Nonlinear ship wake waves as a model of rogue waves and a source of danger to the coastal environment: a review*

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Abstract

A substantial part of the energy of wake waves from high-speed ships sailing in shallow water is concentrated in nonlinear components which at times have a solitonic nature. Recent results of investigations into solitonic wave interactions within the framework of the Kadomtsev-Petviashvili equation and their implications for rogue wave theory are reviewed. A surface elevation four times as high as the counterparts occurs if the properties of the interacting waves are specifically balanced. The slope of the water surface may increase eightfold. The resulting structure may persist for a long time. Nonlinear wake components may exert a considerable influence on the marine ecosystem in coastal areas.

1. Introduction

Concerns related to ship traffic are usually associated with possible accidents (ship collisions or grounding, technical and navigation problems

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caused by severe weather or human error, etc.) that may lead to loss of life or property. These concerns are being effectively managed by international shipping and harbour communities. The basic assumption is that the risks of water surface transport are localised within a small area around the ship.

The continuing introduction of fast ship services during recent decades has created new, major worries. Apart from the massive increase in exhaust emissions (capable of creating substantial changes in the atmosphere – see e.g. Durkee et al. 2000) and the great upsurge in external noise, the most important issue is the wake generated by large, high-speed ships (PIANC 2003, Wood 2000). These by-products of fast ship traffic are not located in small areas any more; they may travel much faster than the ships themselves (ship-generated noise) or may become a part of global problems (exhaust emissions).

The wakes excited by high-powered ships sailing in shallow and moderately deep waters (up to 100 m) exhibit several specific features. Frequently consisting of non-dispersive, highly nonlinear shallow water waves, and often resembling ensembles of Korteweg-de Vries (KdV) solitons (Soomere et al. 2005), the evolution and interaction of these wakes differ fundamentally from the behaviour of linear waves. These large waves can have a significant impact on the safety of people, property and craft (Parnell & Kofoed-Hansen 2001, PIANC 2003).

In this paper, an attempt is made to describe the major aspects of the potential joint influence of this source of solitonic waves and the specific features of their nonlinear interactions. The description of the classical Kelvin ship wave pattern and its changes with increasing ship speeds are sketched first. This is followed by an overview of generic implications of wakes from intensive ship traffic. The central part of the overview is a description of recent developments in the analysis of the specific features of the interactions of (possibly ship-induced) long-crested solitonic waves (interpreted as KdV solitons) within the framework of the Kadomtsev-Petviashvili (KP) equation. In this context, wave dissipation and breaking, and wave-bottom interactions, are ignored. Finally, potential modifications of the wave shape and applications of the results in realistic shallow-water conditions are discussed.

2. Linear wakes

The first description of the stationary wave pattern excited by a moving point source in terms of two sets of waves that move forwards and away from the disturbance (diverging waves), and one set of waves that move in the direction of the disturbance (transversal waves) was given by Froude (1877). This pattern is called the Kelvin wave system (Kelvin wake) after W. Thomson (1887), Lord Kelvin, who constructed the corresponding theory for deep water. The work was expanded by Havelock (from 1908) to resolve discontinuities in the Kelvin model and to include the effects of water depth.

A quick derivation of the Kelvin wave pattern can be found in Lamb (1997, § 256) or Lighthill (1978, § 3.10). The analysis relies on the dispersion relation and needs to apply only three basic ideas: (i) that the wave system is stationary, (ii) that the constant phase curves are perpendicular to the wave vector, and (iii) that the phase velocity c_f of stationary waves is equal to the projection of the ship's velocity V in the wave propagation direction (Yih & Zhu 1989a,b).

Conditions (i) and (iii) simply mean that the pattern of wave crests created by a steadily moving ship can only be stationary if the wave component travelling at an angle θ with respect to the sailing line has the celerity $c_f = V\cos\theta$. The celerity of linear surface waves $c_f \le c_g$, where c_g is the group velocity; hence, the energy of a steady wave system can exist only within a triangular area called the Kelvin wedge. The half-angle α of the wedge satisfies the condition $\sin\alpha = 1/\left(2c_f {c_g}^{-1} - 1\right)$, and is $\alpha = \arcsin(1/3)$ in deep water where $c_f = 2c_g$. The basic features of the steady wave pattern in deep water do not therefore depend on the sailing speed.

If the ship sails in water of finite depth H, the ratio $c_f/c_g = 2/[1 + 2kH \sinh^{-1}(2kH)]$, where k is the wave number (Sorensen 1973). Yet the angle α only depends on the depth Froude number $F_h = V/\sqrt{gH}$, that is, on the ratio of the ship's speed and the maximum celerity of surface waves for this depth.

