

## A METHOD FOR THE ESTIMATION OF THERMAL PROPERTIES OF SOIL

*B. Usowicz*

Institute of Agrophysics, Polish Academy of Sciences, Doświadczalna 4, 20-236 Lublin, Poland

**A b s t r a c t.** The study of the thermal properties of soils, and their determination, is discussed on the basis of the author's statistical-physical model of the thermal conductivity of soils and of de Vries formula concerning the heat capacity of soil.

The theory presented in the paper allows for the determination of the basic thermal characteristics of the soil with relation to the moisture, density, temperature and granulometric composition of the soil and to the water potential in the soil. The error of the estimation does not exceed  $\pm 10\%$  of the measured values.

**Key words:** thermal properties, soils, de Vries formula.

### INTRODUCTION

The principal thermal properties of the soil, characterizing it from the viewpoint of its ability to conduct and accumulate heat, are the thermal conductivity and the heat capacity of the soil. A secondary parameter is the thermal diffusivity of the soil.

Heat capacity per unit of soil volume depends on the heat capacity of the individual components of the soil and is equal to the sum of such component capacities [1,2], while thermal conductivity of the soil is not directly related to the values of thermal conductivity of the individual soil components, but depends also on the distribution of soil particles in space [2,6,7]. The heat capacity per unit of soil volume and the thermal conductivity of the soil increase with increasing water content in the soil [1,2,6,8].

The heat capacity per unit of soil volume is virtually in linear relation to the volumetric moisture content of the soil. Only at very low soil moisture content levels the relation is non-linear, which is related to the lower specific heat of water adsorbed on the surface of solid particles [10].

The thermal conductivity of the soil depends primarily on the soil moisture [1,2,6,8]. In the initial range of moisture content it increases only slightly, since water put in dry soil is first absorbed on the surface of mineral particles. An increase in the soil moisture results in the appearance of water pockets between the particles, which causes the formation of so-called thermal bridges and, in consequence, a strong increase in the thermal conductivity. This increase is observed up to the full capillary water capacity. Further increase in soil moisture causes smaller and smaller increases in thermal conductivity. The lowest values of the thermal conductivity coefficient are those for dry soil, and the highest are reached by a soil at maximum saturation with water. As can be seen, the value of that coefficient depends on the number and surface area of contacts, and therefore it physically depends on the size and shape of soil particles, and on their compaction, i.e., on the structure of

the soil meant as the configuration of the solid phase of the soil. The participation of water vapour in the energy flux and the differentiation of temperature in the soil further complicate the issue. Since there is no possibility of predicting the quantitative relations between the particular components, it is very difficult to assess accurately the relationships between the thermal conductivity coefficient of the soil and the soil structure. This problem is studied indirectly, through parameters related to the soil structure, such as the shape, size and arrangement of soil particles with relation to one another, or soil density [2,6].

The thermal diffusivity of the soil is the quotient of the thermal conductivity and the heat capacity per unit volume of the soil. It determines the capability of the soil to equalize the temperature at all the points of the object under study, and in terms of numbers equals the rate of temperature change at a given point in the soil, caused by a unit change in the temperature gradient. The thermal diffusivity is related primarily to the soil moisture. The relationship is a complex one, as when wetting dry soil the increase in the thermal conductivity is faster than the increase in the heat capacity. Further wetting of the soil results in smaller and smaller increases in the thermal conductivity, while the heat capacity increases steadily at an unchanged rate. When the rate of increase in the heat capacity with increasing soil moisture is greater than the rate of increase in the thermal conductivity, a decrease in the thermal diffusivity is observed. Therefore, the thermal diffusivity reaches its maximum at the soil moisture and density levels characteristic for a given soil. At such soil moisture and density values the rate of 'temperature wave' propagation in the soil is the highest.

The paper presents a method for the estimation of the thermal conductivity, heat capacity, and thermal diffusivity of the soil versus the moisture, temperature, granulometric composition and the water potential of the soil.

