Available online at www.worldscientificnews.com

World Scientific News

An International Scientific Journal

WSN 127(3) (2019) 361-368 EISSN 2392-2192

SHORT COMMUNICATION

Modified Landau-Lifshitz Energy Density in Reboucas-Tiomno-Korotkii-Obukhov Spacetime

M. Korunur[1,a], **M. Salti**[2,b] and **O. Aydogdu**[2,c]

[1]Department of Electricity and Energy, Tunceli Vocational College, Munzur University, Tunceli, TR-62000, Turkey

[2]Department of Physics, Faculty of Arts and Science, Mersin University, Mersin, TR-33343, Turkey

[a-c]E-mail address: mkorunur@munzur.edu.tr , msalti@mersin.edu.tr , oktaydogdu@mersin.edu.tr

ABSTRACT

Energy-momentum (EM henceforth) localization problem is one of the old and unsolved issues theoretical physics. Plenty of studies have been introduced to clarify the localization problem and there are many prescriptions given in gravitational theories such as the general relativity (GR henceforth) and teleparallel gravity (TG henceforth) to deal with this issue. In a recent work, the energy-momentum localization problem has been extended to a modified theory of gravity. In the present work, we calculate energy density associated with the Reboucas-Tiomno-Korotkii-Obukhov (RTKO henceforth) spacetime by making use of the modified gravity version of Landau-Lifshitz (mLL henceforth) formulation. Also, we consider some viable $f(T)$-gravity proposals.

Keywords**:** Energy-Momentum, Modified gravity, Landau-Lifshitz prescription

(Received 08 April 2019; Accepted 23 April 2019; Date of Publication 26 April 2019)

1. INTRODUCTION

Theoretical physicists have interested in the problem of the EM localization over a hundred years. The process of seeking required solutions began with Einstein for the first time and is continuing up to the present day. Unfortunately, this problem has not been removed exactly and it has become a well-known puzzle. Due to working with the localized energy density is very significant to define total energy of our universe, there are many notations introduced in literature: the Einstein [1], Tolman [2], Papapetrou [3] Landau-Liftshitz [4], Bergmann-Thompson [5], Møller [6], Weinberg [7] and the Qadir-Sharif [8] prescriptions. Except for the Møller formulation, all of them should be used in a quasi-cartesian coordinate system in order to get meaningful conclusions. There are many papers about the EM localization issue [9-12]. Vargas [13], using some EM prescriptions in the framework of the TG, calculated the EM density of a Friedman–Robertson–Walker type spacetime and showed that the GR and TG versions of those EM formulations yield same results. Next, after this work, the problematic EM issue has been extended to different perspectives [14-17]. Recently, Abedi and Salti [18] derived an EM prescription in the realm of a modified teleparallel gravity. Furthermore, Ganiou et al. [19], in the framework of $f(T)$-theory, studied the mLL energy formulation to evaluate the corresponding energy densities for some spacetime models.

In the present work, we mainly focus on the mLL EM prescription written by Ganiou et al. [19]. In the next section, we briefly give some theoretical expressions of the mLL energy distribution. Considering some well-known $f(T)$-gravity models, the mLL energy density associated with the RTKO spacetime is obtained in the third section. Subsequently, in the last section, we give concluding remarks. Note that we will use the Greek alphabet $(\alpha, \beta, \gamma, ...)$ for the spacetime indices and the Latin alphabet $(a, b, c, ...)$ to denote tangentspace indices.

2. PRELIMINARIES: MATERIALS AND METHODS

Generally, the metric tensor $(g_{\mu\nu})$ plays very important role to formulate theories of gravity. On the other hand, the tetrad $(h^{a}{}_{\mu})$ provides connection between curved and flat spacetimes and takes crucial roles while describing the structure of spacetime in the TG:

$$h^{a}{}_{\mu} = \partial_{\mu} x^{a} + A^{a}{}_{\mu} \tag{1}$$

where $A^{a}{}_{\mu}$ is translational gauge potential and x^{a} represents the tangent-space coordinates[9]. The tetrad and its inverse version satisfy the following relations:

$$h^{a}{}_{\mu} h_{a}{}^{\nu} = \delta^{\nu}_{\mu}, \qquad h^{a}{}_{\mu} h_{b}{}^{\mu} = \delta^{a}_{b}, \qquad g_{\mu\nu} = \eta_{ab} h^{a}{}_{\mu} h^{b}{}_{\nu}. \tag{2}$$

where, δ^{ν}_{μ} describes the Kronecker delta function and $\eta_{ab} = diag(-1, +1, +1, +1)$ is the Minkowski metric. Moreover, in order to construct the torsion geometry, we need to the Weitzenböck connection which is given [9] by:

$$\Gamma^{\sigma}{}_{\mu\nu} = h_{a}{}^{\sigma} \partial_{\nu} h^{a}{}_{\mu} = -h^{a}{}_{\mu} \partial_{\nu} h_{a}{}^{\sigma}. \tag{3}$$

