HAMAN'S INSPIRATION CONCERNING NEW MECHANICS OF THREE-PHASE MEDIA

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A b s t r a c t. Inspiration and development by J. Haman of a new theory concerning the mechanics of three-phase media is presented. This theory describes the simultaneous motion of the solid phase, water and gas in nonsaturated soil. The final equation obtained is related with the pore geometry. The explanation of the mechanisms of soil deformation and consolidation undertaken by the authors of this article was closely connected with Haman's way of thinking.

K e y w o r d s: mechanics of three-phase media

FOREWORD

All we have done is only following of our Master.

FORESEEING HAMAN'S IDEAS

Haman published his doctoral dissertation entitled 'The Influence of Velocity of Ploughing and Soil-Moisture on the Voluminal Deformation of Body' in Annales Universitatis Mariae Curie-Skłodowska, Lublin, Polonia, Sectio E, vol. X, 8 in 1955.

He focused his clear-sighted theoretical research, laboratory investigations as well as the field experiments on ploughing, which is the most important and the most labourconsuming tillage operation, being at the same time the most incomprehensible process in soil. The title lists all the faults of our knowledge about the process.

Soil mechanics, developed for the civil engineering problems deals with the compact

soil under foundations - sometimes saturated with water. Haman was the first, who recognized that the classical soil mechanics formulated by Boltzman, Coulomb-Mohr, Boussinesq, Terzaghi, Cytovich, Gersvanov, Goldstein, Bernstein, Bekker, Fröchlich, Huber, Biot, Kezdi, Kisiel, Reiner, Rossiński, Wiłun, Zielenin and others is not adequate for agricultural problems.

Perhaps the most important difference between the compact soil under foundation and the agricultural loose soil is high susceptibility of the latter for the large, instantaneous and mostly irreversible changes of the volume. It causes the increase in the number of contact points in the unit soil volume, which will be discussed later [19, 32]. In effect all soil properties are changing abruptly (e.g., strength, cohesion, internal friction, viscoelastic moduli, gas, heat, liquids and nutrients motion coefficients). This was confirmed qualitatively later by Haman [14], Haman and Pukos [19], and Pukos [32].

There was no reasonable theory of volumetric soil deformations in 1955.

The fundamental hypothesis of Terzaghi, Biot, Derski, Ziemba and Cytovich [4,6,7], (the so-called consolidation theory) was based on the assumption, that the soil is fully saturated with water and its filtration is the only mechanism of the voluminal changes.

Haman developed a new theory, which described the simultaneous motion of the water and gas in nonsaturated soil and, furthermore, undertook a trial to relate the obtained final equations with the pore geometry, i.e. the real soil structure. The theory was deduced from the first principles.

In effect, this brave departure from the well established way of thinking about the soil compaction mechanisms forced our further investigations performed by the authors of the present paper and led, at least to certain extent, to the explanation of the mechanisms of soil deformation and consolidation. These studies were continuously supported and verified by their initiator.

In 1977 Haman was the supervisor of the doctoral dissertation of Dr. Konstankiewicz entitled 'The Influence of Pore Distribution on Water Potential in Soils Subjected to Consolidation'. The goal of this study was to investigate:

- the influence of the initial moisture of the examined soils and the applied pressure onto the course of consolidation and the resulting pore radius value distribution,

- the influence of the pressure value and application time on the water characteristics.

To determine the function of the pore radius volume distribution of the soil material, the mercury porosimeter 'Carlo Erba' was used. The investigations carried out confirmed the Haman's hypothesis that stress and initial moisture influences mainly the consolidation of the soil medium as well as the changes of the pore distribution versus pore values.

Meanwhile, some soil scientists dealing with the mechanics of foundations (Tan, Rachmatulin, Kisiel, Kitamura, Litwiniszyn, Mróz, Piłat) published some improvements of classical soil rheology. But the enormous number of papers and books was surprisingly still non-adequate for our loose agricultural soil.

In 1940, Goriachkin [11] published his famous rational equation, which stated that

the driving force during ploughing P is equilibrated by the friction force proportional to the plough weight G, the cutting force proportional to the cross-section of the ridge ab and the force proportional to the kinetic energy of repulsion of the ridge $ab v^2$:

 $P = fG + k a b + e a b v^2 \qquad (1)$

where the coefficients f, k, e are empirical values.