## METHODS

### Heat capacity of the soil

The heat capacity of the soil,  $C_v(\text{J}/\text{m}^3\text{K})$ , was calculated according to de Vries formula [2,3]:

$$C_v = (2.0 x_m + 2.51 x_o + 4.19 x_w) \cdot 10^6 (\text{J}/\text{m}^3\text{K}) \quad (1)$$

where  $x_m$  - mineral part content in a unit of soil volume,  $x_o$  - organic part content in a unit of soil volume,  $x_w$  - water content in a unit of soil volume (volumetric moisture -  $\theta_v = x_w$ ).

### Statistical-physical model of the thermal conductivity of the soil

Thermal conductivity is a phenomenon consisting in spontaneous equalization of temperature in the soil medium under study, not accompanied by any macroscopic movement. From the viewpoint of microscopy, thermal conductivity consists in the equalization of the mean energy of thermal motion through impact between soil particles. The measure of the rate of thermal conductivity is the coefficient of thermal conductivity  $\lambda(\text{W m}^{-1}\text{K}^{-1})$ . In terms of numbers it is equal to the amount of energy  $Q(\text{J})$  flowing in a unit of time  $t(\text{s})$  through a unit of surface area  $S(\text{m}^2)$  at a temperature gradient  $\text{grad } T (\text{K m}^{-1})$  of one:

$$\lambda = \frac{Q}{t S \text{ grad } T} \quad (2)$$

The soil is a multi-component system, and therefore its thermal conductivity depends on a number of factors such as the mineralogical composition and content of organic matter, granulometric and aggregate composition, and moisture. The greatest role in the flow of heat in the soil is that of conductivity through mineral components, organic components, and water. The other components, such as evaporation, condensation, freezing and thawing, water and air convection, water and air movement, and radiation between particles, are of lesser importance. In some cases those secondary

components should be taken into consideration in calculations, primarily evaporation and condensation [2,8].

The statistical-physical model of thermal conductivity (SPMTC) has been designed on the basis of one of the fundamental physical properties characterizing the ability of a given body to conduct heat - the thermal resistance, and a statistical relationship - the polynomial distribution, allowing for the calculation of the probability of the occurrence of all possible configurations of particles taking part in conducting heat [6,7]. The SPMTC model has been designed so that a unit volume of soil (Fig. 1a), consisting of solid particles, water and air, is presented as a system composed of elementary geometrical figures. In this case they are spheres of specific properties, forming overlapping layers (Fig. 1b). It has been assumed that contacts between spheres within a layer and between layers will be represented by a parallel connection of thermal resistors, such as the spheres in a layer, and by series connections between the layers (Fig. 1c). The resultant resistance determined for a system such as in Fig. 1c, was compared with the mean thermal resistance corresponding to a unit of soil volume (Fig. 1a). Having compared the thermal resistance determined in two different manners, and taking into consid-

eration the random character of the distribution of particles in the soil, formulae were developed, allowing for the calculation of the mean value of the thermal conductivity coefficient of the soil [6,7]:

$$\lambda = \frac{4\pi}{m(\theta_v, \phi, T, r, u) \cdot u} \quad (3)$$

where  $\lambda$  - thermal conductivity coefficient of the soil,  $m(\theta_v, \phi, T, r, u)$  - expected value, related to the water content in a unit of soil volume -  $\theta_v$ , soil porosity -  $\phi$ , temperature -  $T$ , sphere radius -  $r$ , and the number of parallel connections of thermal resistors -  $u$ . The expected value is calculated from the following general formula:

$$m(\theta_v, \phi, T, r, u) = \sum_{j=1}^K a_j P(a=a_j) \quad (4)$$

where  $K$  is the number of all possible combinations of particles  $x_{1\alpha}, x_{1\beta}, \dots, x_{k\omega}$ , with  $x_{1\alpha} + x_{1\beta} + \dots + x_{k\omega} = u$ ; the indexes  $\alpha, \beta, \dots, \omega$  indicate how many particles of a given component are present in a given configuration;  $a_j$  - value related to the mineralogical composition, thermal conductivity of the materials, and to the geometry of the particles, calculated from formula (6);  $P(a=a_j)$  - probability of occurrence of  $a=a_j$  - at the same