Next, antisymmetric torsion tensor is defined by using the Weitzenböck connection as

$$T^{\lambda}{}_{\mu\nu} = -T^{\lambda}{}_{\nu\mu} = \Gamma^{\lambda}{}_{\nu\mu} - \Gamma^{\lambda}{}_{\mu\nu}. \tag{4}$$

Using torsion tensor components, one can define Freud's super-potentials:

$$U_{\beta}{}^{\nu\lambda} = hg_{\beta\mu}\left[m_1 T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T^{\beta\nu}_{\beta} - g^{\nu\mu}T^{\beta\lambda}_{\beta})\right], \tag{5}$$

where $h = det(h^{a}{}_{\mu})$ and m_1, m_2 and m_3 are three dimensionless coupling constants of the TG [10]. A specific choice of three dimensionless coupling constants, i.e. $m_1 = \frac{1}{4}$, $m_2 = \frac{1}{2}$, and $m_3 = -1$, yields the equality between the GR and the TG.

The LL EM density in the framework of the TG is written as follows [13]:

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_{\lambda}\left(hg^{\mu\beta}U_{\beta}{}^{\nu\lambda}\right). \tag{6}$$

A generalized form of the LL EM density ($h\tilde{L}^{\mu\nu}$) defined in a modified TG is given [19] as follows:

$$h\tilde{L}^{\mu\nu} = f_T(T)hL^{\mu\nu} + \frac{h}{4\pi G}g^{\mu\sigma}f_{TT}(T)U_{\sigma}{}^{\nu\lambda}\partial_{\lambda}T \ , \tag{7}$$

where $f_T(T) \equiv \frac{df(T)}{dT}$, $f_{TT}(T) \equiv \frac{d^2f(T)}{dT^2}$ and the torsion scalar is written as

$$T = \frac{1}{4}T^{\sigma\mu\nu}T_{\sigma\mu\nu} + \frac{1}{4}T^{\sigma\mu\nu}T_{\nu\mu\sigma} - T^{\sigma}_{\sigma\mu}T^{\nu\mu}_{\nu}. \tag{8}$$

Consequently, one can extract the mLL energy density considering the expression of $h\tilde{L}^{00}$. So, it can be obtained that

$$h\tilde{L}^{00} = f_T(T)hL^{00} + \frac{h}{4\pi G}g^{0\sigma}f_{TT}(T)U_{\sigma}{}^{0\lambda}\partial_{\lambda}T \ . \tag{9}$$

3. CALCULATIONS

The RTKO type spacetime model is given by the following line-element [21-22],

$$ds^2 = A^2(t)[-(dt + me^{x}dy)^2 + dx^2 + e^{2x}dy^2 + dz^2], \tag{10}$$

where m is a positive rotation parameter. The RTKO spacetime represents a spatially homogeneous, rotating and shear-free model with vanishing rotation. Thus, the matrix representations of the metric tensor and its inverse form are written as

$$g_{\mu\nu} = \begin{bmatrix} -A^2(t) & 0 & -me^x A^2(t) & 0 \\ 0 & A^2(t) & 0 & 0 \\ -me^x A^2(t) & 0 & (1-\mathrm{m}^2)e^{2x}A^2(t) & 0 \\ 0 & 0 & 0 & A^2(t) \end{bmatrix}, \qquad (11)$$

$$g^{\mu\nu} = \begin{bmatrix} \frac{\mathrm{m}^2-1}{A^2(t)} & 0 & -\frac{me^{-x}}{A^2(t)} & 0 \\ 0 & \frac{1}{A^2(t)} & 0 & 0 \\ -\frac{me^{-x}}{A^2(t)} & 0 & \frac{e^{-2x}}{A^2(t)} & 0 \\ 0 & 0 & 0 & \frac{1}{A^2(t)} \end{bmatrix}. \qquad (12)$$

Also, using Eqn. (2), the surviving components of tetrads can be obtained as

$$h^a{}_\mu = \begin{bmatrix} A[t] & 0 & \mathrm{m}e^x A[t] & 0 \\ 0 & A[t] & 0 & 0 \\ 0 & 0 & e^x A[t] & 0 \\ 0 & 0 & 0 & A[t] \end{bmatrix}, \quad h_a{}^\mu = \begin{bmatrix} \frac{1}{A[t]} & 0 & 0 & 0 \\ 0 & \frac{1}{A[t]} & 0 & 0 \\ -\frac{\mathrm{m}}{A[t]} & 0 & \frac{e^{-x}}{A[t]} & 0 \\ 0 & 0 & 0 & \frac{1}{A[t]} \end{bmatrix}. \qquad (13)$$

