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EVALUATION OF METHODS FOR SOIL THERMAL CONDUCTIVITY CALCULATIONS

B. Usowicz

Institute of Agrophysics, Polish Academy of Sciences, Doświadczalna 4, 20-236 Lublin, Poland

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Abstract. This paper presents an evaluation of methods for the calculation of the thermal conductivity of soil on the basis of comparison of the thermal conductivity of sand, clay and peat, measured and calculated from the statistical-physical model and de Vries model. For loam, the evaluation was performed on the basis of comparison of results obtained experimentally, models, Laplace transform, and the method of null-alignment, with and without taking into account the thermal conductivity resulting from water vapour movement due to temperature gradient in the soil.

The methods presented can be used alternatively for the determination of thermal conductivity if no high temperature gradients occur in the soil and if it can be assumed that the effect of water vapour on the overall effect of conductivity is slight. With high temperature gradients and high rates of water vapour flow in the soil, the statistical-physical and the de Vries models should be used, the latter requiring considerable care, especially in the determination of weight values.

Keywords: de Vries model, null-alignment and Laplace transform methods, statistical-physical model

INTRODUCTION

The determination of heat flux density in the soil under the conditions of steady and non-steady heat flow requires the knowledge of the basic thermal properties of soil. While it is possible to determine the heat capacity per unit volume of soil with fairly good accuracy, numerous problems are encountered in the determination of thermal conductivity. First of all, it is not always possible to foresee the quantitative role of particular soil components in the transmission of heat, and once we have managed to determine such components, we have to face the problem of averaging those values, as that is what the total thermal conductivity of soil depends upon [2,12-14].

In recent years, certain controversies arose concerning the de Vries model, the method of null-alignment, and the method with Laplace transform [1,2,7]. Many authors show common levels of agreement between values of thermal conductivity obtained from the de Vries model and those obtained experimentally [2,3,5]. A large part of the publications show that the de Vries model overestimates the values of thermal conductivity of soil with relation to the null-alignment method, which determines the thermal conductivity of soil *in situ*, and to the Laplace transform method [1,8-11]. De Vries and Philip [3,4] have a critical attitude towards the null-alignment method, at the same time expressing their doubts concerning the calorimetric method. The paper by Asrar and Kanemasu [1] shows that thermal conductivity values obtained from the de Vries model are underestimated within the range of moisture values from 0 to 0.12 m^3m^{-3}, and beyond that range, towards higher moisture values, they are overestimated compared to measured values.

The proposed new approach, statistical-physical model [13], to the problem of thermal conductivity in soil does not finally decide which model or method is better or the best, but it creates a possibility of quantitative and qualitative comparison among the values of thermal conductivity of soil. It also gives certain hints in the formulation of an answer to the question whether it is true that the de Vries model with the effect of water vapour taken into account does overestimate the thermal conductivity of soil, and whether the null-alignment method and the method with Laplace transform do underestimate the thermal conductivity values.

MATERIAL AND METHODS

Measurement values of the basic physical properties of soil, i.e., soil density, solid phase density, mineralogical composition, organic matter content, water and air content, soil temperature, and thermal conductivity of soil, have been taken from literature [1,2,6,8,9]. Thermal conductivity of soil was determined according to five methods.

The first method - experimental: measurements taken in a laboratory. Thermal conductivity was measured at various water content levels and density values of sand, loam, clay and peat. More information on the experimental methods used can be found in the papers by Kersten [6], de Vries [2], Asrar and Kanemasu [1].

The second method - the null-alignment method [7] - is based on measurements of soil moisture and temperature, taken at short time intervals and at low depths, with simultaneous measurement of thermal conductivity at a given reference level. In the null-alignment method, soil thermal conductivity was calculated by means of the calorimetric method and the temperature gradient method [7], the calculations being started from the point where zero gradient occurred.

The third method - the method with Laplace transform - makes use of the same parameters as the null-alignment method. In this method the equation of thermal conductivity in soil is solved by means of Laplace transform. The transform is used to determine the thermal diffusivity of soil [1], while the heat capacity is calculated from de Vries formula [2]. Thermal conductivity is calculated as a product of the heat capacity and diffusivity of soil.