Shallow-water effects become important when the wavelength is approximately twice as long as the water depth, i.e., when $kH < \pi$. The relevant depth Froude number for diverging waves at the edge of the Kelvin wedge is $\widetilde{F}_{hd} \approx 0.687$. For somewhat longer transverse waves propagating along the sailing line this threshold is $\widetilde{F}_{ht} \approx 0.56$ (Sorensen 1973). Therefore, at F_h above 0.55–0.7 the ship-generated wave system should respond to changes in the water depth.

If the ship's speed $V = \sqrt{gH}$, the angle α reaches the maximum value $\alpha = 90°$. Frequently, it is claimed (perhaps because of a misinterpretation of the results presented by Havelock 1908; Sorensen 1973) that the transverse and the diverging waves form a single large wave with its crest normal to the sailing line, and that this wave travels at the same speed as the disturbance at $F_h \to 1$. Such a description is conceptually imprecise, because what exactly happens at these speeds cannot be described by the linear theory. However, it is true that wave heights do increase considerably at $F_h \to 1$ and wave periods increase gradually as the ship's speed does so.

The threshold $F_h = 1$ serves as a natural basis for the classification of navigational speeds. Operating at speeds of $F_h < 1$ is defined as subcritical, at $F_h > 1$ as supercritical and at $F_h = 1$ as critical. There is a relatively wide transcritical speed range $0.84 < F_h < 1.15$ in realistic conditions, where no clear distinction between sub- and supercritical regimes is possible (Hüsig et al. 2000).

3. Contribution of waves from high-speed ships to wave activity

The generic importance of the contribution of ship traffic to the local hydrodynamic activity in rivers, inland channels and narrow straits was recognised a long time ago. Intuitively, it is clear that heavy ship traffic has a great damaging potential in the neighbourhood of waterways that are sheltered from large wind waves (such as wetlands and low-energy coasts; see e.g. Schoellhamer 1996, Bourne 2000). The influence of ship wakes is presumed to be negligible in coastal areas that are open seawards or exposed to high tides, and where natural waves are frequently much higher than the wakes (Lindholm et al. 2001). Yet wake wash may make a major dynamical contribution also in certain parts of open sea coasts that are exposed to significant natural hydrodynamic loads (Soomere 2005b).

As a model case, the impact of wakes from high-speed ferries on the coastal environment in non-tidal seas is analysed in terms of wave energy and power, and the properties of the largest waves in Tallinn Bay, Baltic Sea (Soomere et al. 2003a,b). This area can have very rough wave conditions (Soomere 2005a). Certain parts of its coasts are exposed to significant wave loads and are already subject to intense beach erosion. However, ship traffic is so intense that ship-generated waves form at least c. 5–8% of the total wave energy and c. 18–35% of the wave energy flux (wave power) in the coastal areas of Tallinn Bay (Soomere et al. 2003a,b, Soomere & Rannat 2003). The reason for such a large contribution of ship waves in the total wave activity is the combination of (i) specific features of the existing hydrodynamic loads (restricted to a particular direction or to a certain frequency interval) with (ii) particularly high anthropogenic wave loads that are qualitatively different from the natural wave loads (Soomere 2005b).

4. Environmental implications

The relatively large contribution of ship waves to the wave power budget in the Tallinn Bay area indicates that the periods of waves from high-speed ships are frequently much larger than the dominating periods of wind waves in this area. The leading wake waves typically have a height of about

1 m and a period of 10–15 s. Such waves seldom occur under natural conditions in many regions of semi-enclosed seas (Soomere 2005a). They were probably first characterised as 'an unknown phenomenon' in the report of the Danish Maritime Authority (1997), which was based on the relevant studies by Kofoed-Hansen & Kirkegaard (1996). Such waves cause unusually high hydrodynamic loads in the deeper part of the nearshore, not only because of their length and height, but also as a result of their nonlinear properties (Soomere et al. 2005). Ship wakes at times contain specific types of disturbances, such as monochromatic packets of relatively short waves (Brown et al. 1989), depression areas penetrating into adjacent basins (Forsman 2001), or a supercritical bore (Gourlay 2001).