Haman introduced additional forces responsible for the energy dissipation related to the work of internal friction in soil and the work of the voluminal deformation. Therefore, the increase in ploughing velocity does not necessarily cause proportional increase of ploughing resistance.

It was very important statement because, according to the Goriachkin's formula the energy consumption for ploughing is proportional to ploughing velocity, which rejected all the possibilities of the ploughing improvement. Haman's hypothesis was oriented to find the optimum ploughing velocity for both the reduction of energy expenses and the minimalization of the negative effects of the soil volumetric deformation during ploughing.

RHEOLOGY AND THERMODYNAMICS

Haman was the first, who recognized that in all practical problems of agricultural soil mechanics the content of gas (30-70 % of the soil volume) is responsible for large, instantaneous, irreversible deformations which cannot be explained within the frames of any classical visco-elasto-plastic theory. It can be easily seen in Fig. 1 that, if the assumption of the linear visco-elasto-plasticity holds, the instantaneous strains must be **reversible** as there is no instantaneous mechanism of energy dissipation.

But there was no other hypothesis at that time. Therefore, he initiated reconsideration of all rheological methods [17]. First a review of rheological models was published in 1968 [22]. General analysis of the all possible configurations and connections of viscous,

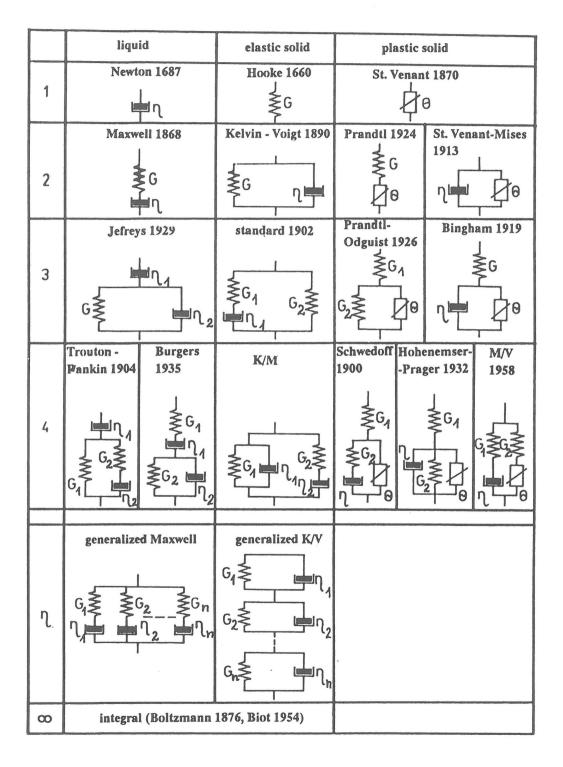


Fig. 1. Classification of the rheological models (after Haman and Zdanowicz [22]).

elastic and plastic analogs (Fig. 1) gave, in effect, a contradiction impossible to be solved: Haman's comprehensive studies of vibrations proved that the parallel connections of viscous and elastic mechanisms were leading to the different coefficient of vibration dumping than connections in series. The same effect was found later by Pukos [32] in case of statical and kinematic loading programs.

It appeared that even the increase in the number of the rheological parameters in classical linear viscoelastic models to infinity (integral models [22]) does not allow to get the proper solution for vibration (especially self-excited ones [14]), energy dissipation and the change of soil structure during deformation.

It follows from the above considerations, that the weekness of the classical soil rheology was the assumed linearity of the equations of state, constructed from the equations of the ideal models of simple bodies. Because of this limitation the method lost its heuristic and creative ability.

The problem why rheology cannot show any way to improve the theory of deformation of the loose soil was really very difficult. The only one assumption was, that the deformation can be subdivided into elastic, plastic and viscous parts. Furthermore, even generalized forms (after the summation or integration) could not help. Haman and Pukos [19] considered two hypotheses:

1. There are some latent assumptions, which should be reconsidered for the media which show the real three-phase structure (considerable air content and following weak structure).

