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INFLUENCE OF HAULING FORCE ON FIRMNESS OF PLURAL STATIONARY MOTIONS OF PASSENGER CAR MODEL

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Summary. The question of influence of hauling force which operates on front axle of passenger car is considered in the article, on the plural of stationary motions of passenger car. It is set that the presence of hauling force changes the diagram of firmness substantially (bifurcation plural).

Key words: car, course firmness of motion, bifurcation plural, trajectory phase, trajectory of motion.

INTRODUCTION

Character of dependence of lateral force as influences the function of slip corner the most substantial rank on descriptions of course firmness of car [Volkov V.P., Kravchenko A.P., 2009, Podrigaylo M.A., Volkov V.P., Boboshko A.A. and others, 2008, Podrigaylo M.A., Volkov V.P., Stepanov V.Y. and others, 2006]. In-process by descriptions course firmness the areas of firmness of plural of steady-states are understood inplane the guided parameters. For the simplest bicycle model in the case of droningly growing dependence of force of the lateral taking any changes of firmness of steady-states, at the change of two guided parameters there are the systems, related to appearance (disappearance) of pair of the special points. In the cases of general, answers these points or branching of fold is the double special point, or branching of collection is the triple special point [Arnold V.I., 1990]. Therefore interest presents determination of areas inplane two guided parameters with the different amount of the stationary modes – limit between them is a critical value (a bifurcation plural has the name) [Arnold V.I., 1990, Poston T., Stewart I., 1980].

The geometrical method of finding of the stationary modes [Pevsner J.M., 1947, Pacejka. H.B., 1978, Verbitskii V.G., Lobas L.G., 1981], complemented the algorithm of construction of bifurcation plural [Verbitskii V.G., 1995, Verbitskii V., Danilenko E., Nowak A., Sitarz M., 2007], is known, enables to conduct the previous analysis of amount of the stationary modes and define limit of firmness inplane the guided parameters: longitudinal rate of movement, corner of turn of the guided module [Verbitskii V., Danilenko E., Nowak A., Sitarz M., 2007, Lobas L.G.; Verbitskii V.G., Boruk I.G., 2000]. From the review of that hauling forces which operate in the contact of wheels with a road [Gillespie Thomas D., 1992, Kosenko A.V., 2010] were not taken into account in these works, but realization of the simplified geometrical method of construction of bifurcation plural at presence of longitudinal force on front wasp is done impossible, the construction of bifurcation plural is in-process conducted on the basis of numeral realization of analytical and geometrical method of lengthening after two parameters [Shinohara Y., 1972, Troger H., Zeman K., 1984, Holodniok M., Klic A., Kubicek M., Marek M., 1991] both at the account of longitudinal force and at its absence (the plane of the guided parameters is broken up on an area with the different amount of the stationary modes; there is a change of firmness of stationary motions on limit of areas). To the points of sharpening (cusps) of critical plural answers branching of collection, and to the regular points is branching of fold [Arnold V.I., 1990, Poston T., Stewart I., 1980, Bruce J., Giblin P., 1988]. The sizes of areas, where proof steady-states will be realized, give a quantitative estimation influence of hauling force on descriptions of course firmness.

A purpose of the article is research of influence of hauling force which operates on front wasp, on firmness of plural of stationary motions of passenger car.

RESULT OF RESEARCHES

For research the «bicycle» model of car is utillized with the fastened steering management, equalizations of motion of which look like:

$$\begin{cases} m(\dot{v} + uw) = X_1 \cos(\theta) - Y_1 \sin(\theta) - X_{fr}; \\ m(\dot{u} + vw) = X_1 \sin(\theta) + Y_1 \cos(\theta) + Y_2; \\ J\dot{w} = a(Y_1 \cos(\theta) + X_1 \sin(\theta)) - bY_2, \end{cases} \tag{1}$$

where: m – mass of car;

Y_1, Y_2 – lateral forces which arise up between a wheel and road;

X_{fr} – force of resistance of car motion;

X_1 – hauling force.

Forces of the lateral taking Y_i, that included in the system of equalizations (1), determined empiric (functions are from the corner of taking δ_i), presented as droningly growing dependence:

$$Y_i = \frac{k_i \delta_i}{\sqrt{1 - \left(\frac{k_i \delta_i}{N_i \varphi_i}\right)^2}}, \tag{2}$$

where: k_i – coefficient of resistance to the lateral taking;

N_i – vertical loading;

φ –coefficient of rolling friction with a road.

