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EVALUATION OF SWEET CORN CUTTING PROCESS BASED ON PROFILE ANALYSIS

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Summary

In the paper profile analysis for evaluation of process of sweet corn cutting is considered. Likelihood ratio test procedures for three hypotheses about "parallelism", "level hypothesis" and "no condition variation" are given. Data on sweet corn cutting process are analysed for unit power consumption ($kW \cdot cob^{-1}$) and weight percentage of kernels cut off (%). Profiles for linear velocities of cob feeder are assumed.

Key words and phrases: profile analysis, testing hypotheses, sweet corn cutting process

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1. Introduction

Sweet corn, whose cultivation area is still increasing, is planted for direct consumption and processing. Its supply growth imposes higher and higher equality requirements concerning this material. To some extent, this problem is sorted out through continuous substitution of the current cultivars with new ones, assuring better quality of the material. Corn quality comprising the physical and chemical properties of a corn-cob and kernels depends on many factors, among others, maturity grade, genotype, variety, storage conditions as well as weather course, fertilization and agrotechnical practices (Wong et al., 1994).

Sweet corn as raw material for the processing industry is harvested in the phase of late-milk ripeness. At this phase a kernel contains the largest amounts of nutrients and is characterized by a low dry mass content between 24-28% (Olsen, 2000). As opposed to physiological maturity, a kernel hasn't got a natural separation boundary. Soft kernels adjoin closely to one another and the corn-cob core, that affects negatively the process of their detachment from the corn-cob core. Sweet corn kernels are obtained for processing (canning, freezing) through their mechanical cut off from corn-cob cores by special machines. This process, however, causes substantial qualitative and quantitative losses of kernels (Hanna et al., 1988).

Sweet corn as material for the food industry is characterized by an unfavorable ratio between the acquired parts (kernel) and refused parts (cover leaves, corn-cobs). Kernel crop reaches only 30-40% of total cob mass. Depending on a variety, a kernel is cut off from a corn-cob in 35-55%, whereas the amount of cut off kernels is strictly connected with the moisture and the physical and morphological properties of a kernel and a cob (Feibert and Shock, 1996). In this paper, evaluation of the cutting process of sweet corn kernels from the corn-cob will be done by using profile analysis. The profile analysis permits deeper analysis of experiment and gives answer for the question whether profiles for some groups of objects are similar. Profile analysis is a well-known method, considered in many papers, for instance, in Srivastava (1987, 2002), Morrison (1967), Greenhouse and Geisser (1959). For statistical analysis of profiles for the different groups, the multivariate analysis of variance is used.

2. Method of experimentation and statistical model

The experimental material was made up by sweet corn-cobs of the standard sugary variety Candle. The cobs for the study were collected by hand from random sites of the plantation at late-milk ripeness phase with the moisture of kernels about 74.6%. The corn-cobs selected for tests were healthy, of straight shape and high degree of kernel filling.

The process of kernel cutting was characterized by the following variable: unit power consumption ($kW \cdot cob^{-1}$) and weight percentage of kernels cut off (%). These variables were determined by using four angular velocities of cutter knife ($rad \cdot s^{-1}$): 167.5, 201.0, 234.6, 268.1 and three linear velocities of cob feeder ($m \cdot s^{-1}$): 0.31, 0.61, 0.92. In the paper, we consider angular velocities of cutter knife as tests, and as groups - linear velocities of cob feeder.

Let us suppose that we would like to compare K tests (angular velocities), containing J groups (linear velocities) with n observations for each combination. Let y_{ijk} $(i = 1, \ldots, n;\ j = 1, \ldots, J;\ k = 1, \ldots, K)$ denote the measured response of the *i*th observation in the *j*th group for the *k*th test. Next, let $\mathbf{y}_{ij} = [y_{ij1}, \cdots, y_{ijK}]'$ be the $(K \times 1)$ vector having *K*-variate normal distribution $N_K(\boldsymbol{\mu}_j, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}_j = [\mu_{j1}, \ldots, \mu_{jK}]'$ $(j = 1, \cdots, J)$ is the expectation but $\boldsymbol{\Sigma}$ is unknown covariance matrix.

According to Srivastava (2002), we concentrate on three problems. The first one is to consider whether profiles for groups are parallel. The second one, assuming parallelism, is to find the distances between profiles and to check the significance between them. The third problem is connected with parallelism of the profiles to the *x*-axis.