Fast ferry traffic may thus form a qualitatively new forcing component with a significant impact on the local ecosystem in certain open sea areas. In Tallinn Bay, the contribution of ship waves to the wave climate is literally comparable with what would happen if the open ocean swell from the North Atlantic were to reach the Gulf of Finland. Ship waves induced at trans- and supercritical speeds may result in violent energy concentrations not only in the vicinity of ship lanes but also in remote sea areas. It does not seem to be unusual any more that holidaymakers are forced to 'flee for their lives when enormous waves erupted from a millpond-smooth sea', or that waves (which caused a fatal accident near Harwich, a port on England's east coast, in July 1999) look like 'the white cliffs of Dover' (Hamer 1999).

This new component of the local ecosystem is a cause for serious concern. An extensive reaction of the fine sediments in the deeper part of the nearshore is conceivable. Ship waves may be responsible for the acceleration of coastal and sea-bottom erosion (Schoellhamer 1996), and may even trigger considerable changes in the existing balance of coastal processes (Soomere & Kask 2003). They also may seriously damage the biological environment. Suspension and re-sedimentation of finer sediments may considerably worsen fish spawning conditions and, under some conditions, may lead to the resuspension of contaminated sediments (Francisco et al. 1999). The accompanying reduced water transparency (Erm & Soomere 2004) may have a suppressing feedback on the bottom vegetation.

5. Solitonic ship waves

The most important feature of the wake of relatively fast ships in restricted waters is that solitary waves can be generated ahead of the ship. John Scott Russell first documented this phenomenon as he watched in 1834 a canal boat pulled by horses stopping suddenly (see his description reprinted, e.g., by Drazin & Johnson 1989). In a more complex form it has been observed in towing tanks where a ship model can radiate waves that move upstream faster than the ship. Helm (1940) was probably the first to report that many solitons may be generated subsequently. At certain speeds the influence of the ship model extended to 4–5 lengths of the model upstream, whereas up to 7 wave crests were detectable. This is a highly intriguing phenomenon, because it is very unusual that 'a forcing disturbance moving *steadily* ... in shallow water can generate, *continuously and periodically*, a succession of solitary waves, propagating ahead of the disturbance' (Wu 1987, my italics). These waves have been named 'precursor solitons'.

This phenomenon may occur in other areas of research and engineering (Wu 1987). It is a generic mechanism of excitation of disturbances in situations where the nonlinear and dispersive effects are specifically balanced, and becomes effective when the group velocity of long waves radiating from the forcing area is close to the velocity of the disturbance. The local waves therefore obtain energy from the source during a relatively long time. In meteorological applications, examples of high waves generated by moving pressure disturbances when the disturbance speed is approximately the critical speed were reported a long time ago (see e.g. Dysthe & Harbitz 1987 and the references therein). The resulting waves sometimes resemble tsunami (Dean & Dalrymple 2004) and are even called 'meteorological tsunami' (Rabinovich & Monserrat 1998).

In confined waters disturbances resembling KdV solitons frequently occur far ahead of a ship (Neuman et al. 2001). The ship's speed is the decisive factor in forming these waves, because for speeds much less than the critical one the linear waves will effectively carry away the wave energy. A ship may generate a sequence of solitary waves starting already from $F_h \geq 0.2$, and such waves are found in numerical computations for $F_h \geq 0.4$, (Ertekin et al. 1984, 1986). They are the largest for transcritical speeds, may be generated also in open sea areas (Li & Sclavounos 2002), and are frequently accompanied by a sudden dropdown of the water surface near the vessel (Forsman 2001, Li & Sclavounos 2002). When breaking, they may form a bore ahead of a ship sailing in a narrow channel within a certain range of Froude numbers (Gourlay 2001).

There exists an opinion that precursor solitons have been responsible for some disasters (Hamer 1999, Li & Sclavounos 2002). A more probable source of high solitonic waves are the long components of diverging waves that become cnoidal (Parnell & Kofoed-Hansen 2001) or take on the shape of KdV solitons (Soomere et al. 2005) in shallow areas.

The theory of the mechanism of precursor soliton generation was given by Akylas (1984) and Cole (1985) for the basically equivalent environments of a moving disturbance and for a flow past a bump in a channel with a horizontal bottom. In these models the dissipation and breaking effects have been omitted. Let $p = p(x + Vt)$ and $b = b(x + Vt)$ respectively represent the moving ship (interpreted as a surface pressure patch) and the topography. Assume that the velocity V is nearly critical, so that $F_h = 1 + \varepsilon\delta$, where $\varepsilon = (H/\lambda)^2 << 1$ for long waves and $\delta = O(1)$. In the coordinate system moving with the pressure patch or topography, the evolution of the water surface $\widetilde{\eta}$ is, with accuracy $O(\varepsilon^2)$, described by the following equation (Wu 1987):

$$\frac{1}{\sqrt{gH}}\widetilde{\eta}_t + \left[(F_h - 1) - \frac{3}{2H}\widetilde{\eta}\right]\widetilde{\eta}_x - \frac{H^2}{6}\widetilde{\eta}_{xxx} = \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{p}{\rho g} + b\right). \qquad (1)$$

