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THE FRACTAL CHARACTERISTICS OF LATTICE MEDIA. PART I

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A b s t r a c t: Fractals can be an instrument of lattice medium description.

K e y w o r d s: topological dimension, probability dimension, fractals characteristics

INTRODUCTION

Lattice media of considerable complexity are difficult to describe with conventional methods of rational mechanics. For cereal corn, for example, we can use a lattice constructed of straight segments where the different index components of contravariant and covariant vectors are orthogonal (Fig. 1). It is ensured by the existence of δ-Kronecker operator in the following scalar product:

$$\vec{e}^{\,j}\,\vec{e}_i=\delta^{\,j}_{\,i} \quad . \tag{1}$$

As far as media containing a lot of hard to define elements are concerned, the product (1) can be stated differently, that is (Fig. 2):

$$\vec{e}^{\,j}_{\blacksquare}\,\vec{e}_i=a^{\,j}_{\blacksquare i} \tag{2}$$

In this way a matrix is obtained whose values will have definite physical and geometrical interpretations. Vectors $\vec{e}^{\,i}_{\blacksquare}$ and $\vec{e}^{\,i}$ are oriented in relation to themselves at angle β^i. The value of angle β^i can be composed, among the other, of the fractals values of co-ordinates.

THE DESCRIPTION OF A LATTICE

The scheme of co-ordinates describing the cereal corn lattice is presented in Fig. 3. The basic aim is to perform a comparison between the co-ordinate line and the straight line (Fig. 4). So that the question is to what extent the form of the line L joining points $A_1, A_2, ..., A_n$ deviates from a straight line. The difference between the line $L(A_1, A_2, ..., A_n)$ and the straight line $l(ab)$ will become investinguishable `if we look from a longer distance'. Then (we will have) $L/l > 1$. The accuracy of detailed differentiation will be determined by the distance of the observation point. So we obtain the function $L=L(c)$ where $c$ is the `observation distance' (Fig. 5). The angle φ is here a fractal number. Each specified volume of corn medium $V_1 \subset V$ will have a different fractal number. The line L joining the contact points and the other characteristic points can be divided into n segments similar to the line L in relation $1/n$. Analogically, the plane can be divided into n^2 elements and accordingly the volume into n^3 parts. Thus the volume of medium can be divided into $N = n^D$ similar elements in the proportion $\psi = 1/n$ to this volume. Then the probability dimension will assume a shape:

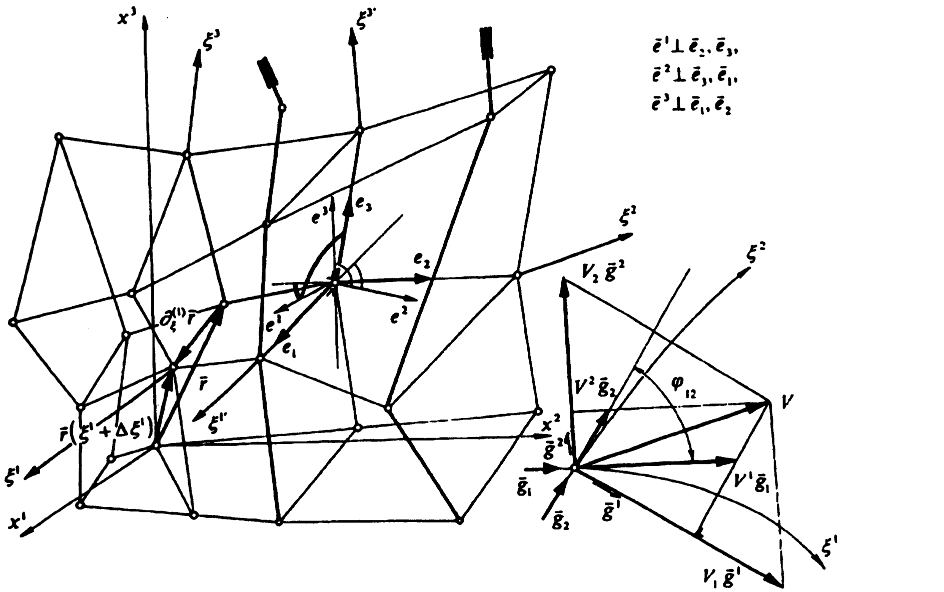

Fig. 1. Co-ordinates ξ^i defining the corn lattice, *1*. *a* - definitions of increments of objects $\partial_\xi^{(1)}$ as well as of the vectors of essential base $\{\vec{e}_i\}$ and mutual base $\{\vec{e}^j\}$, *b* – relationship between contravariant V^i and covariant V_i co-ordinates of vector V.