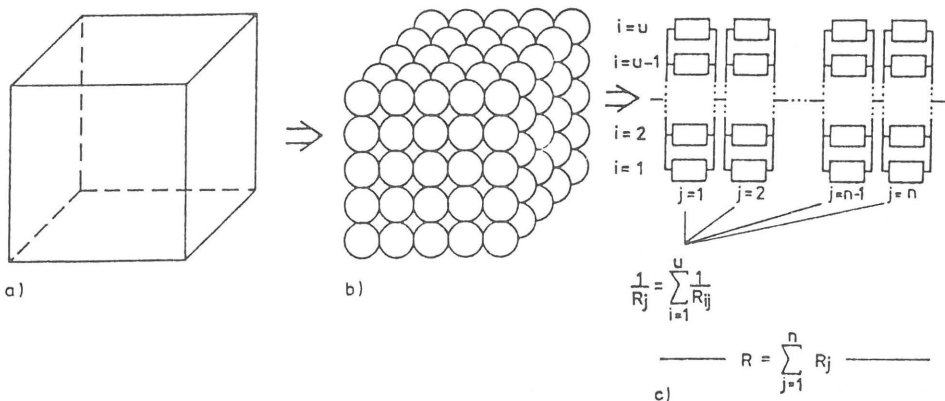


Fig. 1. Schematic diagram of the model of thermal conductivity of soil: a) unit volume of soil, b) system composed of spheres forming overlapping layers, c) parallel connections of the spheres within a layer and series connections between the layers.

time the condition that  $\sum_{j=1}^K P(a=a_j)=1$  must be met. The probability is calculated from a polynomial distribution [4]. The distribution allows for the calculation of the probability of occurrence of all possible configurations of particles  $x_{ij}$  playing a role in thermal conductivity,  $i=1, 2, \dots, k, j=\alpha, \beta, \dots, \omega$  with  $\alpha, \beta, \dots, \omega$  assuming values from the range of 0, 1, ...,  $u$ ,

$$P(x_{1\alpha}, x_{2\beta}, \dots, x_{k\omega}) = \frac{u!}{x_{1\alpha}! \cdot x_{2\beta}! \cdot \dots \cdot x_{k\omega}!} \cdot f_1^{x_{1\alpha}} \cdot f_2^{x_{2\beta}} \cdot \dots \cdot f_k^{x_{k\omega}} \quad (5)$$

where  $f_1, f_2, \dots, f_k$  - the content of particular fractions in a unit of volume, considered as the probability of getting a type  $i$  result in a single test.  $P(x_{1\alpha}, x_{2\beta}, \dots, x_{k\omega})$  is the probability, than  $u$  independent tests will yield exactly  $x_{ij}$  results of type  $j$  if the probability of result  $i$  in a single test is  $f_i$ ,  $i=1, 2, \dots, k, j=\alpha, \beta, \dots, \omega$ .

The composition of the soil includes various chemical compounds, and in a certain unit of the soil volume many particles of the same type can be found. Assuming that a unit of the soil volume contains  $x_1$  particles of one component of a thermal conductivity coefficient  $\lambda_1$  and a particle radius  $r_1, x_2$  particles of another component with  $\lambda_2$  and  $r_2$ , etc., with  $x_1 + x_2 + \dots + x_k = u$ . The thermal conductivity coefficients of the particular components change their values as a function of temperature -  $\lambda(T)$ , and the value of  $a_j$  equals:

$$a_j = \frac{1}{x_{1j} \lambda_{1j}(T) r_{1j} + x_{2j} \lambda_{2j}(T) r_{2j} + \dots + x_{kj} \lambda_{kj}(T) r_{kj}} \quad (6)$$

For a soil, with unchanging mineralogical composition, the value of  $a_j$  will depend on the content of water in a unit of soil volume -  $\theta_v$ , on the soil porosity -  $\phi$ , temperature -  $T$ , sphere radius -  $r$ , and the number of parallel connections of thermal resistors -  $u$ . Therefore, the determination of the thermal

conductivity of the soil requires the determination of the model parameters, i.e. the identification of the model.

