ZESZYTY PROBLEMOWE POSTĘPÓW NAUK ROLNICZYCH 1976, z. 168

SUBSTANCIAL CHARACTERIZATION OF SOILS WHILE BALLASTED

Wacław Kołodziej, Jan Piróg

Warsaw

The variability of existing resistances produced by the soil against agricultural tools, also against the pressing action of tractor-wheels, agricultural machines and all kind moving in these conditions wehicles creates the necessity to undertake the theoretical as well as the experimental investigations in this range.

It is related to the output and performance of agricultural machines as well as to the reguired motional proprieties in the case of terrain vehicles necessary in agricultural cultivations, forestry, mining industry, quarry-plants, building industry etc.

Mathematical dependencies presenting connections between loads and soil deformations, actually used in consideration of all problems of the agricultural tools, agricultural machines, terrainability of vehicles do not consider in substance the soil acting body dimensions or are doing it with insufficient precision. It makes the essential obstancle in elaboration of correct method in computation of vehicles and agricultural mechanisms and causes that there appear difficulties in the theoretical description of influence of some constructional parameters on the working performance of the a.m. vehicles and mechanisms. It results from this that, for instance, wheel-vehicles are aquipped with different in dimensions tyres. The influence of these dimensions on the process of the characterization of wheel reaction on the soil, as essential one, ought to be considered. Independently of this, the publicated mathematical subordinations to the combinations of loads with deformations indicate a considerable number of divergences of processes of the real and shaped ground material characterizations. These problems have been discussed, among others, in papers [12, 14].

On the background of physical description on the phenomenon of ground deformation as result of vertical load operation, in this report there will be presented by one of the authors [12] the theoretical investigations from the sphere of creating the new mathematical formulae.

PHYSICAL PICTURE OF SOIL DEFORMATION UNDER ACTIVITY OF VERTICAL LOAD

Under the idea of soil, we understand in this report exclusively the surfacial coating of the earth, created from matrix as the result of reciprocal interaction of matrix, climate, ground configuration, vegetation and animals more rarely as the result of human activity.

The soils, described by this means, are next divided into:

1. Deformable soils consisted mainly of rocks'decay products;
2. Undeformable soils consisted of solid rocks.

In conducted considerations the only deformable soils are interesting for us. That is why we will name them in shortening — soils.

Assuming that the analysis of interesting us mechanical occurrences that take place in soils is not possible without recognition of many of their physical features, the discussion will be autdistanced by some informations concerning the konwledge of soils.

Generally each soil can de considered as granulated three-phase medium created by:

1) solid substance,
2) water,
3) air.

The solid substance generally consist of so-called mineral structure and organic substances creating together combination of granules and elements, called the soil structure.

According to Priklonski [11] this structure is conditioned by:

a) dimension, shape and kind of surfaces of particular rock's fragments appearing in ledge,
b) dislocation and their reciprocal relation,
c) character of internal elementary forces.

According to Nitzsche [18], the exemplary proportional participation of particular phases for the cultivated soil is shown in Fig. 1.

Three-phase composition of soil is the reason of many mechanical phenomenons occurred. So, for instance, the soil deformations as the result of external forces first of all are the subject to existence of free spaces between mineral grains, while water fulfilling the free spaces is resulting on the course of these deformations in function of time. After complete fulfilling of free spaces with water any deformations could come into being only thanks to removed water, and if the granulation of