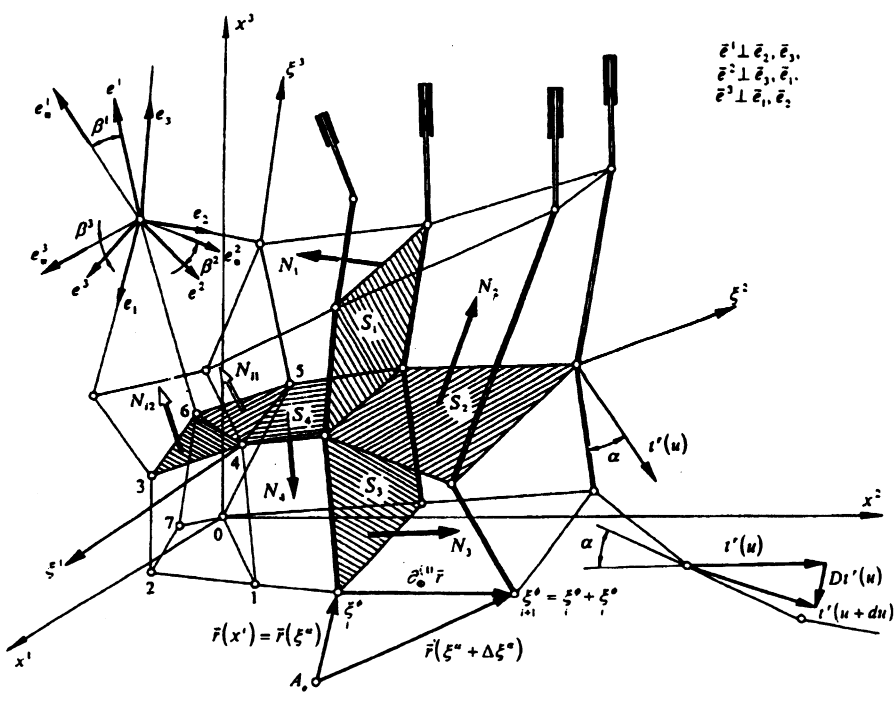

Fig. 2. Co-ordinates ξ^i defining the corn lattice, 2.

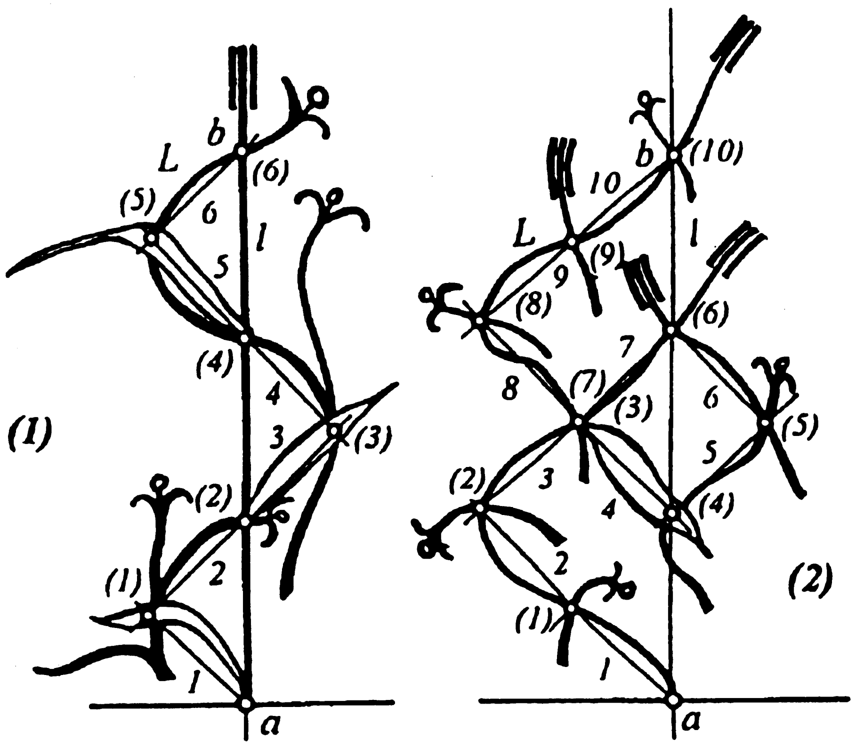

Fig. 3. The state of 'elementary' fragment of corn: before strain (1), after strain (2).