2. There exists another unknown mechanism of deformation (non-elasto-visco-plastic).

The hypotheses were discussed by the authors in the papers [19,21]. The first thermodynamical approach was based on the phenomenological theory of the non-equilibrium processes. Its basic assumptions are:

1. There exists local state in the meaning of the infinitesimally small region for the state functions to be differentiable. 2. Gibbs differential relation is valid (which implies the existence of the temperature, entropy etc.).

3. The entropy balance has an entropy production term which is the sum $X_i Y_i$ of the products of 'fluxes' and 'forces', so that in a product $X_i Y_i$ one factor is even and the other factor is odd with respect to time reversal with both factors vanishing in equilibrium, and the entropy production is never negative.

4. The fluxes are functions of the forces and of those parameters of state which remain independent in equilibrium thermodynamics.

5. For small departures from equilibrium these functions reduce to homogeneous linear functions and satisfy the Onsager's reciprocal relations.

On the basis of the above postulates the laws of conservation of mass, energy, momentum etc. and the law of entropy production are to be formulated. For example, the authors showed [17,19,21] that the energy balance for the deformation or flow in soil in the simplest case should be:

$$\frac{\partial E}{\partial t} = \sum_{j=1}^{n} \sum_{k=1}^{n} (c_{jk} q_j + d_{jk} \frac{\partial q_j}{\partial t}) \frac{\partial q_k}{\partial t}$$
(2)

where the first term describes the reversible process and the second one - irreversible part of the energy balance, p_k and q_k are forces and displacements, respectively, whereas c_{jk} and d_{jk} - coefficients. After transformation the relation for the linear viscoelasticity has the form:

$$\sigma_{k} = \sum_{j=1}^{n} \left(d_{jk} \frac{\partial q_{j}}{\partial t} + c_{jk} q_{j} \right)$$
(3)

This is the constitutive equation of the generalized model of viscoelasic body, which can be generalized further to the most comprehensive integral Boltzmann's and Biot's models [4].

Our conclusion was that the first and last thermodynamical assumptions are invalid for soil. Therefore we decided to refer to the statistical thermodynamics, which insights much deeper into the nature of the considered processes. In statistical thermodynamics the distribution function for the system under consideration is introduced. This function is the density of probability for the system to be in the state of the given values of its parameters as well as the description of evolution of the system in time. The central point of the statistical theory is the most probable distribution, which for the closed system depends on the energy of the system only, and has the form:

$$\rho(q,p) \, dqdp = \frac{1}{Z} \exp\left[\frac{-H(q,p)}{kT}\right] dqdp \quad (4)$$

where (q, p) dqdp is the probability of the finding of the system in a state with values of the generalized coordinates and conjugate forces between the given q, p and q+ dq, p + dp, whereas H(q, p) is the Hamilton function of the system. The function

$$Z = \int \exp\left[\frac{-H(q,p)}{kT}\right] dq dp \qquad (5)$$

is called statistical function, state function, partition function or normalization function. If this function is known, the full characteristics of the body can be obtained.

This method was the subject of Dr. Pukos' dissertation which was naturally fully supported, by his promotor - Haman [19,31]. As far as the visco-elasticity is concerned it was assummed that the function Z depends on temperature, strain and its velocity:

$$Z = Z \left(T, \varepsilon_{ij}, \frac{dE_{ij}}{dt}\right).$$
⁽⁶⁾

Then, starting from the identity for the free energy:

$$F = -k T \ln Z , \qquad (7)$$

the free energy can be written in the form

$$F = -kT \ln Z \left(T, \frac{dE_{ij}}{dt}, \varepsilon_{ij}\right), \qquad (8)$$

whereas the entropy equals

$$S = -\frac{\partial F}{\partial T} = k \ln Z + k T \frac{1}{Z} \frac{\partial Z}{\partial T}$$
⁽⁹⁾

and the internal energy

$$U = F + TS = k T^2 \frac{1}{Z} \frac{\partial Z}{\partial T} .$$
⁽¹⁰⁾

Subsequently, starting from these equations, after transformations [32], the author obtained general equation for the coefficent of elasticity for the nonlinear deformation of the statistical system of elements for any shape of the function of potential between elements in the form:

$$E = \frac{Fd}{\int G(W) H^{-1} \left(-\frac{dU(x)}{dx}\right) dW}$$
(11)

where F is the tensor of external force (in integral and not differential form), d is the mean interparticle distance, G(W) - the Gibbs' distribution of energy and U(x) - the function of the potential shape.