Monotonous approximation of forces of taking is not of principle limitation for the select in-process numeral method of construction of identical curves and біфуркаційних plurals, and enables easily to define influence of longitudinal force (the case of monotonous approximation in default of longitudinal force is well investigational [Verbitskii V.G., 1995, Verbitskii V., Danilenko E., Nowak A., Sitarz M., 2007, Lobas L.G.; Verbitskii V.G., Boruk I.G., 2000]).

Force of resistance of motion X_{op} consists of force of resistance rolling and forces of resistance of air environment [Tarasik V.P., 2006]:

$$X_{fr} = mgf + k_w F v^2, \tag{3}$$

where: $f = f_0(1+(0,0216v)^2)$ - coefficient of resistance to rolling [19];

k_w – coefficient of windage;

F – frontal area of car.

At calculations which will be executed below, the next values of parameters are accepted: φ = 0,8; m = 1325 kg; coefficients of resistance of taking: k_1 = 50 кN/rad, k_2 = 40 кN / rad; f_0 = 0,015; k_w = 0,3 Ns^2/m^4; F = 1,8 m^2; base l = 2,42 m; distance is from front a landmark to the barycenter a = 1,2 m; distance is from back a landmark to the barycenter b.

The analysis of equalizations which determine the plural of steady-states specifies on possibility of direct decision of this system in relation to forces of taking, that allows substantially to simplify procedure of construction of bifurcation plural.

Two last equalizations of the system (4) determine the plural of steady-states:

$$X_1 = \frac{[b(vwmtg(\theta) - uwm + X_{fr}) - a(uwm - X_{fr})]\cos(\theta)}{b+a};$$

$$Y_1 = -\frac{[-vwmctg(\theta) - b(uwm - X_{fr}) - a(uwm - X_{fr})]\sin(\theta)}{b+a}; \tag{4}$$

$$Y_2 = \frac{m\omega va}{b+a}.$$

Utillizing the method of continuation after two parameters [Verbitskii V., Danilenko E., Nowak A., Sitarz M., 2007, Holodniok M., Klic A., Kubicek M., Marek M., 1991], will define the critical values of the guided parameters, that will build a bifurcation plural (graph, which represents parameters which a divergence loss of firmness of steady-states is at). On fig. 1 the got bifurcation plural which specifies on the high-quality changes of bifurcation plural during realization of tractive force on front wasp is resulted: area of the guided parameters θ and v distributed on an area with the different amount (presented numbers in circles) of the stationary modes of motion. In areas with two stationary modes one mode will be proof, and second unsteady. Accordingly in areas with four stationary modes – two proof and two unsteady.

Most dangerous from the point of view course firmness are areas where the stationary modes of motion absent, or there is one unsteady mode.

For the subsequent analysis of firmness of the stationary modes the method of continuation is utillized after one parameter [Shinohara Y., 1972, Lobas L.G., Verbitskii V.G., 1990].

As example an area is considered with four stationary modes of motion (see rice. 1) in the case when longitudinal constituent of speed C.M. car v = 23 m/s.

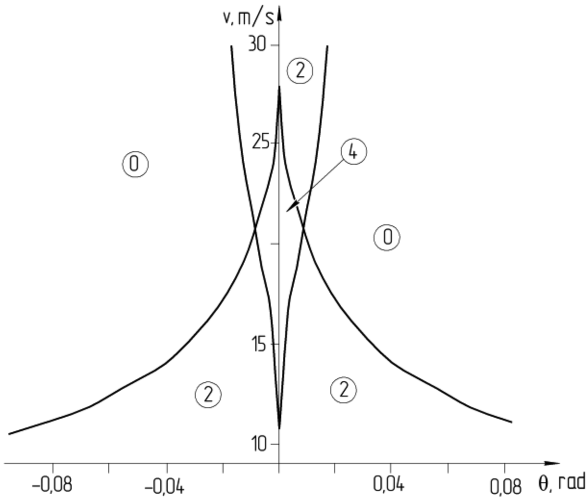

Fig. 1. Bifurcation plural

On fig. 2 dependence of angulator round vertical the landmark of car body ω from the corner of turn of the guided wheels θ is resulted. The points of turn on the graph (1 – 4 on of fig. 2) answer the double stationary modes [Lobas L.G., Verbitskii V.G., 1990, Holodniok M., Klic A., Kubicek M., Marek M., 1991] (divergence loss of firmness) – bifurcation plurals which outline an area with four stationary modes at speed coincide with interfaces 23 m/s (continuous line answers the graph to the proof modes of motion, stroke – unsteady).