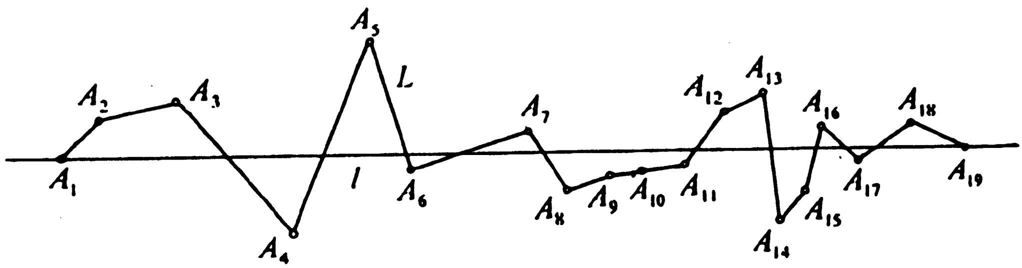

Fig. 4. Co-ordinate L and its relationship with line l.

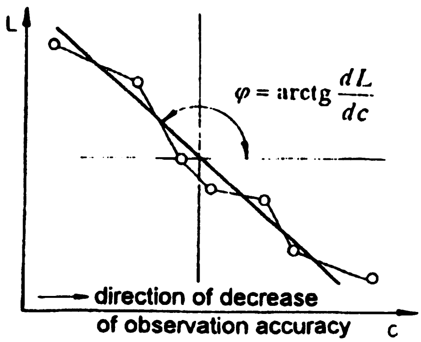

Fig. 5. Geometrical interpretation of fractal number.

$$D = \frac{\ln N}{\ln \frac{1}{\psi}} = -\frac{\ln N}{\ln \psi}. \qquad (3)$$

In Fig. 6 the figures are presented which can be separated from the object when photographed from several directions. In Table 1 the fractal characteristics of the medium are presented. The data in the table have the following meaning. Each case is composed of two fragments - before strain (1) and after strain (2) (the direction of change is indicated by an arrow - Fig. 6). As it results from Fig. 6 (case 1a) $l = (a^2 + b^2)^{1/2} = (1^2+4^2)^{1/2}=17^2 = n$ and the number of broken line segments L amounts $N = 7$ (they are assumed to be the segments of conventional length equal to unity). Accordingly, in the state after strain it will be $n = 17^{1/2}$ and $N = 17$.

The outline L joining the points of contact is divided into straight line segments according to the case investigated. The length of discretization segments is facultative. In order to state the description precisely all the subpictures obtained can be displaced by the discretization segment what will result in occurring new subpictures accompanied by the appropriate new fractal numbers D. In this way it is possible to assign a definite fractal number to each facultatively short segment of co-ordinate. These fractal numbers can be changed into ${}^f\beta^i$ angles with the use of Legendre transform. The quantity ${}^f\beta^i$ will enter the value of β^i angle as a component.

Any deformable real body posses its curvature object $\Re^{\alpha}_{\bullet\beta,\gamma\rho}$ effecting the value of distance $ds \doteq \Delta s$. Since $N = nD$ is related to the distance measure L then $L = L\left(\Re^{\alpha}_{\bullet\beta,\gamma\rho}\right)$ and finally:

$$D = \frac{\ln N\left(\Re^{\alpha}_{\bullet\beta,\gamma\rho}\right)}{\ln n}. \qquad (4)$$

In this way the fractal number and the curvature object can be brought into relationship with use of the length previously defined.

Part II of this paper will treat about the examples of wave motion [1].

Table 1. The fractal characteristics of corn

No.	Fig.	N	n	${}^F D = \frac{\ln N}{\ln n} = \frac{\ln N}{\ln \psi^{-1}}$
1	a	7	$\sqrt{17}$	≈1.37
2	a	17	$\sqrt{17}$	2
3	b	6	$3\sqrt{2}$	≈1.24
4	b	10	$\sqrt{18}$	≈1.59
5	c	6	$3\sqrt{2}$	≈1.24
6	c	18	$\sqrt{18}$	2
7	d	10	$\sqrt{26}$	≈1.41
8	d	26	$\sqrt{26}$	2
9	e	11	$\sqrt{37}$	≈1.33
10	e	37	$\sqrt{37}$	≈2
11	f	10	$\sqrt{20}$	≈1.54
12	f	20	$\sqrt{20}$	2
13	g	9	$\sqrt{17}$	1.55
14	g	21	$\sqrt{17}$	2.15
15	h	6	$\sqrt{18}$	1.24
16	h	18	$\sqrt{18}$	2.00
17	i	6	$\sqrt{18}$	1.24
18	i	36	$\sqrt{18}$	2.48
19	j	10	$\sqrt{26}$	1.41
20	j	36	$\sqrt{26}$	2.20
21	k	11	$\sqrt{37}$	1.33
22	k	53	$\sqrt{37}$	2.20
23	l	10	$\sqrt{20}$	1.54
24	l	28	$\sqrt{20}$	2.23

