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ELASTIC AND STRENGTH PROPERTIES OF ROUND AGRICULTURAL PRODUCTS

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A b s t r a c t. Contact deformation of a round agricultural product can be approximately described using the classical elastic theories. Hertz theory of compression of an elastic spherical body and empirical relation between the strength and the modulus of elasticity of agricultural materials give the relation between the fictive strength and the square root of the correspondent relative compression of a spherical agricultural product. This relation makes clear why the damage of deformed round agricultural products appears at much lower stresses than is usually visible on real compression curves. Compression of berry-like fruits between two plates can be understood on the basis of an elastic model in which the skin tension plays the most important role. Using this model for different berry-like fruits some values of the Young's modulus (50-200 MPa) of their skin have been obtained and these values are in good agreement with the results of previous experiments.

K e y w o r d s: round agricultural products, modulus of elasticity strength

INTRODUCTION

Usually, agricultural products have round shape. The force contact of such a type of products with other round bodies usually causes the products damage [1,3,4,5]. This is why the force contact properties of the agricultural products are understood as the very important properties for keeping the high quality of agricultural products during their harvesting, storage and processing.

Possibilities and limitations of present contact theories for description of force contact properties of agricultural products are briefly discussed in this article.

MODELS

Product compression between two plates will be used as a representative test for contact properties of the round products. This test is schematically described in Fig. 1.

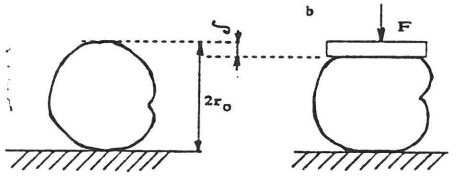

Fig.1. Compression of a round product between two plates, a - initial form of the product with characteristic dimension $2r_0$, b - the product compressed by the force F between two plates, δ - compression of products.

The product of original characteristic dimension $2\,r_0$ is compressed by force F so that the characteristic dimension of the product decreases about δ. This change of fruit dimension corresponds to the relative compression ε, defined by the following equation:

$$\varepsilon = \delta/(2r_0)\,. \tag{1}$$

Sphere is the simplest shape approximation of the round product and this is why the sphere shape of the product will be used for all further discussions.

Contact properties of the sphere product depends also on structure of the product. We use two different models of internal structure of the product: isotropic elastic sphere and closed isotropic elastic membrane filled by fluid (so-called membrane-fluid model). The first model can be used for solid products without important variation in their structure (e.g. seeds). The other one is suitable for fruits and generally for the products with tough skin and liquid or semi-liquid content.

For the first model Hertz contact theory can be used. The effective stress (mean compression stress in the main cross section of the compressed sphere) σ_{ef} is given by the following equation [7]:

$$\sigma_{ef} = \frac{F}{\pi r_o^2} = \frac{4}{3\pi} E_f^{2/3} , \qquad (2)$$

where r_o is an initial radius of the sphere and E_f is a fictive contact modulus of elasticity, defined by the following equation:

$$E_f = E/(1-\nu^2) , \qquad (3)$$

where E is Young modulus and ν is Poisson ratio.

Compression of the membrane-fluid model can be approximately described [3] by the following relation:

$$\frac{F}{2\pi r_o E t} = 0.777\, \varepsilon^{4.255} \qquad (4)$$

with the important membrane properties, i.e., Young modulus E and membrane thickness t.

Equations (2) and (4) are power relations between stress and relative compression, in which modulus of elasticity E takes part in multiplicative constant. There are two important differences between Eqs (2) and (4). The first difference is a different definition of stress: in Eq. (2) σ_{ef} is a mean compression stress in the main cross section of the compressed sphere and in Eq. (4) $F/(2\pi r_o t)$ is a fictitious stress in the main cross section of the model membrane. The second difference consists in different values of the power exponent: 3/2 in Eq. (2) and 4.255 in Eq. (3).