Eq. (1) is an example of a forced Korteweg-de Vries (fKdV) equation with a singular forcing function for a point disturbance. For $F_h = 1$, $p = b =$ const, this equation is the classical homogeneous Korteweg-de Vries (KdV) equation. This framework intrinsically contains only one spatial dimension.

The first two-dimensional numerical results showing the existence of precursor solitons were probably presented by Wu & Wu (1982). They calculated nonlinear long waves forced by a pressure patch moving with a near-critical speed $V \approx \sqrt{gH}$ in a two-dimensional tank with the use of the generalised Boussinesq model of Wu (1981). A solitary wave emerges ahead of the pressure disturbance and propagates upstream.

Numerical experiments based on the Green-Naghdi fluid sheet equation also demonstrated that a series of upstream-propagating soliton-like disturbances appeared ahead of the ship at transcritical speeds ($F_h = 0.9 - 1.2$, Ertekin et al. 1984, 1986). Lee et al. (1989) established that the fKdV model and the generalised Boussinesq model give similar predictions of this phenomenon and show a satisfactory agreement with experiments. The agreement is especially reasonable when $F_h \approx 1$ and the height of the disturbance is small compared with the water depth. A comparison between the fully nonlinear model and the two models above was carried out by Casciola & Landrini (1996) with the use of a more accurate boundary integral approach.

6. Interaction of solitonic wake components

It has been suggested by many authors that an appropriate nonlinear mechanism is responsible for extreme waves. In this context, analysis of the propagation and interactions of KdV solitons has intriguing applications within the framework of abnormally high waves in shallow coastal areas as well as in the general theory of rogue waves.

Nowadays, the mechanisms of interaction of unidirectional KdV solitons are well understood. They do not create any drastic increase in wave amplitudes (Drazin & Johnson 1989). However, considerable amplitude amplification may occur when KdV solitons propagating in slightly different directions meet each other (Hammack et al. 1989, 1995). The 'collision' of two solitons is one of the few mechanisms able to create long-living, extremely high wave humps in shallow water (Kharif & Pelinovsky 2003).

A suitable mathematical model for describing the interaction of nearly unidirectional solitonic shallow water waves is the Kadomtsev-Petviashvili (KP) equation that admits explicit multi-soliton solutions. Such interactions may lead to spatially localised, extreme surface elevations that can be up to four times as high as the incoming waves.

This mechanism has long been known as the Mach reflection (also called the Mach stem; Miles 1977, Freeman 1980) of solitary waves from a wall. It has only recently been proposed as an explanation of the freak wave phenomenon (Peterson et al. 2003). It may become evident only (i) provided long-crested shallow water waves can be associated with solitons and (ii) provided the KP equation is a valid model for such waves. These conditions are not common for storm waves in deep water; however, they are often satisfied when two systems of swell approach a shallow area from different directions. Groups of solitonic waves propagating at a small angle also appear if wakes from two ships intersect in shallow water. Their interaction may be responsible for the dangerous waves along shorelines mentioned in Hamer (1999).

The nondimensional KP equation for surface gravity waves in shallow water reads (Segur & Finkel 1985)