The fourth method - the method of de Vries model obtained on the basis of the potential theory [2], makes use, in the calculation of thermal conductivity, of data on the mineralogical composition and organic matter content, the content of water and air, soil temperature, soil water potential, and gas pressure in the soil. This model takes into consideration the thermal conductivity of solids, liquids and gas, and permits the inclusion of the thermal conductivity due to water vapour convection resulting from temperature gradient and water pressure gradient in the soil at constant temperature.

The fifth method - the statistical-physical model obtained on the basis of the concept of thermal resistance and statistical polynomial distribution [12-14], makes use of the same data and the same components of thermal conductivity as the second, third, and fourth methods. In the calculation of thermal conductivity taking into account the effect of water vapour, the calculations of the thermal conductivity of air and water vapour were performed on the basis of the theory presented in the paper by Kimball *et al.* [8].

Comparisons were performed for the values of thermal conductivity of soil obtained on the basis of the methods presented. The following values, known from the error calculus, were adopted as the measure of approximation accuracy of the values calculated to values measured:

a) mean square error:

$$\sigma = \left[\sum_{i=1}^{n} \frac{(\lambda_{mi} - \lambda_{ci})^2}{k} \right]^{1/2}$$

if $n<30$ then $k=n-1$
if $n>30$ then $k=n$

b) maximum relative error:

$$\eta_o = \max_{i=1.2n} \left\{ ((\lambda_{mi} - \lambda_{ci}) / \lambda_{mi}) 100\ \% \right\}$$

where n - number of comparisons, λ_{mi} - measured thermal conductivity, λ_{ci} - calculated thermal conductivity.

RESULTS AND DISCUSSION

The comparisons of the thermal conductivity values of various soils, measured and calculated from the models, are presented in Table 1. The values of thermal conductivity for loam, measured, calculated from the models, the method with Laplace transform, and the null-alignment method, with and without taking into account the thermal conductivity resulting from water vapour movement under the effect of temperature gradient in the soil, are presented in Table 2. The data in Table 1 indicate that the results obtained from the de Vries model and the statistical-physical model are on a similar level of agreement with the measured values. Only in the case of clay the maximum relative error for values from the de Vries model is nearly twice as high as for values from the statistical-physical model. The results presented in Table 2, without taking into account the effect of water vapour, indicate that the statistical-physical model gives the best approximation of the calculated values to the measured ones. The other methods also provide fairly good approximations. However, their mean square error is nearly twice as high, or even higher than in the case of the statistical -physical model. When the effect of water vapour is taken into account, the de Vries model significantly increases thermal conductivity

T a b l e 1. Experimental (Method 1) and calculated values of thermal conductivity from de Vries model (Method 4) and the statistical-physical model (Method 5) for Fairbanks sand, Healy clay, Fairbanks peat

Thermal conductivity ($\mathrm{W\ m^{-1}\ K^{-1}}$)								
Methods								
1	4	5	1	4	5	1	4	5
	Sand			Clay			Peat	
2.30	2.27	2.23	1.54	1.53	1.52	0.45	0.48	0.50
2.54	2.64	2.45	1.23	1.19	1.18	0.45	0.44	0.45
2.08	1.97	2.06	0.83	0.98	1.00	0.40	0.39	0.41
2.19	2.24	2.24	1.35	1.29	1.33	0.37	0.38	0.40
2.03	1.91	1.97	0.91	1.05	1.12	0.25	0.24	0.24
2.22	2.06	2.10	1.63	1.50	1.55	0.25	0.25	0.26
1.57	1.63	1.42	1.19	1.18	1.27	0.13	0.16	0.14
1.79	1.81	1.84	0.95	0.95	1.04	0.14	0.15	0.13
1.44	1.49	1.45	0.88	0.97	1.11	0.10	0.13	0.09
1.22	1.27	1.15	0.64	0.82	0.74	0.09	0.10	0.06
1.20	1.17	1.29	0.60	0.77	0.76	0.06	0.07	0.05
0.92	1.05	1.08	0.41	0.70	0.55	0.05	0.05	0.04
0.81	0.84	0.78	0.30	0.36	0.25			
0.58	0.71	0.58	0.21	0.27	0.16	σ	0.017	0.022
0.52	0.50	0.44	0.16	0.20	0.11	η_o(%)	30.0	33.3
0.38	0.40	0.31						
0.33	0.34	0.23	σ	0.127	0.123			
			η_o(%)	70.7	34.1			
σ	0.083	0.087						
η_o(%)	22.4	30.3						

σ - mean square error, η_o - maximum relative error.

