nodal tide in these seas requires further study. The theoretical interval of occurrence of the equilibrium tide amplitude in the Baltic Sea was 5.7–8.9 mm; the actual phenomenon most probably lies within a similar interval.

References

Bendat J. S., Piersol A. G., 1986, *Random Data: analysis and measurement procedures*, Wiley, New York, 566 pp.

Currie R. G., 1981, *Amplitude and phase of the 11-year term in sea level: Europe*, Geophys. J. R. Astr. Soc., 67, 547–556.

Doodson A., 1921, *The harmonic development of the tide generating potential*, Proc. R. Soc., 100, 305–329.

Dziadziuszko Z., 1995, *Annual mean sea level at Świnoujście*, Mater. Oddz. Mor. Inst. Meteor. i Gosp. Wod., Gdynia, 10 pp., (in Polish).

Efron B., 1981, *Nonparametric standard errors and confidence intervals*, Can. J. Statist., 9, 139–172.

Efron B., Gong G., 1983, *A leisurely look at the bootstrap, the jack-knife and cross-validation*, Am. Statist., 37, 36–48.

Ekman M., 1988, *The world's longest continued series of sea level observations*, Pageoph, 127, 1, 73–77.

Maksimov I. V., Sarukhanyan E. I., Smirnov N. P., 1970, *Ocean and Cosmos*, Gidrometeoizdat, Leningrad, 216 pp., (in Russian).

Proudman J., 1960, *The condition that a long period tide shall follow the equilibrium law*, Geophys. J. R. Astr. Soc., 3, 244–249.

Pugh D. T., 1987, *Tides, surges and mean sea level*, Wiley, 472 pp.

Rossiter J. R., 1967, *An analysis of annual sea level variations in European waters*, Geophys. J., 12, 259–299.

Spencer N. E., Woodworth P. L., 1993, *Data holdings of the permanent service for mean sea level*, Perm. Serv. Mean Sea Level, Birkenhead, 81 pp.

Trupin A., Wahr J., 1990, *Spectroscopic analysis of global tide gauge sea level data*, Geophys. J., 100, 441–453.

Vose R. S., Schmoyer R. L., Steurer P. M., Peterson T. C., Heim R., Karl T. R., Eischeid J. K., 1992, *The global historical climatology network: long-term monthly temperature, precipitation, sea level pressure and station pressure data*, ORNL/CDIAC–53 Rep., Oak Ridge, 99 + 189 pp.

Wiśniewski B., 1978, *Seasonal and inter-annual oscillations of the Baltic sea level*, Wyż. Szk. Pedag., Szczecin, 91 pp., (in Polish).

Wróblewski A., 1998a, *Inter-annual oscillations of Baltic water volumes and sea level*, Oceanologia, 40 (3), 183–203.

Wróblewski A., 1998b, *The effect of the North Sea on oscillations of the mean monthly sea levels in the Baltic Sea*, Cont. Shelf Res., 18, 501–514.