$$(\eta_t + 6\eta\eta_x + \eta_{xxx})_x + 3\eta_{yy} = 0. \tag{2}$$

Here nondimensional variables (x, y, t, η) are related to physical variables $(\widetilde{x}, \widetilde{y}, \widetilde{t}, \widetilde{\eta})$ as follows: $x = \sqrt{\varepsilon}\left(\widetilde{x} - \widetilde{t}\sqrt{gH}\right)/H$, $y = \varepsilon\widetilde{y}/H$, $t = \sqrt{\varepsilon^3 gH}\widetilde{t}/H$, $\eta = 3\widetilde{\eta}/(2\varepsilon H) + O(\varepsilon)$, $\varepsilon = |\widetilde{\eta}_{\max}|/H << 1$ and, as above, the effects of dissipation and breaking are omitted, and the water depth is constant. The two-soliton solution to eq. (2) can be decomposed into a sum $\eta = s_1 + s_2 + s_{12}$ of incoming solitons $s_{1,2} = A_{12}^{1/2} k_{1,2}^2 \Theta^{-2} \cosh\ (\phi_{2,1} x + \ln A_{12}^{1/2})$ and the residue $s_{12} = 2\Theta^{-2}[(k_1 - k_2)^2 + A_{12}(k_1 + k_2)^2]$. Here $\Theta = \cosh \frac{1}{2}(\phi_1 - \phi_{-2}) + \cosh \frac{1}{2}(\phi_1 + \phi_2 + \ln A_{12})$, $\phi_i = k_i x + l_i y + \omega_i t$, $\kappa_i = (k_i, l_i), a_{12} = \frac{1}{2}k_{1,2}^2$, $i = 1, 2$, are the wave vectors and amplitudes of the incoming solitons, the 'frequencies' ω_i satisfy the dispersion relation $k_i\omega_i + k_i^4 + 3l_i^2 = 0$ of the linearised KP equation, $A_{12} = [\lambda^2 - (k_1 - k_2)^2]/[\lambda^2 - (k_1 + k_2)^2]$ is the phase shift parameter and $\lambda = l_1 k_1^{-1} - l_2 k_2^{-1}$ (Peterson & van Groesen 2000).

The interaction may result in either the positive or the negative phase shift (defined by the sign Δ_{12}, where $\Delta_{12} = -\ln A_{12}$) of the

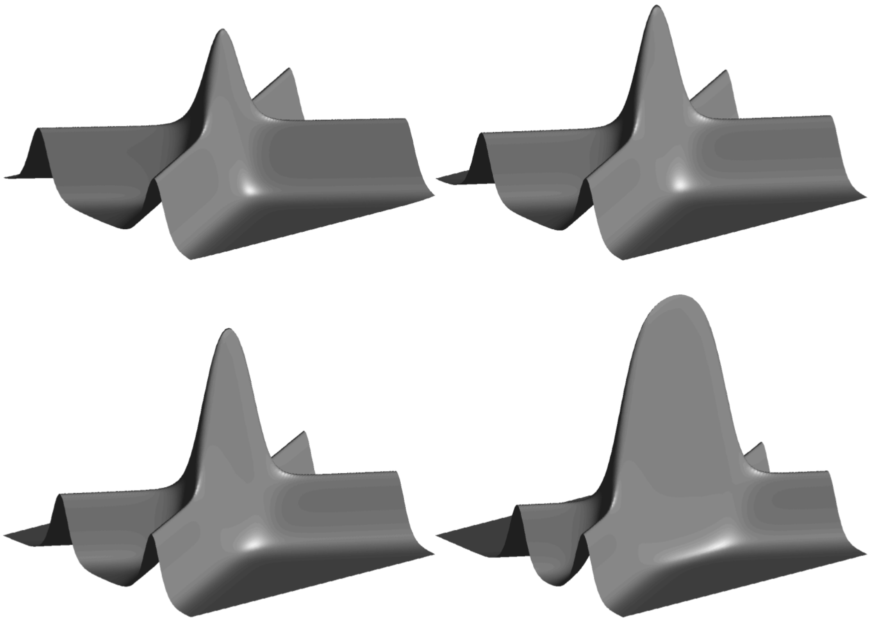

Fig. 1. Surface elevation in the vicinity of the interaction area, corresponding to incoming solitons with equal amplitudes $a_1 = a_2$, $l = -l_1 = 1/3$, $k_{res} = \sqrt{1/3}$ corresponds to the resonant case, and $k = 0.85k_{res}$ (upper left panel), $k = 0.95k_{res}$ (upper right), $k = 0.99k_{res}$ (lower left), $k = 0.9999k_{res}$ (lower right). The area $0 \le z \le 4a_1$, $|x| \le 30$, $|y| \le 30$ in normalised coordinates is shown on each panel

counterparts. The interaction pattern (Fig. 1) is always symmetric with respect to a particular point called the interaction centre, and is stationary in a properly moving coordinate frame. The phase shifts $\delta_{1,2}$ of the counterparts (Fig. 2) only depend on the amplitudes of the incoming solitons and the angle between their crests. The relations for the phase shifts $\delta_{1,2} = \ln A_{12}/|\kappa_{1,2}|$ and for the intersection angle $2\tan\frac{1}{2}\alpha_{12} = \lambda$ can be simplified to one transcendental equation with respect to either of the amplitudes of the interacting solitons (Peterson & van Groesen 2000)

$$\delta_1\sqrt{2a_1(1+\lambda^2/4)} = \pm\ln\frac{\delta_2^2\lambda^2 - 2(\delta_2-\delta_1)^2a_1^2}{\delta_2^2\lambda^2 - 2(\delta_2+\delta_1)^2a_1^2}. \qquad (3)$$

This angle α_{12} and the magnitudes of the phase shifts $\delta_{1,2}$ can be estimated, e.g., from aerial photos. If the sign of the phase shift is known, eq. (3) uniquely defines the heights of the interacting solitons. The sensitivity of this method and several simplifications of eq. (3) are discussed in Peterson & van Groesen (2001).