The coefficient is dependent on the temperature and stress (gradient of the potential). In this way this is the nonlinear elastic process, which for the case of the parabolic potential

$$U(x) = M (x_0 - x)^2$$
(12)

can be reduced to constant coefficient of elasticity, dependent on the temperature, soil particle geometry and the shape of the potential (Hooke's law) [32]: Using the relation (5) and (7) one gets:

$$E = \frac{2M}{dkT} \exp\left(\frac{Es}{kT}\right).$$
(13)

As far as the real intergranular potential in soil is concerned the author proposed the exponential potential and the elasticity coefficient appeared to be dependent on stress:

$$E(F,T) = \frac{F a d \exp\left(\frac{Es}{kT}\right)}{kT ar \sin h \frac{Fd^2}{Ma}}$$
(14)

where F is the tensor of external force, T temperature, M and a are the parameters of the potential function, Es - the free energy and d - the intergranular distance, and $ar \sin h = (\sin h)^{-1}$.

Similar considerations for the viscous (time dependent) process had already existed [31] and the author adapted them for the soil in the form:

$$\eta = \frac{f l_1 h \exp{\frac{Es}{kT}}}{2 l kT \sin{h} \frac{f l_2 l_3}{2 kT}}$$
(15)

where η is the viscosity coefficient, l - the distance between two subsequent positions of the equilibrium of the molecule in the liquid, l_1, l_2, l_3 - the distances between subsequent molecules in three mutually perpendicular directions, Es - the free energy of the water molecule, F - the forementioned integral tensor of external force, T - the temperature, sin h denotes the hiperbolic sine, and k, h - stand for the Boltzmann's and Planck's constants, respectively.

These (much more realistic) nonlinear (dependent on stress) elastic and viscous processes were introduced by the author into rheological constitutive equations.

They appeared to fit much better to the experimental characteristics. They confirmed that the viscous resistance of soil is decreasing with the increase of deformation velocity, which was predicted by Haman's theory. It was possible to explain disagreements in the experiments performed by Oida [32] before the theory was developed. Wolski *et al.* [37] were able to use it in the problems of soil subsidence under damm.

The Eq. (15) for the case of constant temperature in an isotropic medium, i.e.:

$$l = l_1 = l_2 = l_3 \tag{16}$$

can be written in the form:

$$\sin h \, \frac{FV}{2kT} = \frac{FV}{2kT}, \quad \eta = \frac{h}{V} \exp \frac{Es}{kT} \quad , (17)$$

where A is constant and V is the volume of the element of soil layer which undergoes the viscous deformation.

The simplifying assumption (16), though natural for soil, is not necessary and its only

goal is to get the easier estimation of V. The experimental results of the shearing tests in sand, loam and loess were compared to the Eq. (17), [32].

The diameter of the elementary viscous deformation appeared to be of the order of 10^{-7} to 10^{-9} m in value. It means that the size of the elementary deformation of soil is of the order of 10 to 100 molecular layers. As the measurements were performed with water content of 6%, 15% and 21% for sand, loess and loam respectively, there could be just 10 to 100 layers of water molecules between the grains of these soils. This confirms the statistical method of the creation of the constitutive equations for soil and Haman's hypothesis that the water content is of primary importance for the control of the velocity of voluminal deformations. The mechanism of the deformation can be seen in Fig. 2.

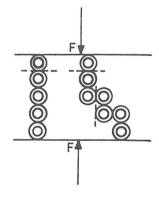
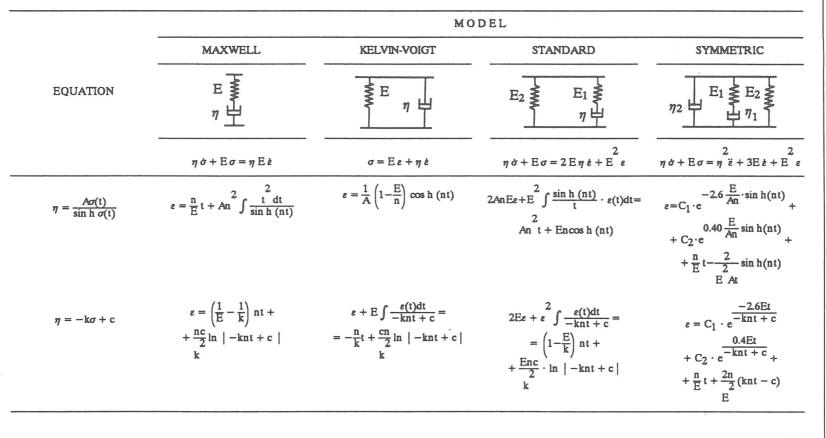