The model under study was identified as a model which modifies the number of parallel connections of thermal resistors with changes in the ratio of the water content in a unit of soil volume to the porosity of the soil (soil saturation with water),  $u$  in the function of saturation ( $\theta_v/\phi$ ) (Fig. 2), and the sphere radii - one radius for mineral soils and one for organic soils.

A common characterization of the number of parallel connections of thermal resistors  $u$  was determined as a function of saturation for all the soils under study. However, no common radius was found for the spheres of the soils. It turned out that it is possible to determine a common sphere radius,  $r_{ij}=0.044$  m, for all the mineral soils, while for the organic soils the sphere radius was  $r_{ij}=0.08$  m.

Within the ranges marked in the figure with a double line the value of the thermal conductivity coefficient is calculated as the arithmetic mean of the value calculated for  $u$  and for  $u-1$ . Such an approach attenuates the jump-like transition in the value of  $u$ . In the presentation of thermal characteristics in the function of moisture, density, etc., the courses obtained must be smoothed out by means of  $n$ -th degree polynomials or other

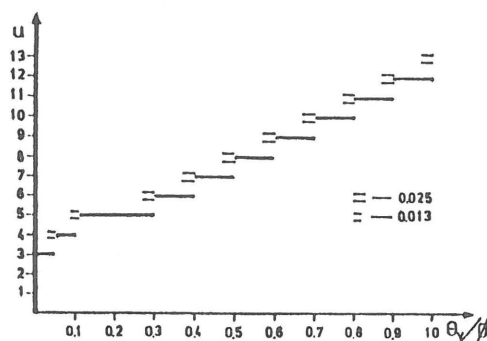


Fig. 2. Characterization of the number of parallel connections of thermal resistors  $u$  in the function of soil saturation with water  $\theta/\phi$ .

functions. The objective of the smoothing out is to eliminate slight fluctuations appearing with a change in the number of parallel connections of thermal resistors in the function of soil saturation with water.

### Thermal vapour diffusivity

In a soil not saturated with water, and with the presence of a temperature gradient, a part of the energy flowing through the soil is transmitted by water vapour. Water vapour flow through soil causes an increase in the thermal conductivity of the soil. The thermal conductivity component related to water vapour and soil air is referred to by de Vries [2] by the general term compound thermal conductivity and expressed as a sum of the thermal conductivity of air  $\lambda_a$  and the thermal conductivity of water vapour  $\lambda_v$ :

$$\lambda_{app} = \lambda_a + \lambda_v. \quad (7)$$

The thermal conductivity of water vapour can be calculated from the equation formulated by Kirscher and Rohalter (after de Vries [2]), or from the equation modified by Philip and de Vries [2,3], expressed by formula:

$$\lambda_v = h l D_a \nu (d\rho_o/dT) \quad (8)$$

where  $h$  - relative humidity which can be calculated from the Kelvin equation (formula 9) or from an empirical relation [5],  $L$  ( $J kg^{-1}$ ) - latent heat of vapourization,  $D_a$  ( $m^2 s^{-1}$ ) - diffusion coefficient for water vapour in air,  $\nu$  (dimensionless) - mass flow,  $T$  (K) absolute temperature, and  $\rho_o$  ( $kg m^{-3}$ ) - saturated vapour density. De Vries considers two ranges of water moisture in which relative humidity equals one, or varies according to the Kelvin equation:

$$h = \exp(\psi M_w / \rho_w R T) \quad (9)$$

where  $\psi$  (kPa) - soil water pressure head,  $M_w$  ( $kg mol^{-1}$ ) - molecular weight of water and equal to 0.018,  $\rho_w$  ( $Mg m^{-3}$ ) - density of liquid water and equal to 1.0,  $R$  ( $J mol^{-1} K^{-1}$ )

universal gas constant, equal to 8.3143. Within the range between field water capacity and full saturation with water, the relative humidity of the soil air equals one. Below the field water capacity the relative humidity can be calculated from the Kelvin formula, or from an empirical Eq. (5). Kimball *et al.* [5] propose another division of the moisture range into two intervals. The first of these is from zero to a moisture value corresponding to the wilting point, and the second - from the wilting point to soil saturation with water.