For the negative phase shift case $A_{12} > 1$ (which is typical in interactions of solitons with comparable amplitudes) an interaction pattern emerges,

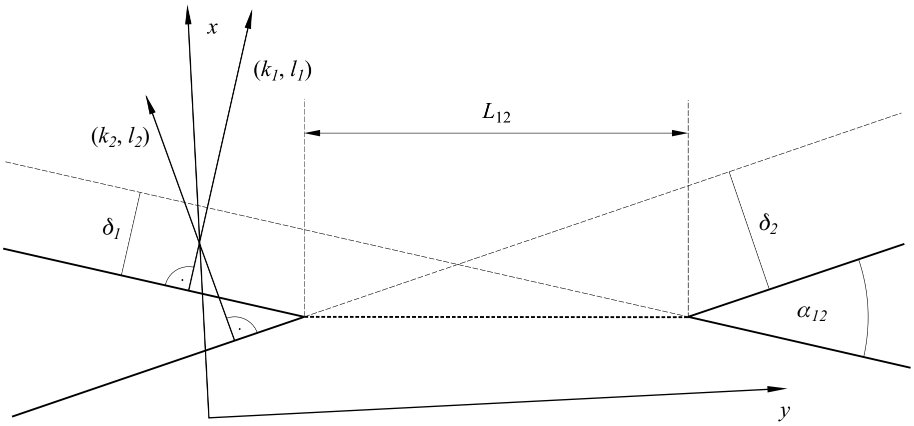

Fig. 2. Idealised patterns of crests of incoming solitons (bold lines), their position in the absence of interaction (dashed lines) and the common crest (bold dashed line) in the negative phase shift case

the height of which exceeds the sum of the amplitudes of the incoming solitons (e.g. Miles 1977, Tsuji & Oikawa 2001). When two waves of arbitrary amplitudes a_1 and a_2 meet, the maximum amplitude M of their superposition can be written as $M = m(a_1 + a_2)$, where the 'nonlinear amplification factor' m may depend on both a_1 and a_2 and their intersection angle. The maximum surface elevation for equal amplitude solitons is $a_{\max} = 4a_{1,2}/\left(1 + A_{12}^{-1/2}\right)$ (Miles 1977, Soomere & Engelbrecht 2005a). Thus, the nonlinear superposition of two equal amplitude solitons may lead to a fourfold amplification of the surface elevation in the resonance case $A_{12} \to \infty$. In the highly idealised case of the interactions of five solitons, the surface elevation may exceed the amplitude of the incoming solitons by more than one order (Peterson 2001).

Extreme water level elevations occur if the solitons intersect at a physical angle $\widetilde{\alpha}_{12} = 2\arctan\sqrt{3\widetilde{\eta}/h}$ (Peterson et al. 2003). This angle is about 36° for waves with heights $\widetilde{\eta} = 1.8$ m (the maximum ship wave height mentioned in Soomere & Rannat 2003) meeting each other in an area with a depth of 50 m, and about 70° for waves with heights $\widetilde{\eta} = 0.8$ m in a coastal zone with a depth of 5 m.

For unequal amplitude solitons the maximum elevation $a_{\max}$ for finite A_{12} and the amplitude of the resonant soliton a_∞ at $A_{12} = \infty$ are

$$a_{\max} = a_{12} + 2A_{12}^{1/2}\frac{a_1 + a_2}{\left(A_{12}^{1/2} + 1\right)^2}, \qquad a_\infty = \frac{(k_1 + k_2)^2}{2}. \tag{4}$$

The expression for a_∞ was probably first obtained for the exact resonance of ion-acoustic solitons in a field-free plasma (Gabl & Lonngren 1984) directly from the resonance conditions, assuming that the resonant structure is a KdV soliton. It was re-derived from the conditions for stationary points of the explicit two-soliton solution of the KP equation in Duan et al. (2004). A simple derivation of expressions (4) is given in Soomere (2004). The amplification factor $m = 1 + 2k_1k_2\left(k_1^2 + k_2^2\right) \approx 2$ when the amplitudes of the interacting solitons are close to each other, and is near to 1 when they are fairly different.