Fig. 2. Mechanism of visco-elastic process. F - external force.

Konstankiewicz [27,28] improved the rheological model. After introducing the relations (14) and (15) into several simplest rheological analogs (Table 1), she found the best equation for the case of the constant deformation velocity, which was the case she was concentrated on:

$$\frac{dF(t)}{dt} + AE_1 \sin h F(t) \frac{E_1 E_2 n \sin h F(t)}{A F(t)} t = (E_1 + E_2) \frac{A F(t)}{\sin h F(t)}$$
(18)

where E_1, E_2 are elasticity coefficients.



HAMAN'S INSPIRATION

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For the experimental verification of theoretical models the author used the triaxial experiment with preselected various steady rates of deformation and various spherical strains. None of the characteristics obtained could be described by means of the linear rheological equation.

In view of the impossibility of solving a nonlinear equation in elementary functions, approximate methods by means of the minimization of functions were used, and numerical values were obtained for the parameters of the equation, for specific, taken form the experiment, values of stress and strain deformation. Numerical analysis was used for this purpose. The parameters calculated permit an accurate experimental verification of the relations (14) and (15), and this has been done for the weak loamy sand, heavy loamy sand and light silty loam.

The results permitted the formulation of the conclusions that the stress-strain relationship in the case of constant strain rate in soil can be described by means of a nonlinear viscoelastic rheological equation postulated above. The initial status of the soil deformed decisively affected the relation of the viscosity and elasticity coefficients to stress. The rate of the applied deformation determined the quantitative changes in the coefficients of viscosity and elasticity.

PROBABILISTIC APPROACH

Subsequent improvements of the statistical theory of soil deformation following the Haman's conceptions made every point and surface of the soil state during deformation to be reached (Fig. 3).

But it is not Haman to be easily satisfied. He stressed many times that the cruxial verification of the soil mechanics in agriculture is connected with the effect of time, velocity, dynamical effects and irreversibility of the voluminal changes.

Despite successfull verification of our statistical rheology the problem of irreversible instantaneous compaction was still beyond our ability for explanation. It can be seen in

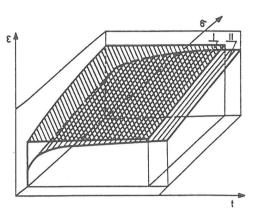


Fig. 3. Mechanical states of a medium under deformation. E - strain, σ - stress, t - time.

Fig. 4, which shows the result of the stressstrain experiment in triaxial cell for a loamy soil. One can easily see how small are the reversible deformations at any stage of load application (even at the very beginning).

Till now we dropped out the assumption concerning a small distance between the state of equilibrium (Onsager's principle) and introduced a new definition of the non-local stress and strain in the integral form:

$$f = \int \frac{\Delta F}{\Delta S}, \ d = \int \frac{\Delta L}{L}$$
 (19)

instead of a continuous, differential classical form, to free ourselves from the assumptions of continuity and homogeneity and small stresses, strains and their time derivatives.

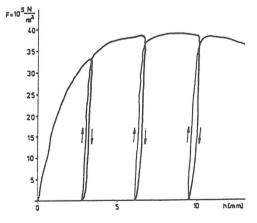


Fig. 4. Cyclical loading of a loamy soil. F-force, h-displacement.

 $g_{q}(Ds)$ $g_{2}(f)$ $g_{3}(Dp)$ $g_{4}(DV)$ P_{mln} X=Aexp(Bf)t Dp, DV $f = f'(1-\mu)-C$

However, it did not help us to find a mechanism of instantaneous, irreversible voluminal changes which, in turn, decide about the structure of a three-phase medium and its changes.