With the help of Eqns. (3) and (13), non-zero components of the Weitzenböck connection are calculated as

$$\Gamma^0{}_{00} = \Gamma^1{}_{10} = \Gamma^2{}_{20} = \Gamma^3{}_{30} = \frac{\dot{A}}{A}, \ \Gamma^2{}_{21} = 1 \ , \qquad (14)$$

where the dot means time-derivative, i.e. $\dot{A} \equiv \frac{dA(t)}{dt}$. Subsequently, the surviving components of anti-symmetric torsion tensor can be found by considering Eqn. (4):

$$\mathrm{T}^1{}_{01} = -\mathrm{T}^1{}_{10} = \mathrm{T}^2{}_{02} = -\mathrm{T}^2{}_{20} = \mathrm{T}^3{}_{03} = -\mathrm{T}^3{}_{30} = \frac{\dot{A}}{A}, \ \mathrm{T}^2{}_{12} = -\mathrm{T}^2{}_{21} = 1. \qquad (15)$$

Hence, Freud's super-potential yields the following results

$$U_0{}^{01} = -U_0{}^{10} = \frac{1}{4}e^x(2-m^2)A^2,$$

$$U_0{}^{02} = -U_0{}^{20} = -\mathrm{m}A\dot{A},$$

$$U_0{}^{12} = -U_0{}^{21} = -\frac{1}{4}\mathrm{m}A^2,$$

$$U_1{}^{01} = -U_1{}^{10} = e^x(1-m^2)A\dot{A},$$

$$U_1{}^{02} = -U_1{}^{20} = \frac{1}{4}\mathrm{m}A^2,$$

$$U_1{}^{12} = -U_1{}^{21} = -\mathrm{m}A\dot{A},$$

$$U_2{}^{01} = -U_2{}^{10} = \frac{1}{4}e^{2x}\mathrm{m}(1-m^2)A^2,$$

$$U_2{}^{02} = -U_2{}^{20} = e^{x}(1-m^2)A\dot{A},$$

$$U_2{}^{12} = -U_2{}^{21} = -\frac{1}{4}e^{x}\mathrm{m}^2A^2,$$

$$U_3{}^{03} = -U_3{}^{30} = e^{x}(1-m^2)A\dot{A},$$

$$U_3{}^{13} = -U_3{}^{31} = -\frac{1}{2}e^{x}A^2,$$

$$U_3{}^{23} = -U_3{}^{32} = \mathrm{m}A\dot{A}. \tag{16}$$

where $h = e^x A^4$.

So, the LL energy density can be found by making use of the relation of Freud's super-potentials. Then, it is found that

$$hL^{00} = \frac{e^{2x}(\mathrm{m}^2-1)A^4}{4\pi}. \tag{17}$$

Moreover, we can use some well-known specific $f(T)$-gravity models to calculate the mLL energy density.

- The first model is defined [23] by

$$f_I(T) = aT + \frac{b}{T}, \tag{18}$$

where a and b are represent two positive reel numbers. For this case, after using Freud's super-potentials and Eqn. (9), the corresponding energy density is computed as

$$h\tilde{L}_I^{00} = \frac{1}{4\pi}e^{2x}(\mathrm{m}^2-1)A^4\left\{a - \frac{4bA^8}{[\mathrm{m}^2A^2+12(\mathrm{m}^2-1)\dot{A}^2]^2}\right\}. \tag{19}$$

Here, we used the following expression of the torsion scalar

$$T = \frac{6(1-\mathrm{m}^2)\dot{A}^2}{A^4} - \frac{\mathrm{m}^2}{2A^2} \tag{20}$$

- The second model is given [23] by

$$f_{II}(T) = aT + bT^{\mathrm{n}}, \tag{21}$$

were n is a constant. For this case, the corresponding energy density can be found as

$$h\tilde{L}_{II}^{00} = \frac{1}{4\pi} e^{2x}(\mathrm{m}^2 - 1)A^4 \left\{a + bn\left[\frac{6(1-\mathrm{m}^2)\dot{A}^2}{A^4} - \frac{\mathrm{m}^2}{2A^2}\right]^{n-1}\right\}. \quad (22)$$

- As a final model [23], we take the following expression

$$f_{III}(T) = aT + bT^{\delta}\ln(T), \quad (23)$$

where δ is another constant. So, the mLL energy is obtained as

$$h\tilde{L}_{III}^{00} = \frac{e^{2x}(\mathrm{m}^2-1)A^4}{4\pi}\left\{a + 2^{1-\delta}b\left[1 + \delta\ln\left(\frac{12(1-\mathrm{m}^2)\dot{A}^2 - \mathrm{m}^2A^2}{A^4}\right)\right]\left[\frac{12(1-\mathrm{m}^2)\dot{A}^2 - \mathrm{m}^2A^2}{A^4}\right]^{\delta-1}\right\}. \quad (24)$$