2.1. Test for similarity of profiles

The profiles for different groups are parallel if the following hypothesis is true

$$H_1^0 : \begin{cases} \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \gamma_1 \mathbf{1}_K \\ \vdots \\ \boldsymbol{\mu}_{J-1} - \boldsymbol{\mu}_J = \gamma_{J-1} \mathbf{1}_K \end{cases},$$

where $\mathbf{1}_K$ is a vector of K ones, γ_i $(i = 1, \ldots, J-1)$ represents the distance between the *i*th and (*i*+1)th group. To test the hypothesis H_1^0 we use the following statistic (see Srivastava, 2002, p.233):

$$\lambda_1 = \frac{|\mathbf{C}\,\mathbf{SSE}\,\mathbf{C}'|}{|\mathbf{C}(\mathbf{SSE} + \mathbf{SSTR})\mathbf{C}'|}, \tag{2.1}$$

where $\mathbf{C}$ is a $(K-1)\times K$ contrast matrix of the form $\mathbf{C}=\begin{pmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -1 \end{pmatrix}$, satisfying condition $\mathbf{C1}_K=\mathbf{0}_{K-1}$, $\mathbf{0}_{K-1}$ is null vector of the size (K-1), $\mathbf{SSTR}=\sum_{j=1}^{J} n\overline{\mathbf{y}}_{.j}\overline{\mathbf{y}}'_{.j}-nJ\,\overline{\mathbf{y}}_{..}\overline{\mathbf{y}}'_{..}$, $\mathbf{SSE}=\sum_{j=1}^{J}\sum_{i=1}^{n}\mathbf{y}_{ij}\mathbf{y}'_{ij}-\sum_{j=1}^{J}n\overline{\mathbf{y}}_{j.}\overline{\mathbf{y}}'_{j.}$, and $\overline{\mathbf{y}}_{.j}=n^{-1}\sum_{i=1}^{n}\mathbf{y}_{ij}$, $\overline{\mathbf{y}}_{..}=(nJ)^{-1}\sum_{j=1}^{J}\sum_{i=1}^{n}\mathbf{y}_{ij}$.

Using Corollary 6.2.1 in Srivastava (2002, p. 180), we know that $-[J(n-1)-\frac{1}{2}(K-J+1)]\ln\lambda_1$ has asymptotically χ^2 distribution with (K-1)(J-1) degrees of freedom.

2.2. Tests for profile distances and confidence intervals

If the hypothesis H_1^0 is not rejected, then the profiles for J groups are parallel. In such a case, we test a hypothesis that distances between profiles are not significant. This hypothesis can be described as follows

$$H_2^0:\boldsymbol{\gamma}=[\gamma_1,\dots,\gamma_{J-1}]'=\mathbf{0}_{J-1}.$$

The hypothesis H_2^0 can be tested using the following statistics (Srivastava, 2002, p. 233)

$$\lambda_2=\frac{1}{\lambda_1}\frac{|\mathbf{SSE}|}{|\mathbf{SSE}+\mathbf{SSTR}|}. \tag{2.2}$$

After the transformation (see, Srivastava, 2002, p.180) of λ_2, we get

$$\frac{(J(n-1)-K+1)}{J-1}\frac{1-\lambda_2}{\lambda_2}=F_{J-1,J(n-1)-K+1}.$$

When the hypothesis H_2^0 is rejected then we can estimate $\boldsymbol{\gamma}$ and calculate confidence intervals for each coordinate γ_j $(j = 1, \cdots, J-1)$. The maximum likelihood estimate of $\boldsymbol{\gamma}$ is given by the formula (see, Srivastava, 2002, p. 233):

$$\tilde{\boldsymbol{\gamma}} = \frac{\mathbf{Z}'\mathbf{SSE}^{-1}\mathbf{1}_K}{\mathbf{1}'_K\mathbf{SSE}^{-1}\mathbf{1}_K}, \tag{2.3}$$

where $\mathbf{Z} = \left(\overline{\mathbf{y}}_{.1} - \overline{\mathbf{y}}_{.2}, \ldots, \overline{\mathbf{y}}_{.(J-1)} - \overline{\mathbf{y}}_{.J}\right)$.

A simultaneous $(1-\alpha)100\%$ confidence interval for γ_j has a form (Srivastava, 1987)

$$\mathbf{a}'_j\tilde{\boldsymbol{\gamma}} \pm T_\alpha \sqrt{\frac{\mathbf{a}'_j\left[\mathbf{A} + \mathbf{Z}'\mathbf{C}'\left(\mathbf{C}\,\mathbf{SSE}\,\mathbf{C}'\right)^{-1}\mathbf{CZ}\right]\mathbf{a}_j}{\mathbf{1}'_K\,\mathbf{SSE}^{-1}\mathbf{1}_K}}, \tag{2.4}$$

where $T_\alpha = \frac{J-1}{J(n-1)-K+1} F_{J-1,J(n-1)-K+1,\alpha}$, $\mathbf{A} = \frac{1}{n}\mathbf{1}_{J-1}\mathbf{1}'_{J-1} + \frac{1}{n}\mathbf{I}_{J-1}$, $\mathbf{I}_{J-1}$ is an identity matrix of the size (J-1), and $\mathbf{a}_j$ denotes a (J-1) vector having the only jth coordinate equal to one and the rest are zeros.