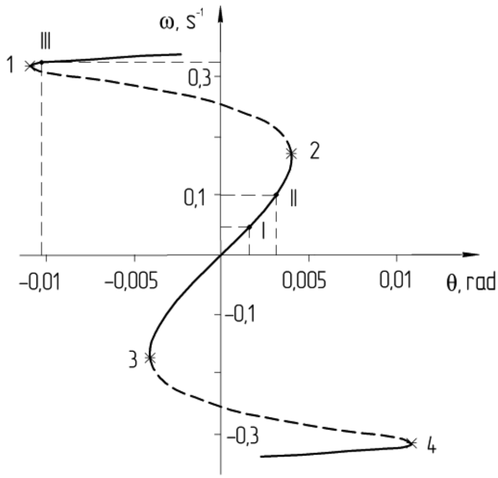

Fig. 2. Dependence $\omega = f(\theta)$

For clarification of parameters of the got stationary modes will consider three from them, that to in obedience to fig. 2 is proof. The modes have the followings parameters:

u_I = -0,496 m/s; $\omega_I = 0{,}065\ s^{-1}$; θ_I = 0,0021 rad;
u_{II} = -1,092 m/s; $\omega_{II} = 0{,}133\ s^{-1}$; θ_{II} = 0,0037 rad;
u_{III} = -7,387 m/s; $\omega_{III} = 0{,}319\ s^{-1}$; θ_{III} = -0,0108 rad.

For each of these modes will define the corners of taking of front and back axes [Pevsner J.M., 1947]of, which can serve as the certain criterion of practical realization of the proper withstand motions:

$$\delta_1 = \theta - \frac{u + a\omega}{v}; \qquad \delta_2 = \frac{-u + b\omega}{v}.$$

The next values of corners of taking are got:
Mode I – δ_1 = 1,16˚; δ_2 = 1,43˚.
Mode II – δ_1 = 2,54˚; δ_2 = 3,12˚.
Mode III – δ_1 = 16,83˚; δ_2 = 19,37˚.

Obviously, that realization of the third mode needs certain experimental confirmations, that it is related to the large enough values of corners of taking. It should be noted that in opposition to the cases turn of car to the right, for a case III, needs turn of steering wheel. For the first two modes the trajectories of motion of centre-of-mass car are numeral built (fig. 3).

Thus radiuses of trajectories of C.M. it is possible approximately to define after a formula $R = v/\omega$: for the mode I – R_I = 354 m; for the mode II – R_{II} = 173 m, III – R_{III} = 72 m (radius of trajectory of point D of longitudinal ax which has a rate of longitudinal movement v = 23m/s). Distance of this point is from C.M. determined correlation L = u/ω: I – L_I = -7,6 m, II – L_{II} = - 8,2 m, III – L_{III} = - 23,2 m, that explains the numeral coinciding of radiuses of C.M. and points of D, or their divergence, and the radius of CM is determined by $R_C = \sqrt{R^2 + L^2}$: R_{CI} = 353,9 m; R_{CII} = 173,1 m; R_{CIII} = 75,7 m.

The values of radiuses are resulted higher coincide with those which are got on the basis of numeral integration. For finding out of influence of longitudinal force of X1 on fig. 3 the fragments of basic branches of bifurcation plurals are resulted for two cases: hauling force of X_1 = 0; hauling force of X_1 = 0. Will notice that the second and third equalizations of the system (1) were utilized in last case only, in what X_1 = 0.

Fig. 3 illustrates differences between the bifurcation plurals of frontpulling and rearpulling cars. But it follows to notice that in this case influence of hauling force is not taken into account on the size of coefficients of resistance of taking, that is why have «unchanging» critical speed of rectilineal motion $v_{cr}^2 = k_1 * k_2 * l * g/(k_1 - k_2)$; ($v_{cr}$ = 28,1 m/s).