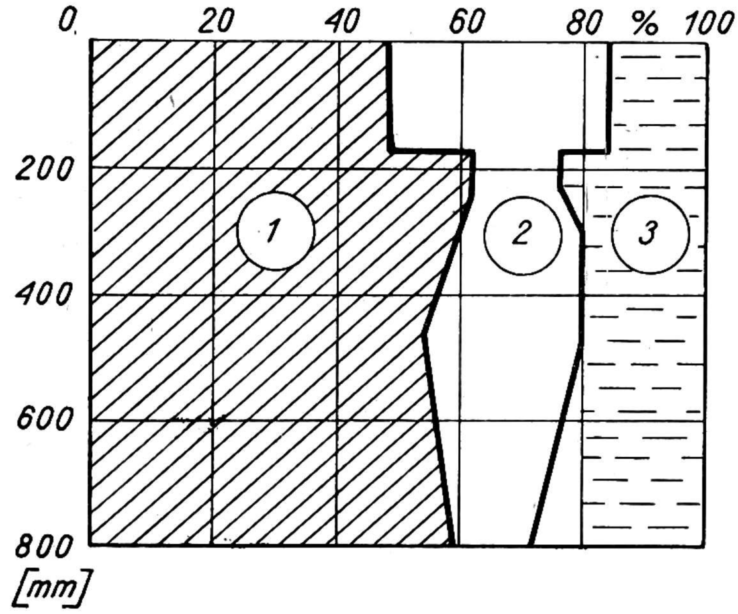


Fig. 1. Three-phase composition of soil: *1* — solid substance, *2* — air, *3* — water

soil is smaller, the process of coming out of water will be slower. Slower will be also the process of deformations which arise as result of decreasing of the free spaces volume. In pedology the volume of free spaces between granules is measured by porosity of soil and the contents of water is described with the help of so-called humidity of soil. As it results from the above, the soils in quite real manner are differentiating from continuous mediums, because both mineral grains and granules of soil do not make the homogeneous compact mass but they are separated by air and water. The experience shows that this fact does not eliminate the possibility of application in mechanics of soil the methods om analysis based on mechanics of continuous mediums.

It demands only the assumption to be made that investigated elementary volumes of subsoil are extremely large in comparision with dimensions of particles.

This conditions in examination of influences of rolling mechanism of vehicles and cultivating tools on soils is realised in full and that is why this method will be willingly exercised by us in further considerations. This standpoind is also confirmed by fact that, against appearances, it is the macroscopical method based on physical measurable dimensions without penetrating into microscopical phenomenons. The measurable physical dimensions are appearing here as resultants of average interactions of microscopical unmeasurables. From other hand side we have to realise that not all methods of the mechanics of continuous mediums are usefull for analysis of soil propriety. It becomes evident that the method of linear elasticity theory cannot be concretely adopted for these purposes. So, this fact is influencing the choice of method of further procedure.

From this, what was said, above it results undoubtly that further considerations must proceed towards the construction of physical combinations connecting tensions with deformations, presented normally under the form of so-called material characterizations with the respective mathematical formulation. Classical methods of building of this kind combinations are based on subjecting of the material sample to the examination of the homogenouos state of tension and deformation and establishing on this way dependencies between tension and deformation in the form of appropriate diagrams which nextly are subject to theoretical elaborations. Of course, for the full knowing the character of seeked physical rules the additional experimental investigations are realised in which the infliction of deformations or tensions is the certain function of time elapsed, for instance, the investigations at constant velocity of deformation, investigations at established value of tension (creep), or at last investigations at established deformation (relaxation).

However, we must be conscionus of the fact that the method of procedure used here is often (for instance: for sandy soils) impossible for realisation. That is why in mechanics of soils we have to go in direction of investigations of their behaviour in concrete boundary conditions what in this consideration means external interaction on the half-space of the investigated medium. So, there appears the problem of necessity to build the material characterization of soil in case of three-axial state of tension. The only way conducting to this target is preparation of the diagram on the basis of comparision of the intensivity of tensions and deformations in possibly simple and homogeneous state of compressing tensions that take place in three-axial conditions.

These investigations in laboratory conditions have not been made unfortunately many times. However, these investigations clearly indicate the possibility of using them in other investigations conducted directly in natural conditions, based on pressing the respective props into soil and registration of dependence of their sinking upon the loading forces. The last investigations have also the additional virtue that describe the soil attributes in natural state, so they reflect better the real state.