The geometric features of the composite structure created by the interactions of two solitons within the framework of the KP equation have been analysed in Peterson et al. (2003), Soomere (2004), Soomere & Engelbrecht (2005a), and Soomere & Engelbrecht (2006). The area where a high hump potentially emerges may be associated with the area where the interacting waves have a common crest (Fig. 2). Its length $L_{12} \sim \ln A_{12}$ (Peterson et al. 2003) and is therefore modest unless $A_{12} \to \infty$, that is, unless the interacting solitons are near-resonant. For equal amplitude solitons, the extent of the area where the elevation in the composite structure exceeds the sum of amplitudes of the counterparts may considerably exceed the estimates based on the geometry of the wave crests (Soomere & Engelbrecht 2005a); however, the length of this area is also roughly proportional to $\ln A_{12}$.

A part of the analysis of the geometric features of the high wave humps is generalised to the case of interacting solitons with unequal amplitudes in Soomere (2004) and Soomere & Engelbrecht (2006). The spatial extent of the high hump in interactions of solitons with considerably different amplitudes is roughly as large as if the amplitudes were equal. Such interaction mostly leads to bending of the crests of both the counterparts (Fig. 3; cf Duan et al. 2004) and may be one of the reasons for hits by high waves arriving from an unexpected direction.

The process of high wave hump formation is studied numerically in Porubov et al. (2005). These authors simulate the collision of semi-infinite structures within the framework of the KP equation. Transversal energy flow along the crests of such a structure presumably takes place, and the results are not directly comparable with the ones presented above. However, a high wave hump, the height of which considerably exceeds the sum of the heights of the counterparts, is formed quite fast in a certain interaction region. Interaction of solitary waves with the crest localised in one half-plane is studied numerically in Tsuji & Oikawa (2004) in terms of the modified KP (mKP) equation, in which the quadratic term of the KP equation is replaced by the cubic term $6\eta^2\eta_x$. The mKP equation admits both positive and negative solitary wave solutions. The interaction of positive solitary waves results either in structures containing a very high and narrow wave

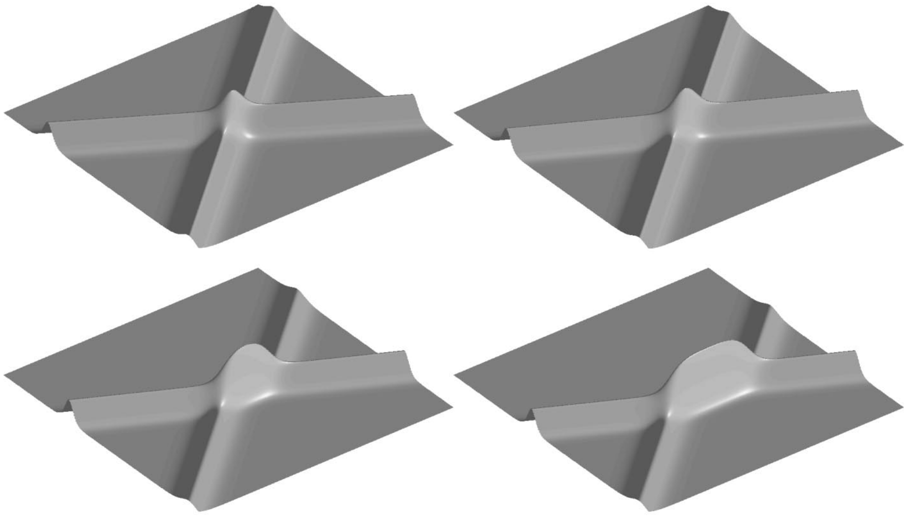

Fig. 3. Surface elevation in the vicinity of the interaction area, for $k_2 = 1/3$ $l = -l_1 = 0.2$, $k_{res} = 0.6$ and $k_1 = 0.85k_{res}$ (upper left panel), $k_1 = 0.95k_{res}$ (upper right), $k_1 = 0.999k_{res}$ (lower left), $k_1 = 0.9999k_{res}$ (lower right) in normalised coordinates (x, y). The area $| x | \leq 60, | y | \leq 90$ is shown on each panel

hump or in transforming the incoming waves into a sequence of much smaller waves.