In the statistical-physical model of the thermal conductivity of soil the thermal conductivity related to water vapour can be considered together with the thermal conductivity of air or separately, but in the latter case it is necessary to determine what volume is occupied by the individual components.

## RESULTS

The method presented above was used to perform a sample determination of the thermal characteristics in the function of soil moisture and density. In the data relating to the soil from Felin (soil lessive developed from a loess-like formation) five main components were identified: quartz, other minerals, organic matter, water and air (the mineralographic composition was simplified to the division into two classes: quartz and other minerals - meaning all the other minerals which occur in a given soil) which are practically utilized in the calculation of the thermal conductivity. The values of the thermal conductivity coefficients of these soil components are quoted in Table 1. In the Felin soil the content of quartz was 67 %, that of other minerals - 31.5 %, and 1.5 % of organic matter; the density of the solid phase was  $2.65 Mg m^{-3}$ .

The thermal characteristics of the soil, determined on the basis of the statistical-physical model of thermal conductivity and the equation of the heat capacity of the soil, are presented in Fig. 3. The thermal conductivity of the soil (Fig. 3a) increases with soil mois-

Table 1. Values and expressions for parameters used in calculating the thermal conductivity of soils

Source <sup>a</sup>	Parameters	Value/expression
	$\lambda_q, \text{W m}^{-1} \text{K}^{-1}$	$9.103 - 0.028 T$
2	$\lambda_m, \text{W m}^{-1} \text{K}^{-1}$	2.93
2	$\lambda_o, \text{W m}^{-1} \text{K}^{-1}$	0.25
1	$\lambda_w, \text{W m}^{-1} \text{K}^{-1}$	$0.552 + 2.34 \cdot 10^{-3} T - 1.10^{-5} T^2$
1	$\lambda_a, \text{W m}^{-1} \text{K}^{-1}$	$0.237 + 0.000064 T$
1	$L, \text{J kg}^{-1}$	$2490317 - 2259.4 T$
1	$D_a, \text{m}^2 \text{s}^{-1}$	$0.0000229 \cdot [(T = 273) / 273]^{1.75}$
2	$D_a, \text{m}^2 \text{s}^{-1}$	$21.7 \cdot 10^{-6} (101.325/P) (T/273)^{1.88}$
1	$\nu$ , dimensionless	$P/[P - (h_o R (T + 273) / 1000 Mw)]$ P - barometric pressure, kPa
1	$\rho_o, \text{kg m}^{-3} \text{K}^{-1}$	$10^{-3} \exp [19.819 - 4975.9 / (T + 273)]$
1	$d\rho_o/dT, \text{kg m}^{-3} \text{K}^{-1}$	$4975.9 \rho_o / (T + 273)^2$

<sup>a</sup> 1. Kimball et al. [5]; 2. de Vries [2].

ture, and the increase is higher for a soil of greater density. The relationship between the soil thermal conductivity and soil moisture is non-linear in character. The inclination of the characteristics: thermal conductivity versus moisture, decreases with decreasing moisture, and the values of the thermal conductivity coefficient at a given moisture are higher at higher soil density levels.

In this case, the characteristics of heat capacity per unit of soil volume in the function of volumetric moisture of the soil is linear (Fig. 3b), because de Vries' equation does not provide for non-linearity occurring at low moisture levels. One can say that the heat capacity increases with increasing moisture, and that, for a given soil at a certain moisture content, the heat capacity increases also with increasing soil density. However, the increase in heat capacity with increasing density is observed up to the point of soil saturation with water, which means that increase in the content of solid phase in a unit of soil volume can take place at the expense of a decrease in the content of soil air. When the point of saturation is reached, further increase in the content of solid phase in a unit of soil volume results in a decrease in the water content in that unit of volume, and therefore in a reduction in the heat capacity per

unit of soil volume, and conversely, when a decrease in the content of solid phase occurs in compacted soil, and the solid phase is replaced with water, the heat capacity of the soil increases.