To results of this kind of experimental investigations we reffer on schematic diagram in Fig. 2 and we mention them in Figs. 3-12. From comparision of Fig. 3 and Fig. 10 results that the curves of diagram have common features of geometrical visual similarity, and the soil has classical non-linear characterization. The features of similarity, that ought be presented firstly, are:

1) tangent of the pitch angle of curve $\sigma_i = f_1(\varepsilon_1)$ of of curve $p = f(z)$ to obscissa axle has always finite value decreasing, as the axial deforma-

Material characteristics of load	MECHANIC INDICATORS OF SOIL	SORT OF SOIL
actual	diagram $\sigma_i = f(\varepsilon_1 = \varepsilon_z)$ — R.L. Kondner [15], Fig. 3;	Loam, structure disturbed
	diagram $\sigma_i = f(\varepsilon_1 = \varepsilon_z)$ — R.L. Kondner [15], Fig. 4;	loam, variable preconsolidation coefficient
	diagram $\sigma_i = f(\varepsilon_1 = \varepsilon_z)$ — K.Y.Lo [17] Fig. 5	loam, sample raliet test
	diagram $\tau_{okt}/\sigma_{śr} = f(\gamma_{okt})$ — A.S. Stroganow [24], Fig. 6;	sands, differents degree of condensation
	diagram $\tau_{okt}/\sigma_{śr} = f(\varepsilon_i)$ — A.S. Stroganow [25], Fig. 7;	sand
	diagram $\sigma_{śr} = f(3\varepsilon_{śr} = I_1)$ — A.S. Stroganow [24], Fig. 8;	sands, different degree of condensation
	diagram $I_1 = 3\varepsilon_{śr} = f(\gamma_{okt})$ — A.S. Stroganow [24], Fig. 9;	sands, different degree of condensation
	diagram $p = f(z)$ — S.S. Saakjan [22] Fig. 10	clayey
	— S.S. Korczunow [1]	peat subsoils
	diagram $z = f(b)$ — Press [9], Fig. 11;	clayey, sand
	diagram $z = f(\sqrt{F})$ — M.H.M. Azmi [4], Fig. 12	fluffed dry sand
	— W.A. Andrejew [3],	snow, depth 1 [m]
modelated	diagram $\sigma_i - f(\varepsilon_i)$ — elasticly — plastic model, rigidly — elastic model. Fig. 13	
	$p = k \cdot z$ — Gerstner [10] — 1813;	uny
	$p = (a_1 \cdot u + a_2 \cdot F) \cdot \sqrt{Z} = k \cdot \sqrt{Z}$ — R. Bernstein [8] — 1913;	uny
	$p = k \cdot z^n$ — M.N. Letoszniew [16] — 1929;	uny
	$p = (k_1/b + k_2) \cdot Z^n$ — M.G. Bekker [5, 6, 7] — 1956;	uny
	$p = k(z/b)^n$ — S.S. Saakjan [22] — 1956;	uny
	$p = p_0[1 - \exp(-Z/k)]$ — S.S. Korczunow [20] — 1963;	wet soils and peats
	$p = p_0 th(k/p_o)Z$ — W.W. Kacygin, M.E. Macepuro [20, 21] — 1963;	uny
	$p = p_o/1 + p_a k_1 \operatorname{arc\,tg} k_2$ — Ja S. Agiejkin [2] — 1968	uny
	$p = m_o \cdot Z/\sqrt{1 + m_o^2/p_o^2 Z}$ — W. Kołodziej [12] — 1971	uny