2.3. Test a hypothesis about condition variation

Let us suppose that profiles for all groups are parallel (the hypothesis H_1^0 is not rejected). The hypothesis about parallelism of profiles for all groups to x-axis can be described as follows

$$H_3^0 : \boldsymbol{\mu}_J = \delta\mathbf{1}_{K\text{-}1},$$

where δ is unknown constant. The hypothesis H_3^0 is rejected if

$$\frac{nJ(nJ-K+1)}{K-1}\overline{\mathbf{y}}'_{..}\mathbf{C}'\left[\mathbf{C}\left(\mathbf{SSE} + \mathbf{SSTR}\right)\mathbf{C}'\right]^{-1}\mathbf{C}\overline{\mathbf{y}}_{..} \geq F_{K-1,nJ-K+1,\alpha}. \tag{2.5}$$

3. Numerical example

For the experiment described in Section 2 we consider three groups of linear velocities of cob feeder ($\mathrm{m \cdot s^{-1}}$): 0.31, 0.61, 0.92 and four tests as angular velocities of cutter knife ($\mathrm{rad \cdot s^{-1}}$): 167.5, 201.0, 234.6, 268.1 repeated on 60 corn-cobs. Then we have $J = 3$, $K = 4$, $n = 60$. The averages of the unit power consumption of kernel cutting, calculated over 60 replications are shown in Table 1 and the profiles for three linear velocities of cob feeder are illustrated in Figure 1.

Table 1. The average power of consumption for the kernel cutting ($\mathrm{kW \cdot cob^{-1}}$)

Group Linear velocity	Angular velocity of cutter knife (**test**)				**Group Mean**
	167.5	201	234.6	268.1	
0.31	0.71	0.61	0.51	0.38	**0.55**
0.61	0.67	0.58	0.48	0.36	**0.52**
0.92	0.62	0.53	0.44	0.31	**0.47**
Test mean	**0.66**	**0.57**	**0.48**	**0.35**	**0.52**

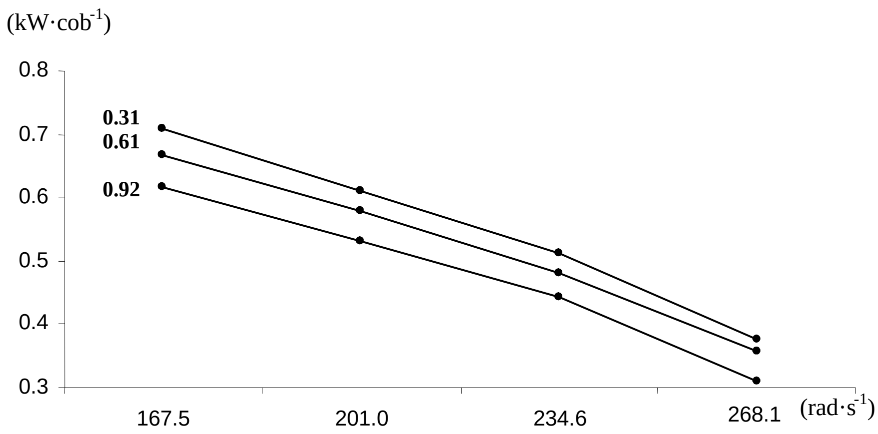

Fig. 1. The profiles of groups for linear velocity

For the whole data set, the matrices appeared in test functions described in (2.1), (2.2), (2.3), (2.4) and (2.5) are equal, respectively,

$$\mathbf{SSE} = \begin{pmatrix} 0.537 & 0.024 & 0.024 & -0.094 \\ 0.024 & 0.388 & -0.011 & -0.045 \\ 0.215 & -0.011 & 1.556 & -0.068 \\ -0.094 & -0.045 & -0.068 & 0.385 \end{pmatrix}, \quad \mathbf{SSTR} = \begin{pmatrix} 0.245 & 0.215 & 0.169 & 0.181 \\ 0.215 & 0.215 & 0.148 & 0.162 \\ 0.169 & 0.148 & 0.118 & 0.123 \\ 0.181 & 0.162 & 0.123 & 0.141 \end{pmatrix}.$$

The proper test functions for the hypotheses, critical values and decisions about rejection of the hypotheses are presented in Table 2.