7. Modification of the wave shape

A pronounced feature of freak waves is that they are particularly steep. It has been claimed that the shape of actually measured freak waves cannot be explained with the use of the existing wave physics, and 'it is concluded that new physics, not incorporated in standard approaches to offshore engineering design, may have played an important role in the generation of this [Draupner's New Year 1995] freak wave' (Walker et al. 2004).

Nonlinear interactions may form a part of this new physics in shallow areas. Plots of two-soliton solutions (Haragus-Courcelle & Pego 2000, Peterson & van Groesen 2000, Peterson et al. 2003) suggest that the front of the near-resonant wave hump is very steep. This feature is also recognisable in the experiments with the Mach reflection of supercritical ship wakes (Chen et al. 2003), where the highest part of the wave hump is narrower than the incoming solitons.

The maximum slope of the front of the two-soliton solution may be eight times as large as the slope of the incoming solitons, giving the relevant maximum 'nonlinear slope amplification factor' equal to 4 (Soomere & Engelbrecht 2005a). For unequal amplitude solitons, the slope amplification may be twice as great as the amplitude amplification (Soomere & Engelbrecht 2005b). This result, although intriguing, is not totally unexpected, because a new soliton is formed by the nonlinear interactions (Miles 1977b, Freeman 1980, Soomere & Engelbrecht 2006). It is higher and therefore narrower than the incoming solitons.

8. Discussion: soliton interactions and compact wakes in realistic conditions

It is not clear whether the above-discussed features can be recognised in isolated form in open sea conditions. There is, however, increasing evidence that they may become evident under certain specific conditions.

The extraordinary steepness of the front of the near-resonant structure makes a hit by such a structure exceptionally dangerous. The dimensional profile of solitary waves is $\widetilde{\eta} = a\cosh^{-2}(\beta x)$, where a is the amplitude of the soliton and $\beta \approx \frac{1}{2}\sqrt{3a/h^3}$ (Drazin & Johnson 1989). The steepness of even quite high solitonic waves, although proportional to a^3, is moderate for deeper areas and they are far from breaking. For example, for the depth $h \approx 70$ m of the Draupner area, the maximum steepness of 3, 5 and 8 m high solitary waves is 0.006, 0.013 and 0.026, respectively (Soomere & Engelbrecht 2006). Such waves are hardly distinguishable on the open sea. However, the maximum steepness of an 18 m high solitary wave, which may be formed through the interaction of two long and long-crested waves with a height of about 5 m, is about 0.087. This is somewhat less than the actual slope of the front of the Draupner wave.

Since a KdV soliton breaks when its height reaches about 80% of the water depth, the breaking limit is not usually reached in deeper areas. Yet the high wave hump may break when it propagates into an area where the conditions for the existence of the two-soliton solution are not satisfied (Peterson et al. 2003).

Breaking wave humps may occur in more shallow areas, where groups of solitonic waves intersecting at a small angle may appear if wakes from two ships meet each other. The crossing of solitonic ship waves with heights of $\widetilde{\eta} = 0.8$ m and maximum slopes of 0.043 (which frequently occur in the coastal zone of Tallinn Bay at a depth of 5 m; Soomere & Rannat 2003) may produce extremely steep structures which have a height of over 3 m, a maximum slope of about 0.34, and which are close to breaking.

The particularly high hump in the nonlinear interaction of KP solitons has a considerable length only when the heights of the incoming waves, their intersection angle and the water depth are specifically balanced. Consequently, the fraction of sea surface occupied by extreme elevations is apparently small compared with the area of a wave storm or the area covered by ship wakes.

There is an important difference between high waves potentially excited by the described mechanism and those arising owing to the focusing of transient and directionally spread waves. In the latter case a number of waves with different frequencies and propagation directions are focused at one point at a specific instant in time to produce a time-varying transient wave group that normally does not propagate far from the focusing area (Kharif & Pelinovsky 2003). A wave hump from nonlinear interaction within the framework of the KP equation, theoretically, has an unlimited lifetime and in favourable conditions may cross large sea areas. The potentially long lifetime of nonlinear wave humps may greatly increase the probability of being hit by such waves.

A generic feature of long, long-crested and high ship wakes is that they may cause ship traffic to have a considerable remote impact owing to their low decay rates and their exceptional compactness (Soomere 2005b). This feature should be addressed in the analysis of the impact of harbours and associated ship traffic in the neighbourhood of vulnerable areas. In particular, it may be necessary to extend the definition of pollution (which is commonly interpreted as the release of certain substances or noise into the environment) towards including the release of energy in general into the marine environment.

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