STRENGTH OF ROUND SOLID PRODUCTS

For the maximal pressure p_o in the contact circle Hertz theory gives the following equation [7]:

$$p_o = 2\, E_f\, \varepsilon^{1/2}/\pi . \qquad (5)$$

It means that at rupture point (compression of sphere), pressure p_o gains critical value $p_{op} = 2\, E_f\, \varepsilon_p^{3/2}/\pi$. The strength of sphere material is proportional to p_{op}:

$$\sigma_p = k\, p_{op} , \qquad (6)$$

where k is constant parameter. For most agricultural materials simple power relation between strength σ_p and Young modulus E was obtained [2]:

$$\sigma_p = 2\sqrt{10}\, \sigma_o\, (E/E_o)^{3/4} \qquad (7)$$

where $\sigma_o = E_o = 1$ Pa are the dimensional constants. Right sides of the Eqs (6) and (7) with using the Eq. (5) gives the following relation for modulus E_f:

$$E_f = 100\, (\pi\, \sigma_o/k)^4\ [(1-\nu^2)/E_o]^3\, \varepsilon_p^{-2} . \qquad (8)$$

Inserting right side of Eq. (8) instead of E_f into the Eq. (2), the final relation for the critical value of the effective stress σ_{efp} in the compressed sphere is obtained:

$$\sigma_{efp} = 400\pi^3\, (\sigma_o/k)^4\ [(1-\nu^2)/E_o]^3 \varepsilon_p^{-1/2}/3 . \qquad (9)$$

For $\nu=0$ and $k=1$ Eq. (1) is simplified to the following form:

$$\sigma_{efp} = 4134/\varepsilon_p^{-1/2}. \qquad (9a)$$

Equations (9) and (9a) show the power decrease of the critical value of effective stress with increasing critical value of the relative sphere compression ε_p.

Experimental values of σ_{efp} and ε_p for different agricultural round products and steel spheres are plotted in Fig. 2. For this figure only such data were selected that indicate the first violation of the power form of the corresponding compression curve. It means that critical values σ_{efp} and ε_p are understood as coordinates at which anelasticity of the product begins, i.e., where plasticity starts to operate (steel), apples begin to crack and/or berry flesh begins to flow. It seems the data are in relatively good agreement with the Eq. (9a). The values from Fig. 2 are relatively low and much more higher values can be observed for big ruptures of agricultural round products. For example the values σ_{efp} about 0.3-0.4 MPa and ε_p about 0.15-0.2 were for macroscopic rupture of potato tubers [5].

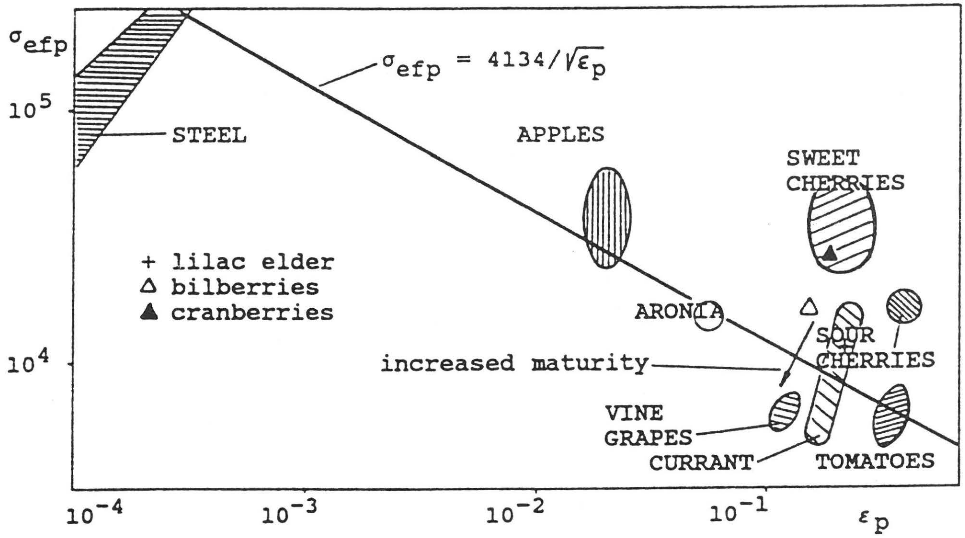

Fig. 2. Critical values of effective stress (σ_{efp}) in relation to critical values of relative compression (ε_p) for different agricultural round products and steel spheres. Critical point is identified with the beginning of anelastic behaviour of the compressed product and/or sphere in this case.