Fig. 2. Schematic diagram of experimental investigation

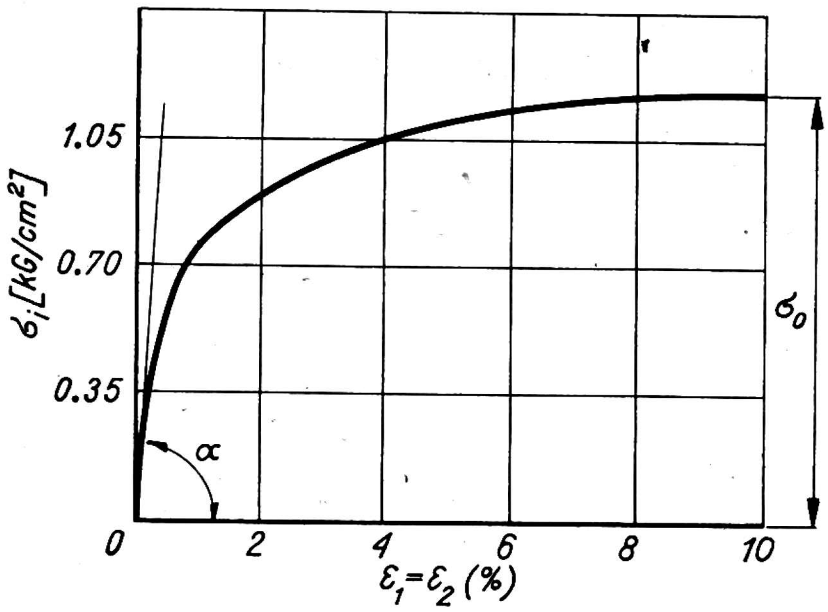


Fig. 3-12. Experimental results according to Fig. 2

Fig. 3.

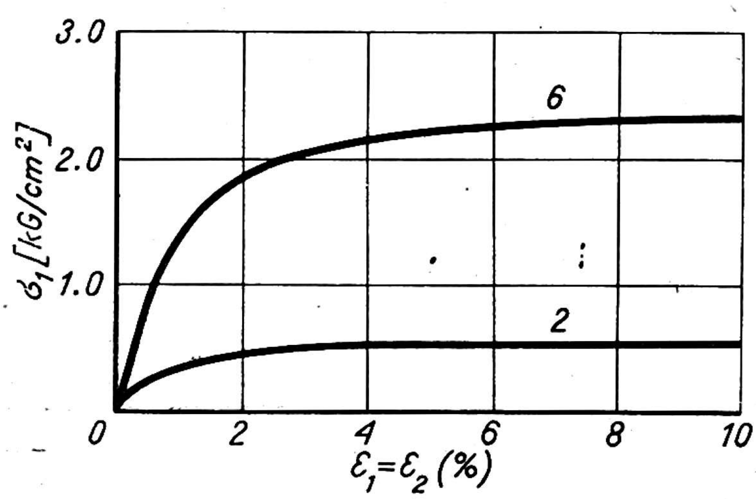


Fig. 4.

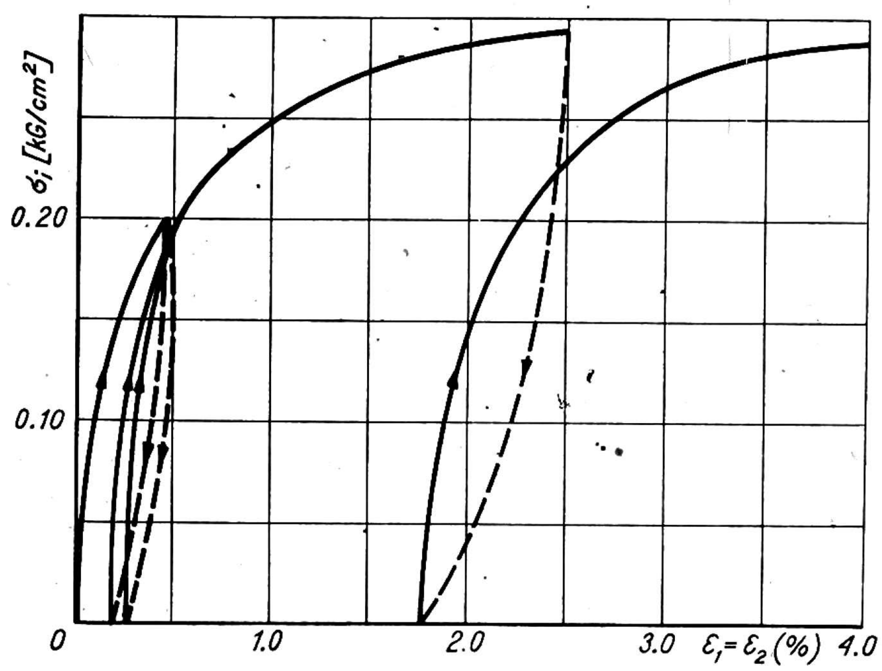


Fig. 5.