At last, it appeared that it is necessary to have a quantitative way of the description of soil structure.

It is generally known that the flows and deformations in agricultural materials are measured on the surface of samples, blocks or profiles. In this way the structure and its changes are not considered and 'black box models' can be proposed only, because the sample is treated as homogeneous by the experimentator and the changes of its properties during the process are averaged and almost neglected.

It is the fault of experiments that nonlinearity inherent to the structure of three-phase agricultural material cannot be introduced into theoretical consideration.

As the soil consists of a great number of elements different in size and shape (pores, grains, aggregates) and the interactions between them are extremely complex, it is natural to consider the parameters influencing the soil deformation as random variables. In this case the complexity of the soil medium is an advantage, which makes the consideration of its elements as statistical populations (sets, ensambles) possible.

A probabilistic solution was proposed in the paper [32]. It was assumed that the soil structure can be described by 4 random variables the values of which are (Fig. 5):

- diameters of the skeleton particles and aggregates, which describe the geometry of a solid phase,
- pore diameters, deciding which soil grains or aggregates can enter into a given pore during deformation,
- pore volumes, informing about the soil volume which can enter into a pore under consideration,
- contact forces, responsible for the stress heterogeneity and instantaneous, irreversible effects.

Fig. 5. Soil structure and random variables.

Then, the probabilistic equation of deformation for a single pore was derived and integrated over all sizes of pores, aggregates, particles and contact forces giving the soil deformation as the function of soil structure and its initial condition (Fig. 6, Table 2). The effect of stress and time on the soil strains was taken into consideration as well as dry friction (dependent on normal force), viscous friction (dependent on deformation velocity and tangent force), cohesion and the initial state of the soil structure naturally. Subsequent steps can be seen in Fig. 6.

This method made it possible to answer the following fundamental questions:

1. Which physical variables (deterministic and random) are responsible for the volumetric deformation of granular media?

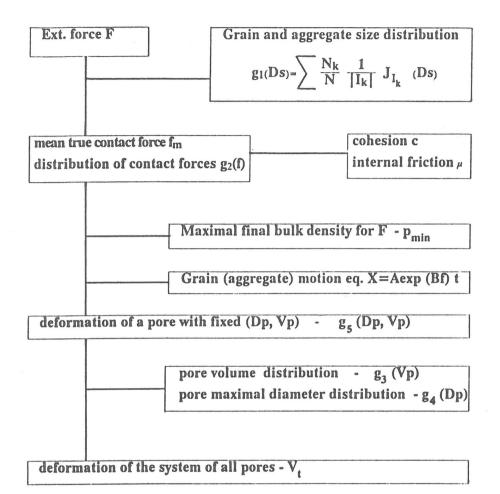
2. What are the mechanisms of this process and corresponding equations?

3. Why and how the quantity of bigger pores is decreasing during deformation abruptly?

4. Why the instantaneous deformation is almost completely irreversible?

5. How to formulate the same equations for all types, kinds and varietes of soil?





where

$$f'_{m} = \frac{F}{\text{numb. cont. points/volume}} = \left[\frac{18 F(1 - P_{min})}{\pi (E Ds)^{3}}\right]^{\frac{2}{3}}$$

dry friction $f = f' - \mu f' - c$

deformation of a pore with fixed (Dp, Vp) g_{5} (A, B, Ds, Dp, Vp, f, t) = (Dp•Vp) g_{1} (Ds) C_{3} (fm) $\int_{h}^{2fm} g_{2}$ (f) df

Fig. 6. Scheme of probabilistic theory.

$$V_{t} = \sum_{k=0}^{n} V_{0k} \frac{2}{3} \left[\frac{E V_{pk}}{E Ds} (1 - p_{min}) \right] c_{3} (f_{m}) \int_{h(a, A, B, t)}^{2f_{m}} g_{2} (f) df \otimes$$

$$\lim_{\substack{min (Dp, Ds_{max}) \\ \int Ds_{min}}^{Ds_{max}} (Ds)^{3} g_{1} (ds) d (Ds) \\ \sum_{Dp \in II_{k}}^{Ds_{min}} Dp - g_{3} (Dp) \frac{N_{p}}{N_{k}}$$

$$c_{3} (f_{m}) = \frac{V_{t}}{\sum_{k=1}^{n} V_{0k} \left[\frac{Vpk}{EDs} (1 - p_{min}) \right]_{Dp} \int \frac{E V(Dp, V)}{Dp} g_{4} (Dp) d(Dp) \frac{Np}{Nk}}$$

Fig. 6. Continuation.