4. CONCLUSIONS

Studying the EM localization problem is very interesting and yields important conclusion both in the GR and the TG. As we mentioned in the first section, one of the important prescriptions is the LL EM complex. In the present study, using the mLL formulation, we discuss the modified energy density in the RTKO spacetime model for three viable $f(T)$-gravity cases. After assuming a suitable set of constants, for instance $a = 1, b = 0$, in the $f(T)$-gravity models, one can easily see that our results can be reduced to their TG versions:

$$hL^{00} = \frac{e^{2x}(\mathrm{m}^2-1)A^4}{4\pi}.$$

If it can be defined an exact form of the cosmic scale factor $A(t)$, we can understand the evolutionary behavior of energy density and describe effects of the modified gravity for the selected spacetime model.

Acknowledgement

This study was presented as an oral presentation at the 2nd International congress of vocational and technical sciences held in Batumi/Georgia on 10-13 May 2018.

Reference

[1] Einstein, Sitzungsber. Preus. Akad. Wiss. Berlin (Math. Phys.) 778 (1915).

[2] R.C. Tolman, Relativity, thermodynamics and cosmology. Oxford University Press, London, (1934) 227.

[3] A Papapetrou, Einstein's theory of gravitation and flat space. *Proc. R. Irish. Acad.* A 52 (1948) 11-13.

[4] L.D. Landau and E.M. Lifshitz, The classical theory of fields, 4th edition. Pergamon Press, Oxford, (1987)

[5] P.G. Bergmann and R. Thomson, Spin and angular momentum in general relativity. *Phys. Rev*. 89 (1953) 400.

[6] Møller, On the localization of the energy of a physical system in the general theory of relativity. *Ann. Phys.* 4 (1958) 347-371.

[7] S. Weinberg, Gravitation and cosmology: Principle and applications of general theory of relativity. John Wiley and Sons, Inc., New York, (1972).

[8] Qadir and M. Sharif, General formula for the momentum imparted to test particles in arbitrary spacetimes. *Phys. Lett. A* 167 (1992) 331-334.

[9] V.C. de Andrade and J.G. Pereira, Gravitational Lorentz force and the description of the gravitational interaction. *Phys. Rev. D* 56 (1997) 4689.

[10] K. Hayashi and T. Shirafuji, Gravity from Poincaré Gauge Theory of the Fundamental Particles. I: General Formulation. *Prog. Theor. Phys.* 64 (1980) 866-882.

[11] F.I. Mikhail, M.I. Wanas, A. Hindawi and E.I. Lashin, Energy-momentum complex in Møller's tetrad theory of gravitation. *Int. J. Theoretical Phys.* 32 (1993) 1627-1642.

[12] K.S. Virbhadra, Energy associated with a Kerr-Newman black hole. *Phys. Rev. D* 41 (1990) 1086.

[13] T. Vargas, The energy of the universe in teleparallel gravity. Gen. Relativ. Gravit. 36 (2004) 1255-1264.

[14] M. Salti, Different Approaches for Møller's Energy in the Kasner-Type Spacetime. *Modern Phys. Lett. A* 20 (2005) 2175-2182.

[15] O. Aydogdu, M. Salti, and M. Korunur, Energy in Reboucas-Tiomno-Korotkii-Obukhov and Gödel-type Space-times in Bergmann-Thomson's Formulations. *Acta Physica Slovaca* 55 (2005) 537-548.

[16] M. Sharif and M.J. Amir, Teleparallel energy–momentum distribution of lewis–papapetrou spacetimes. *Mod. Phys. Lett. A* 22 (2007) 425-433.

[17] S. Aygun and I. Tarhan, Energy–momentum localization for Bianchi type-IV Universe in general relativity and teleparallel gravity. *Pramana* 78 (2012) 531-548.

[18] H. Abedi and M. Salti, Multiple field modified gravity and localized energy in teleparallel framework. *Gen. Relativ. and Grav.* 47 (2015) 93.

[19] M.G. Ganiou, M. J. S. Houndjo, and J. Tossa, f(T) gravity and energy distribution in Landau–Lifshitz prescription. *Int. J. Mod. Phys. D* 27 (2018) 1850039.

[20] K. Hayashi and T. Shirafuji, New general relativity. *Phys. Rev. D* 19 (1979) 3524.

[21] M.J. Rebouças and J. Tiomno, Homogeneity of Riemannian space-times of Gödel type. *Phys. Rev. D* 28 (1983) 1251.

[22] S. Carneiro and G.A.M. Marugan, Anisotropic cosmologies containing isotropic background radiation. *Phys. Rev. D* 64(8) (2001) 083502.

[23] R. Myrzakulov, Accelerating universe from F(T) gravity. *Eur. Phys. J. C* 71(9) (2011) 1752.