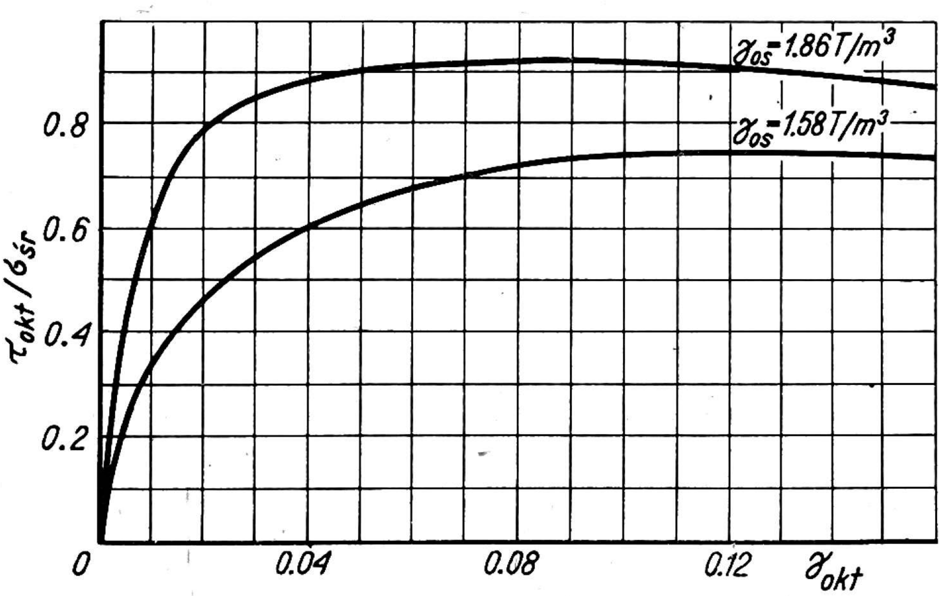


Fig. 6.

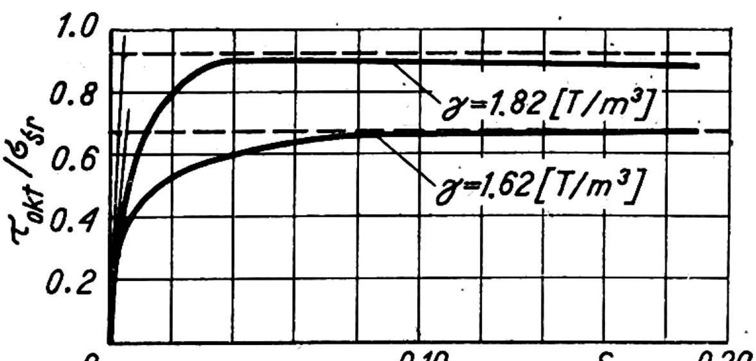


Fig. 7.