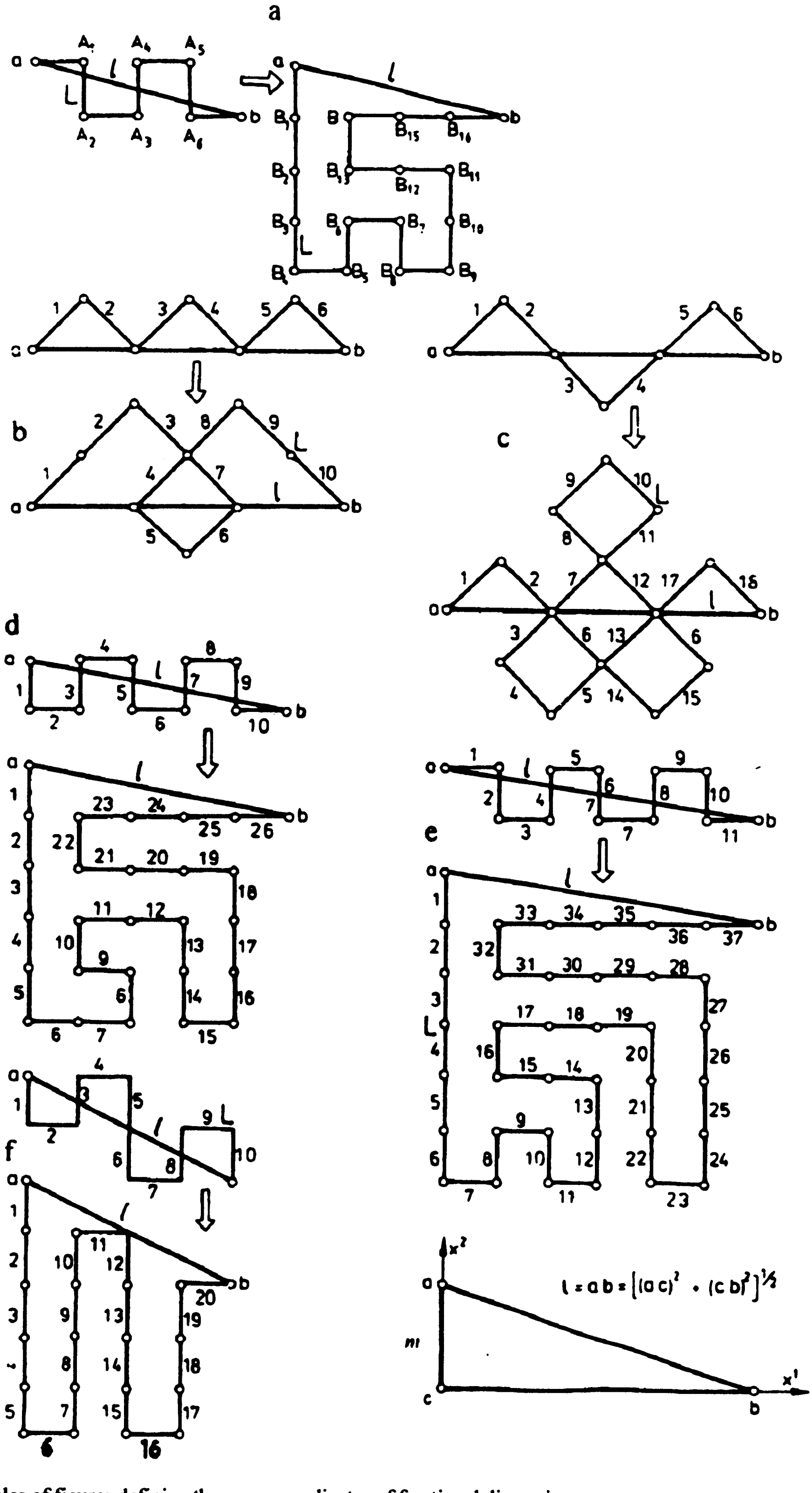

Fig. 6. Examples of figures defining the corn co-ordinates of fractional dimension.

CONCLUSION

Different definitions of covariant components result in different geometry having its interpretation in appropriate physical and geometrical characteristics of the medium. Fractal description can be treated as a certain interpretation of the state of the medium. Finally, the fractal number and the curvature object can be brought into relationship.

LITERATURE

1. **Marciniak A., Fijołek A., Kwiecień S., Podgórski J.: The fractal characteristics of lattice media. Part II. Int. Agrophysics 10, 283-288, 1996.**