Table 2. Average soil temperature, moisture content, heat capacity, and thermal conductivity for 0 to 10 cm of Avondale loam at an average bulk density of 1.4 Mg m^{-3}, quartz fraction of 0.373 g g^{-1} and particle density of 2.71 Mg m^{-3}

Date	Aver. temp. (°C)	Moist. cont. ($m^3 m^{-3}$)	Heat cap. $\left(\frac{MJ}{m^3 K}\right)$	Thermal conductivity ($W m^{-1} K^{-1}$) Methods						
				1	2	3	4	4*	5	5*
17 May 1973	25.2	0.259	2.34	1.13	1.13	1.04	1.21	1.55	1.15	1.17
11 July 1973	32.3	0.282	2.43	1.20	0.96	1.09	1.34	1.63	1.17	1.19
20 Sept. 1973	26.9	0.276	2.40	1.18	1.13	1.14	1.34	1.59	1.17	1.19
21 Sept. 1973	26.9	0.266	2.36	1.15	1.05	1.01	1.26	1.55	1.16	1.17
2 Oct. 1973	27.2	0.225	2.20	1.00	0.88	1.01	1.00	1.51	1.12	1.14
7 Dec. 1973	11.6	0.261	2.34	1.14	1.17	1.09	1.17	1.46	1.16	1.17
				σ	0.131	0.094	0.114	0.456	0.057	0.067
				η_o(%)	20.0	12.2	13.6	51.0	12.0	14.0

Data: 2,4,4* from Kimball *et al.* [9] and 1,3 from Asrar and Kanemasu [1].
Method: 1 - Measured in the laboratory [1,6]; 2 - Null-alignment (Kimball *et al.* [9]); 3 - Laplace transform (Asrar and Kanemasu, [1]); 4 - de Vries model without vapour (Kimball *et al.* [8]; 4* - de Vries model with vapour (Kimball *et al.* [8]); 5 - statistical-physical model without vapour (Usowicz [13]); 5* - statistical-physical model with vapour (Usowicz [14]).

while the statistical-physical model only slightly, though the thermal conductivity due to water vapour was calculated identically for both the models [8]. The high contribution of thermal conductivity due to water vapour in the overall thermal conductivity of soil in the de Vries model was caused, most probably, by poor selection of the shape coefficient in the function of moisture. And the question of how significantly the shape coefficient affects the calculated values of thermal conductivity of soil is discussed in the papers by Kimball *et al.* [8,9].

As follows from the comparisons and analyses, the values of thermal conductivity of soil obtained from the models (Tables 1 and 2), without taking into account the effect of water vapour, from the method with Laplace transform, and from the null-alignment method, are in fairly good agreement over a wide range of moisture and soil density values. Greater differences were observed at very low moisture values. If no high temperature gradients occur in the soil and it can be assumed that the effect of water vapour on the overall effect of conductivity is negligible, the methods presented can be applied alternatively for the determination of thermal conductivity.

As it was to be expected, the values of thermal conductivity of soil obtained from the null-alignment method and from the method with Laplace transform are somewhat underestimated with relation to values calculated from the statistical-physical model and considerably underestimated with relation to values obtained from the de Vries model with the conductivity due to the effect of water vapour taken into account (Table 2). This situation is caused by the fact that a part of the energy is transmitted through the latent heat which is hidden in water vapour. The null-alignment method and the method with Laplace transform do not fully provide for that fact in their calculations. It is known, however, that when water vapour flows through the soil some of the latent energy is released during vapour condensation, which is manifested in an increase in the temperature of the soil layer in which the vapour condensed. That part of the latent heat which passes through the object freely has a decreasing effect on the total thermal conductivity of soil. And thus, with high temperature gradients and high water vapour flow rates it is recommended to use the statistical-physical model and the de Vries model, with special emphasis on correct weight determination. The application of the null-alignment method and the method with Laplace transform is not recommended in this case.