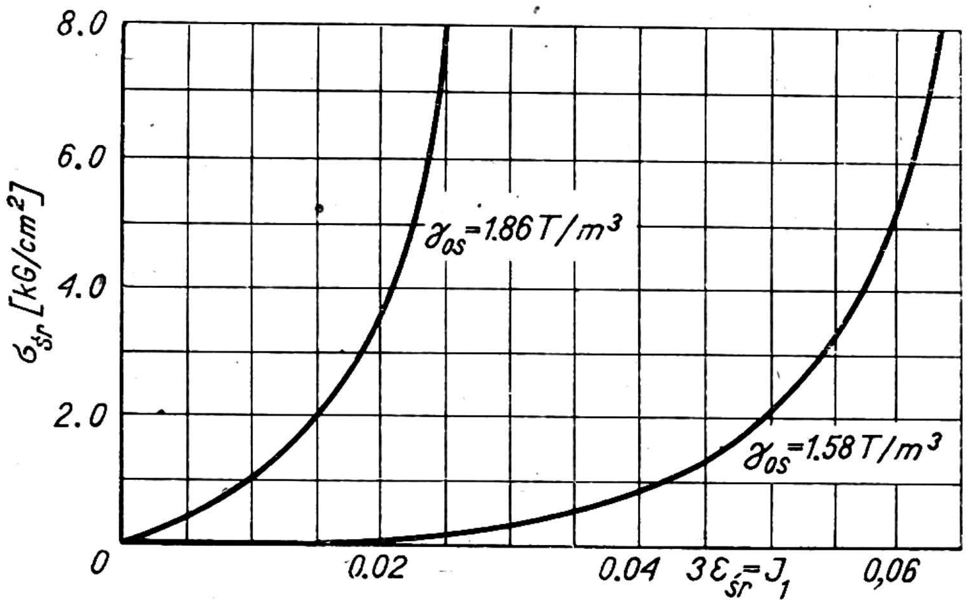

8.0
6.0
4.0
2.0
0
0.02
0.04
0,06
$\sigma_{śr}$ [kG/cm²]
γ_{os}=1.86 T/m³
γ_{os}=1.58 T/m³
$3\varepsilon_{śr}=J_1$

Fig. 8.

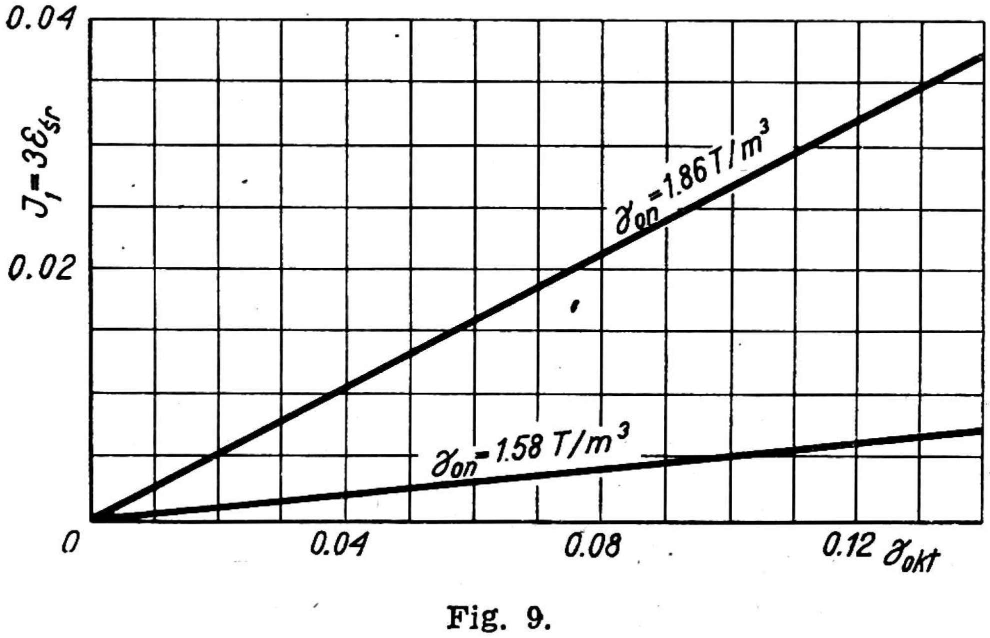

0.04
0.02
0
0.04
0.08
0.12
$J_1=3\varepsilon_{śr}$
γ_{on}=1.86 T/m³
γ_{on}=1.58 T/m³
γ_{okt}

Fig. 9.