Table 2. Different probabilistic methods in mechanics

Statistical approach	Probabilistic approach
discrete syst. of equal elements, discrete values of stress and strain (for one element), 'motion' in phase space of generalized coordinates q, p, t	discontinuous unequal structural elements, true stresses and strains, 'motion' in space of random variables (distribution functions)
linear thermodynamics of irreversible processes, linear deterministic constitutive theory (Fick, Darcy, Ohm, linear elasto-visco-plasticity)statistical thermodynamics, ergodic theorem, nonlinear statistical constitutive theory (nonlinear elasto-visco-plasticity, nonlinear diffusion and flow theory)	probabilistic theory (random state variables obtained experimentally)
	stochastic processes
	discrete syst. of equal elements, discrete values of stress and strain (for one element), 'motion' in phase space of generalized coordinates q, p, t statistical thermodynamics, ergodic theorem, nonlinear statistical constitutive theory (nonlinear elasto-visco-plasticity, nonlinear

The equations were compared with the experimental characteristics and were found compatible.

Similar considerations were made for the deformation of plant cellular and fibrous material, it means materials of biological origin. In this case the structure is considered quantitatively using random variables, the values of which are related to the cell or fibre geometry and strength.

CONCLUSIONS

It is no accident than that the classical mechanics has not been able to establish a functional formulation of physical relations (constitutive equations) and, in order to obtain their linearity, it has restricted itself to formulate local relations (theory of plasticity, viscoelasticity, laws of diffusion, water flow, heat flow). To recognize the structure of the threephase agricultural materials, it is necessary to introduce integral conditions between forces and flows or deformations_respectively, together with the quantitative measure of structure in the form of random variables.

As it was shown in [32], the probabilistic equations can be reduced to the deterministic non-linear relations, which can be further reduced to the linear equations for small gradients, deformations, flows, and their derivatives.

When the structure can be well approximated by the system of equal elements (like in crystals, liquids and gases), it is possible to use statistical thermodynamics (Table 2). This enables to define stresses, gradients, strains and flows as the sum of the effects for single element and to get some deterministic non-linear constitutive equations.

Probabilistic micromechanics is concerned with the formulation of the stressstrain response with the inclusion of microstructural effects that are due to the inherent geometrical and physical differences between the structural elements.

Most of the significant field quantities involved in any formulation of the material behaviour are, by nature, random variables or function of such variables.

Recently Haman stated that our knowledge about three-phase media is still nonadequate. And he is right because we are looking for a causal mechanics having the effective causes (stresses and gradients) unknown to a full extent. Until we are not able to measure the intergranular forces.

REFERENCES

- Bekker M.G.: Off the road locomotion. The University of Michigan Press, Ann. Arbor., 1956.
- Bernacki H., Haman J., Kanafojski C.: Agricultural machines theory and construction. U.S. National Science Foundation, 1972.
- 3. Byszewski W., Haman J.: Gleba, maszyna roślina. PWN, Warsaw, 1977.
- Biot M.A.: General theory of three-dimensional consolidation. J. Appl. Phys., 12, 35-71, 1941.
- 5. Boussinesq, J.: Application des potentiels. Paris, 1885.
- 6. Cytovich N.A.: Soil mechanics (in Russian). Moscow, 1951.
- Derski W., Ziemba S.: Analiza modeli reologicznych. PWN, Warsaw, 1968.
- Gersvanov N.M.: Osnovy dinamiki gruntovoy massy. Moscow, 1937.
- Goldstein M.N.: Mechanical properties of soils (in Russian). Moscow, 1952.
- Fröchlich O.K.: Drückverteilung im Baugrunde. Wien, 1934.
- Goriachkin W. P.: Sobranny sochinienia. Moscow, 1940.
- Haman J.: The influence of the tractor wheel velocity on the conditions of plant growing. Proc. Vth Conf. ISTVS, Detroit, 92-107, 1975.
- Haman J.: Studium nad dwoma przypadkami powstawania drgań samowzbudnych korpusu pługa. Ann. UMCS, Sect. E, 14, 353 - 381, 1959.
- Haman J., Pukos A.: A mezogazdasagi technika befolyasa a talaj allapotara. A lengyel kutatasok attekintese, Sci. Rep. Univ. Agric. Sciences, Gödöllö, 71, 301-310, 1975.
- Haman J., Pukos A.: Aktualne zagadnienia mechaniki gleb. Probl. Agrofizyki, 15, 1975.
- Haman J., Pukos A.: The influence of agricultural engineering technics on the condition of soils. A survey of the Polish studies. Zesz. Probl. Post. Nauk Roln., 168, 21-37, 1976.