The statistical-physical model which takes into account the thermal conductivity due to water vapour requires further studies, even though it gives values which fall in between those obtained by the methods under consideration. The objective of such studies would be to provide data for comparisons which could lead to modification of the characteristics of the number of connections between thermal resistors and to the substitution of the determined sphere radius with true radii originating from the granulometric distribution, and from the distribution of pores and capillaries formed by water between soil particles.

CONCLUSION

The comparisons of values of thermal conductivity of soils, obtained from experiment, from models without taking into consideration the effect of water vapour, from the method with Laplace transform, and from the null-alignment method, show fairly good agreement with relation to measured values. If it can be assumed that the effect of water vapour on the overall effect of thermal conductivity is slight, then the methods presented can be used alternatively. Distinct differences in the values of thermal conductivity calculated from the de Vries model with relation to the values obtained from the statistical-physical model were observed if in both the models the thermal conductivity due to the movement of water vapour was taken into account. These differences, as well as earlier studies by other authors, suggest that the application of the de Vries model requires considerable care, and especially correct determination of weights. One should also be aware that the null-alignment method and the method with Laplace transform underestimate the values of thermal conductivity of soil, as they do not take into account the thermal conductivity due to the movement of water vapour under the effect of temperature gradient. The application of these methods is limited to systems of small temperature gradients and low water pressures. The statistical-physical model takes into account the thermal conductivity due to water vapour, but even though it gives the best agreement with measured values, it still requires further studies, and primarily the performance of measurement experiments which will supply suitable data for comparisons and, possibly, for the modification of the model.

REFERENCES

1. **Asrar G., Kanemasu E.T.**: Estimating thermal diffusivity near the soil surface using Laplace transform: Uniform initial conditions. Soil Sci. Soc. Am. J., 47, 397-401, 1983.
2. **de Vries D.A.**: Thermal properties of soils. In: Physics of Plant Environment (Ed. W.R. van Wijk). North-Holland, Amsterdam, 210-235, 1963.
3. **de Vries D.A., Philip J.R.**: Soil heat flux, thermal conductivity and the null-alignment method. Soil Sci. Soc. Am. J., 50, 12-18, 1986.
4. **de Vries D.A.**: A critical analysis of the calorimetric method for determining the heat flux in soils. Proc. 8th Int. Heat Transfer Conf., Hemiphen Publ. Corp., Washington, 2, 473-476, 1986.
5. **Hopmans J.W., Dane J.H.**: Thermal conductivity of two porous media as a function of water content, temperature and density. Soil Sci., 142(4), 187-195, 1986.
6. **Kersten M.S.**: Thermal properties of soils. Bull. 28, University of Minnesota. Inst. Technology, Engineering Experiment Station, 52, 21, 1949.
7. **Kimball B.A., Jackson R.D.**: Soil heat flux determination: A null-alignment method. Agric. Meteorol., 15, 1-9, 1975.
8. **Kimball B.A., Jackson R.D., Reginato R.J., Nakayama F.S., Idso S.B.**: Comparison of field-measured and calculated soil-heat fluxes. Soil Sci. Soc. Am. J., 40, 18-25, 1976.
9. **Kimball B.A., Jackson R.D., Nakayama F.S., Idso S.B., Reginato R.J.**: Soil-heat flux determination: Temperature gradient method with computed thermal conductivities. Soil Sci. Soc. Am. J., 40, 25-28, 1976.
10. **Pikul J.L. Jr., Allmaras R.R.**: A field comparison of null-alignment and mechanistic soil heat flux. Soil Sci. Soc. Am. J., 48, 1207-1214, 1984.
11. **Sikora E., Kossowski J.**: Thermal conductivity and diffusivity estimations of uncompacted and compacted soils using computing methods. Polish J. Soil Sci., 26(1), 19-26, 1993.
12. **Usowicz B.**: Studies on the dependence of soil temperature on its moisture in field. Ph.D. Thesis (in Polish). Academy of Agriculture, Lublin, 1991.
13. **Usowicz B.**: Statistical-physical model of thermal conductivity in soil. Polish J. Soil Sci., 25(1), 25-34, 1992.
14. **Usowicz B.**: A method for the estimation of thermal properties of soil. Int. Agrophysics, 7(1), 27-34, 1993.