COMPRESSION OF BERRY-LIKE PRODUCTS

Compression curves of berry-like fruits are composed usually from two power parts, that have a linear form with different slopes in logarithmic coordinates (Fig. 3). The first part can be simply termed as Hertz's part, compression of fruit flesh plays an important role in this case and the relation between force and relative compression is approximately given by the Eq. (2), so that the slope of the line in Fig. 3 is about 1.5. Second part of the compression curve is determined

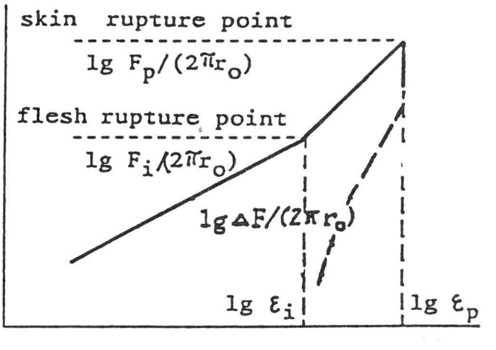

Fig. 3. Schematical drawing of a typical compression curve of a berry-like fruit compressed between two plates (full line). Function log $[(F-\Delta F)/(2\pi r_0)]$ is denoted by the dashed line.

mainly by a tension behaviour of the tested fruit skin and can be simply described by membrane-fluid model (Eq. (4)) when instead of force F is taken the difference F-F_i, only. This arrangement is based on the idea that supporting force of the fruit flesh is constant for the second part of the compression curve. This idea can help us to determine modulus of elasticity of fruits skin through the following equation:

$$E = (F_p - F_i)/(1.555\,\pi\, r_o\, t\, \varepsilon_p^{\;4.255}). \quad (10)$$

Some values of E that have been obtained in previous experiments [1], are collected in Table 1. When the values obtained for the unripe currant fruits are excluded then the modulus of elasticity of different berry-like fruits move approximately in range 50-150 MPa. These values are in very good agreement with the values that have been obtained by direct tension of fruits skin: secant modulus about 10 MPa for apples [6] and 30-80 MPa for tomatoes [4].

Table 1. Modulus of elasticity of fruit skin (E). The values obtained by evaluation of the compression curves - Eq. (10)

Sort and variety of fruit		Date	E (MPa)
Currant	- JVT	19.06	80±18
		25.06	140±14
		01.07	90±25
		07.07	140±24
	- Heineman	19.06	250±62
		25.06	160±43
		01.07	120±36
		07.07	120±35
	- Othelo	19.06	125±27
		25.06	70±18
		01.07	40±16
		07.07	80±18
	- Bohemia	19.06	380±95
		25.06	60±11
		01.07	140±20
		07.07	200±39
Aronia	- Nero		60± 9
Lilac elder			50±10
Bilberries			80±18
Cranberries			110±26
Vinegrape	- Thurgau-Miller		120±45
	- Burgund White		95±17
	- Ryzlink Rhine		160±77
Rowanberry	- Sweet		50±17
	- Red		130±26

CONCLUSION

Compression of solid round agricultural products between two plates can be simply described by Hertz's theory for low quasi-elastic deformations. When deformation overcomes the critical value σ_p simple power shape of deformation curve must be modified. Critical values of effective stress in the compressed products decrease with increasing critical values of relative compression (it is proportional to the square root of relative compression). For berry-like fruits the critical behaviour is connected with a beginning of fruit flesh flow and/or fruit skin tension. Analysis of the real compression curves of berry-like fruits gives the following approximative values for the elastic moduli of their skins: 50-150 MPa. These values are in very good agreement with the values given by previous direct measurements.

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