For additional research of bifurcation plurals and their verification it is necessary to choose control parameters θ and v, values of which are in near of border of bifurcation plurals.

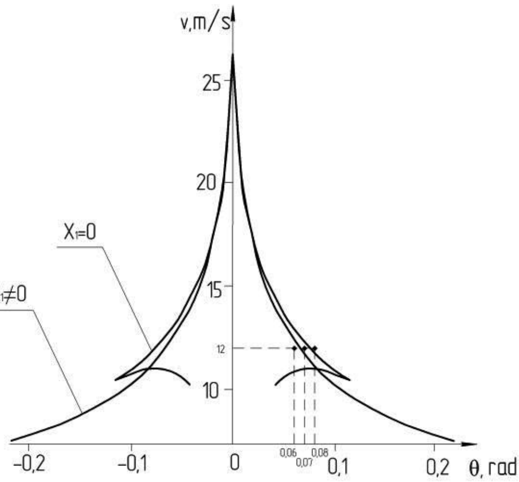

Fig. 3. Comparison of bifurcation plurals

Will consider the graph $X_1 \neq 0$ (look on fig. 3), in quality an example elect speed of car v = 12 m/s but value of corner of turn of the guided wheels θ = 0,06 rad (it is under a curve) and θ = 0,07 rad (it is outside the graph which is examined).

For the select guided parameters numeral integration is build phase trajectories narrow-mindedness of which specifies on realization of proof stationary motions; the proof stationary modes are answered by the circular trajectories of centre-of-mass car inplane road (fig. 4).

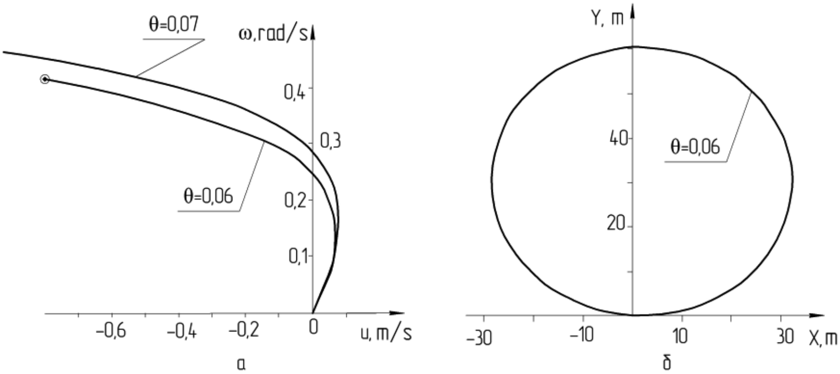

Fig. 4. Phase trajectories (a) and trajectories of motion of centre-of-mass (b) at v = 12 m /s and $X_1 \neq 0$

From fig. 4, *a* evidently, that at θ = 0,06 advices have the proof mode of motion with parameters (u = - 0,699; ù = 0,397), this mode is answered by the circular trajectory of C.M., which is resulted on rice. 4; at the value of θ = 0,07 advices a phase trajectory is unlimited, it specifies or on absence of the proof modes, or on that beginning of co-ordinates does not belong to the area of attracting of the proof mode (from lines. 1 swims out in actual fact, that for the proper guided parameters steady-states absent in general).

In case $X_1 = 0$ elect the guided parameters: v = 12 m/s, θ = 0,07 рад (it is under the basic branch of bifurcation of curve); v = 12 m/s, θ = 0,08 rad (it is outside of bifurcation plural).

On fig. 5 a phase trajectory (a) and trajectory of centre-of-mass (a), which are built for the case of v = 12 m/s, θ = 0,07 rad.