- Haman J., Pukos A.: The relationships between mechanical soil properties and yield of plants. Zesz. Probl. Post. Nauk Roln., 220, 45-51, 1980.
- Haman J., Pukos A.: Problems of physical equations in agricultural soil mechanics. Zesz. Probl. Post. Nauk Roln., 220, 337-345, 1980.
- Haman J., Horabik J., Pukos A.: Mechanical investigations of the agricultural materials in the Institute of Agrophysics. Zesz. Probl. Post. Nauk Roln., 304, 9-17, 1985.
- Haman J., Pukos A.: Probleme d'interpretation physique des deformations des sols. Zesz. Probl. Post. Nauk Roln., 312, 217-228, 1986.
- Hannan J., Zdanowicz A.: O potrzebie rozszerzania studiów nad reologią materiałów w rolnictwie. Roczn. Nauk Roln., 68 -C-2, 195-217, 1969.
- 22. Huber M.T.: Teoria plastyczności. PWN, Warsaw, 1954.
- Kisiel I.: Zastosowanie reologicznego ciała M/V w mechanice gruntów. Ossolineum, Wrocław, 1967.
- Kitamura R.: Analysis of soil deformation as a Markov process. University Press, Kyoto, 1982.
- Konstanklewicz K.: The influence of pore distribution on water potential in soils subjected to consolidation. Zesz. Probl. Post. Nauk Roln., 197, 68-71, 1977.
- Konstankiewicz K.: The experimental verification of the mechanical model of soil medium. Proc. IIIrd European Conf. ISTVS, Warsaw, September, 1986.
- Konstankiewicz K.: Wpływ prędkości odkształcenia na charakterystyki mechaniczne gleb. Probl. Agrofizyki, 51, 1987.
- Litwiniszyn J.: The model of a random walk of particles adapted to research of problems of mechanics of loose media. Bull. Acad. Pol. Sci., ser. Sc. Techn., XII, 5 -28, 1964.
- Piłłat M.: Rheological properties of a model soil medium, Zesz. Probl. Post. Nauk Roln., 220, 356 - 365, 1983.
- Pukos A.: Thermodynamical interpretation of soil medium deformation. Zesz. Probl. Post. Nauk Roln., 220, 367 - 389, 1980.
- Pukos A: Odkształcenia gleby w zależności od rozkładów wielkości porów i cząstek fazy stałej. Problemy Agrofizyki, 61, 1990.
- Rachmatulin C. A., Demianov J.A.: Prochnost pri intensivnych kratkovriemiennych nagruzkach. Izd. Fiz. Mat. Lit., Moscow, 1961.
- 33. Reiner M.: Reologia teoretyczna. PWN, Warsaw, 1958.
- 34. Rossiński B.: Mechanika gruntów. Szczecin, 1952.
- 35. Tan Tiong Kie.: Determination of the rheological and hardening parameters of cohessive soils and a new nonlinear theory of consolidation. Proc. Symp. REMESO, Grenoble, 1964.
- 36. Wiłun Z.: Gruntoznawstwo drogowe. Warsaw, 1947.
- Wolski W., Szymański J.: Embarkments of organic soils. Warsaw Agric. Univ., Dept. of Geotechnics, 1983.
- Vyalov S.S.: Rheological fundamentals of soil mechanics. Elsevier, 1986.
- Zielenin A.N.: Fizicheskiye osnovy tieorii riezania gruntov. Moscow, 1950.