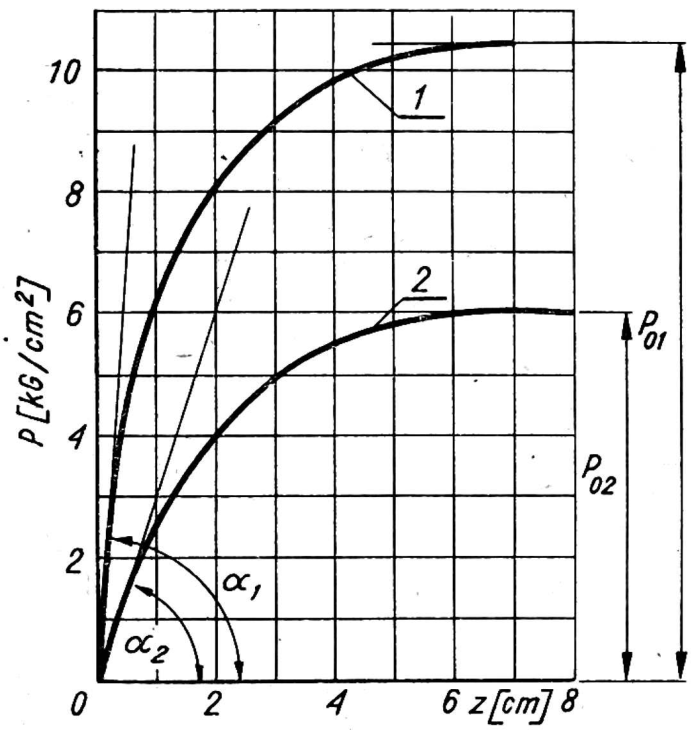


Fig. 10. 1 — acc. to Saakjan's research, for clayey soils, 2 — acc. to Korczunow's, for peat subsoils

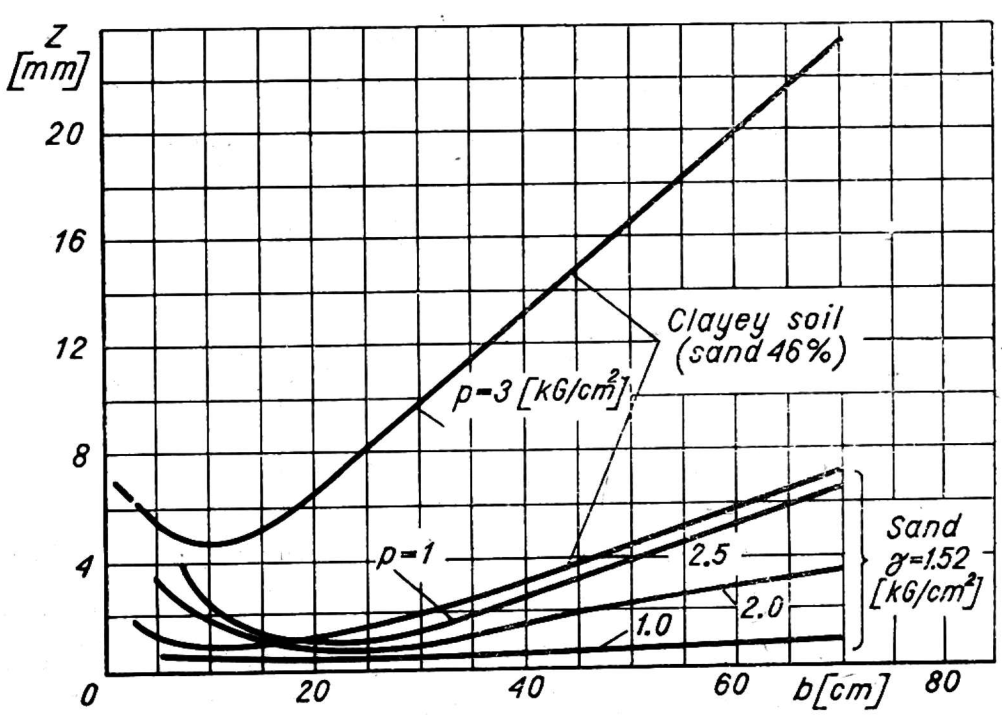


Fig. 11.