The thermal diffusivity of the soil, as it was to be expected, attains its maximum at a certain level of soil moisture and density (Fig. 3c). It is clearly seen that the thermal diffusivity of a soil of constant moisture is higher at higher density levels, and that maxima of the thermal diffusivity occur at lower moisture contents for higher densities. This last statement is very important, as it shows without question that with an increase in the soil density the maxima of the thermal diffusivity tend towards lower moisture content values. In many studies reported in literature [1], maxima of thermal diffusivity (for a given soil) at increasing soil density occur at the same moisture content values, and even tend towards higher moisture contents. This situation is the result of forcing the curve to match the measurement points, and in such cases it is not always clear where the actual maximum is located and what is the source of the discrepancy. The error in the assessment of the thermal properties of soils does not exceed  $\pm 10\%$  of the values measured [2,6].

## CONCLUSIONS

The theory presented in the paper allows for the determination of the basic thermal characteristics of soils with relation to the moisture content, density, temperature, mineralogical composition, water potential, and other physical properties of the soil. The error of the assessment of the thermal properties of the soil does not exceed  $\pm 10\%$  of the values measured.

The application of de Vries' theory and of the model presented by the author allows for the elimination of the time and labour consuming laboratory or field methods for the determination of heat capacity per unit of soil volume and of the thermal conductivity coefficient of soils.

The studies carried out showed that the thermal diffusivity of a soil of a constant moisture content is higher at higher density levels, and that maxima of the thermal diffusivity occur at lower moisture contents for higher density levels.

## AUTHOR'S NOTE

The author will be happy to provide more detailed information on the method, the algorithm for the determination of the thermal properties of soils, and on the software used in the study, upon request, to anybody interested in the subject.

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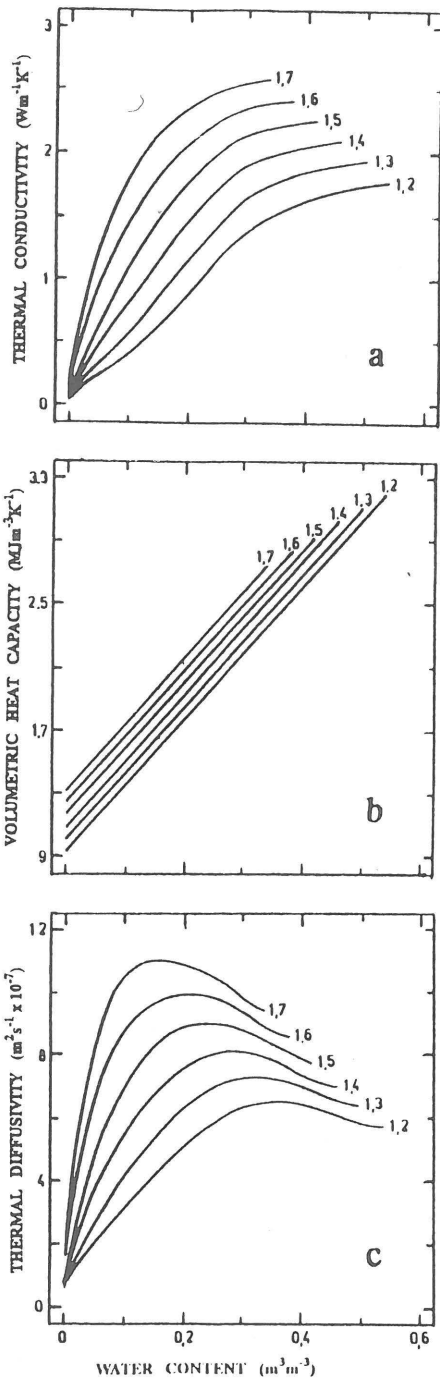


Fig. 3. Characteristics of the thermal properties of soil in the function of moisture at various levels of density: a - thermal conductivity, b - heat capacity per unit of soil volume, c - thermal diffusivity of soil.

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