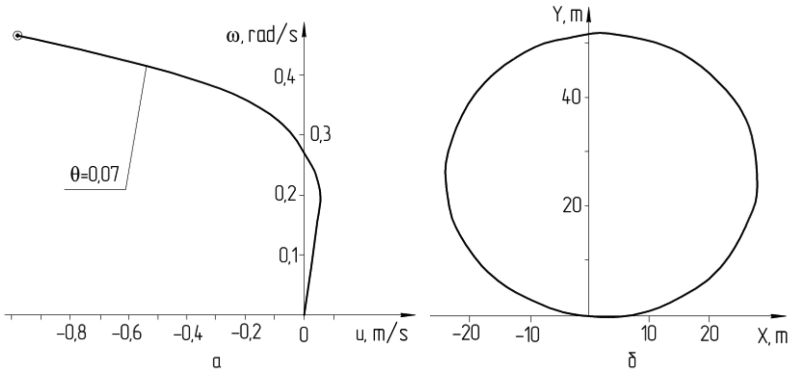

Fig. 5. Phase trajectory (a) and trajectory of motion
of centre-of-mass (b) at v = 12 m/s, θ = 0,07 rad ($X_1 = 0$)

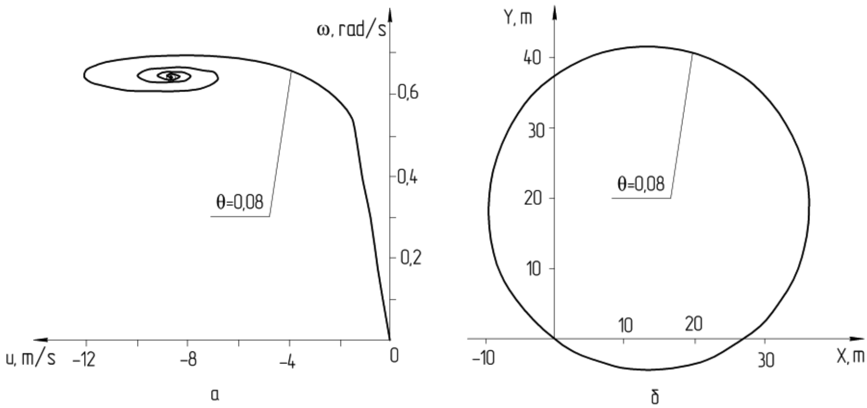

Fig. 6. Phase trajectory (a) and trajectory of motion of centre-of-mass (b) at
v = 12 m/s, θ = 0,08 rad ($X_1 = 0$)

From fig. 5, a evidently, that at the fixed guided parameters v = 12 m/s, $\theta = 0{,}07$ rad a car goes out on the proof circular mode (u = - 0,998 m/s; $\omega = 0{,}465$ rad/s), inplane road he is answered by the circular trajectory of CM (resulted on fig. 5, б).

On fig. 6 a phase trajectory and trajectory of centre-of-mass, which are built for a case is resulted v = 12 m/s, $\theta = 0{,}08$ rad.

Fig. 6 specifies on realization of the proof mode (u = - 8,6800 m/s; $\omega = 0{,}6454$ rad/s), although coming from bifurcation plurals (fig. 3), it would follow to expect the unsteady mode. A similar situation can testify to the presence of additional (or additional) branches of bifurcation plurals and in case X1 = 0.

These additional stationary modes do not have a practical value, because the corners of taking on axes exceed 40 degrees ($\delta_1 = 0{,}7388, \quad \delta_2 = 0{,}7889$).

CONCLUSIONS

Hauling force influences on descriptions of course firmness of passenger car – changes the type of bifurcation plural (integral description of firmness of all to pluralness of the stationary modes of motion). For the analysis of influence of hauling force on descriptions of firmness additional research of phase portraits (task of estimation of areas of attracting of the proof stationary modes) which would answer the areas of plane of the guided parameters with the different amount of the stationary modes is needed in large.

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ВЛИЯНИЕ ТЯГОВОЙ СИЛЫ НА УСТОЙЧИВОСТЬ МНОЖЕСТВА СТАЦИОНАРНЫХ ДВИЖЕНИЙ МОДЕЛИ ЛЕГКОВОГО АВТОМОБИЛЯ

Владимир Сахно, Александр Кравченко,
Андрей Костенко, Владимир Вербицкий

Аннотация. В статье рассмотрен вопрос влияния тяговой силы, которая действует на передней оси легкового автомобиля, на множество стационарных движений легкового автомобиля. Установлено, что наличие тяговой силы существенно изменяет диаграмму устойчивости (бифуркационное множество).

Ключевые слова: автомобиль, курсовая устойчивость движения, множество бифуркационное, траектория фазовая, траектория движения.