Table 2. The results of testing the hypotheses

Hypothesis	Test function	Critical value	Decision
H_1^0	5.44 ($\lambda_1 = 0.97$)	$\chi^2_{(K-1)(J-1)}$ =12.59	not reject
H_2^0	136.09 ($\lambda_2 = 0.39$)	$F_{J-1,n-J-K+1} = 3.05$	reject
H_3^0	1075.33	$F_{K-1,nJ-K+1,0.05} = 2.66$	reject

The results presented in Table 2 show that the hypothesis about parallelism, H_1^0, is not rejected. We can not conclude that profiles for different linear velocities of cob feeder are not parallel. Moreover, the second hypothesis is rejected, so the vector of distances between profiles is not null. It is useful to estimate $\boldsymbol{\gamma}$ using formula (2.3) and to calculate confidence intervals for the coordinates of $\boldsymbol{\gamma}$ using (2.4). The estimate of $\boldsymbol{\gamma}$ and 95% confidence intervals are equal: $\tilde{\boldsymbol{\gamma}} = [0.029, 0.049]$, $\gamma_1 \in (0.017, 0.040)$ and $\gamma_2 \in (0.036, 0.058)$.

In the experiment, the weight percentage of kernels cut off (%) was also examined. For the data set we have got the averages shown in Table 3 and profiles illustrated in Fig. 2.

Table 3. The average weight percentage of kernels cut off (%)

Group Linear velocity	Angular velocity of cutter knife (**test**)				**Group Mean**
	167.5	201	234.6	268.1	
0.31	52.64	52.99	62.37	65.58	**58.40**
0.61	50.8	51.22	60.2	63.52	**56.43**
0.92	46.07	50.09	60.78	59.45	**54.10**
Test mean	**49.84**	**51.43**	**61.12**	**62.85**	**56.31**

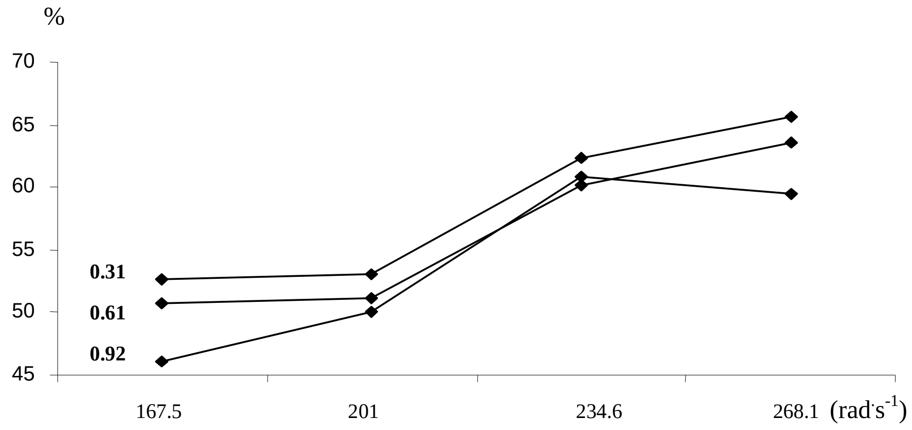

Fig. 2. The profiles of groups for linear velocity

However, the test function for the hypothesis H_1^0 reached the value 22.17, which is greater than critical value 12.17. Then we conclude that the profiles for three groups of linear velocities of cob feeder are not parallel. This conclusion does not allow for testing the other hypotheses.

4. Conclusions

Profile analysis can be used to describe cutting process of corn-cobs. Using this analysis, we have shown that the profiles for groups of linear velocities of cob feeder are similar. This means that greater linear velocity causes a constant difference of the unit power of consumption. Moreover, we also proved that the differences between profiles are statistically significant. We have shown that increasing linear velocity of cob feeder and increasing angular velocity of cutter knife causes greater unit power of consumption. Furthermore, we proved that linear velocities are not parallel to the x-axis. Confidence intervals for the distances of profiles give the detailed analysis between the linear velocities of cob feeder. However, profile analysis applied to the weight percentage of kernels cut off has shown that profiles for groups of linear velocities of cob feeder are not similar.

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OCENA PROCESU CIĘCIA KUKURYDZY OPARTA NA ANALIZIE PROFILOWEJ

Streszczenie

W pracy wykorzystujemy analizę profilową do opisu procesu cięcia kolb kukurydzy. Podajemy funkcje testowe do weryfikacji trzech hipotez o równoległości profili, o istotnych odległościach pomiędzy profilami oraz o równoległości profili do osi odciętych. Dane dotyczące procesu cięcia kukurydzy analizujemy dla jednostkowego zużycia mocy ($kW \cdot cob^{-1}$) i procentu odciętej masy ziarna (%). Jako profile przyjmujemy liniową prędkość podajnika kolb.

Słowa kluczowe: analiza profilowa, testowanie hipotez, proces cięcia kukurydzy

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