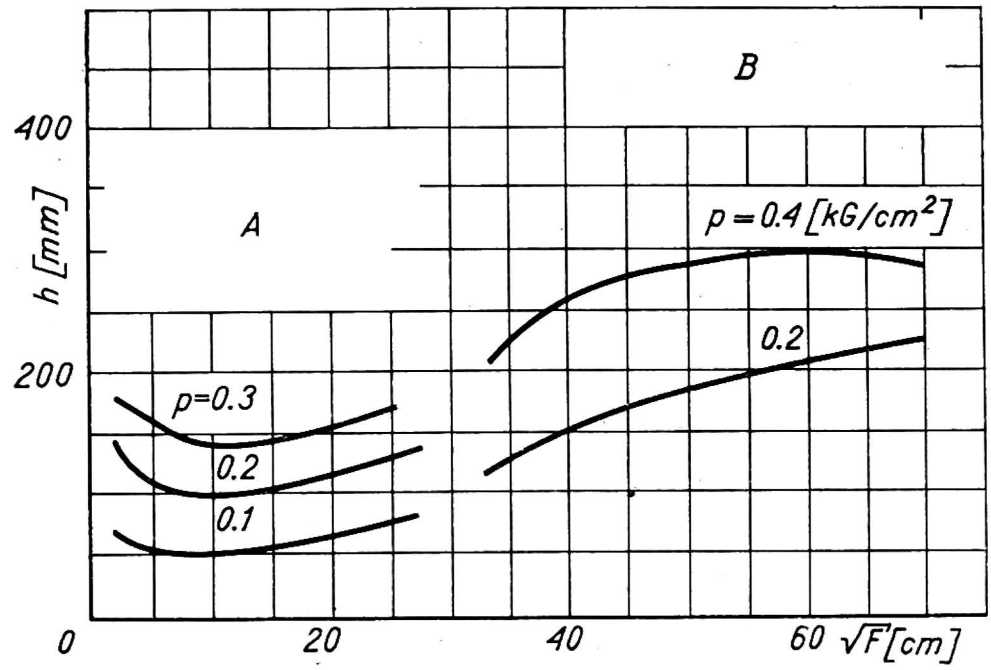


Fig. 12. *A* — fluffed dry sand acc. to Azmi's research [4], *B* — snow, of 1 m depth acc. to Andrejew's research [3]

tion increases, or the prop sinks, from the maximum value received on the beginning of the coordinate system up to the zero value reached at the unfinitively big deformations or sinkings of the pressed prop;

2) diagrams $\sigma_i = f_1\ (\varepsilon_1)$ or $p = f\ (z)$ are asymptotically convergent to limits σ_o or p_o, respectively, reached at infinite values of axial deformations ε_1 or sinking of the prop — z.

Beside this, the maximum magnitudes of tension σ_o or of unitary load p_o received during investigations can be accepted as the measure of soil durability in given conditions. By the formulation „given conditions" we understand in the case of diagram presented on Fig. 3, the side pressure existing in the chamber of measuring apparatus, and in case of diagrams given of Fig. 10 — the side push on halfspace exerted by the pale of soil situated under the pressed prop.

During analysis of diagrams there comes an idea to choose material constants of soil, that are determining the mathematical form of function.

$$\sigma_1 = f\ (\varepsilon_1) \text{ or } p = f\ (z).$$

We are of the opinion that these constants should be connected with values of pitch angle tangent curve of the diagram to abscissa axle in the begining of co-ordinate system and with maximum value of the permissible external load, what further will be exploitated. The interesting supplementary confirmation of shape of the material characterization diagram, presented on Fig. 3 and Fig. 10, are investigations of loam samples (Fig. 5) elaborated by K. Y. Lo [17], proceeded in more varying conditions as there were used by turns load relief.

As it results from Fig. 5, material characterization of particular cycles of the secondary loads fully preserve the shape described above. Changed are only: slightly — value of pitch angle tangent curves, and more clearly — reached values of permissible external loads, and at the same time increase of permissible external loads value vas easily foreseeable in next cycles.

From Fig. 5 results additionally that the material reliefing characterization has also non-linear complexion. Due to explain the course of deformation process we are analysing the physical phenomenons concomitant with soil deformations under influence of load, caused by a rigid prop which has described dimension of circular section. All indicates for this, from the macroscopic point of view, that there are three main phenomenons:

1) elastic deformation,
2) condensation of soil,
3) plastic streaming (displacement).

The last two from above mentioned phenomenons cause permanent deformations. The difference between them is based on fact that volume of deformable soil, in the case of plastic streaming is fixed, but in the case of soil condensation is changed irreversibly after taking of the load. It must be also underlined that although the three above metioned phenomenons are alvays accompanying to any process of soil deformation, the proportion of their influence on this process depends not only on sorts of soil (clay, sand etc.), but also is the subject of change during the process of deformation. It is clearly visible on the model of spatial placing of phenomenons determining the deformation of soils.

From examinations made by Rankine, Terzaghie and others results

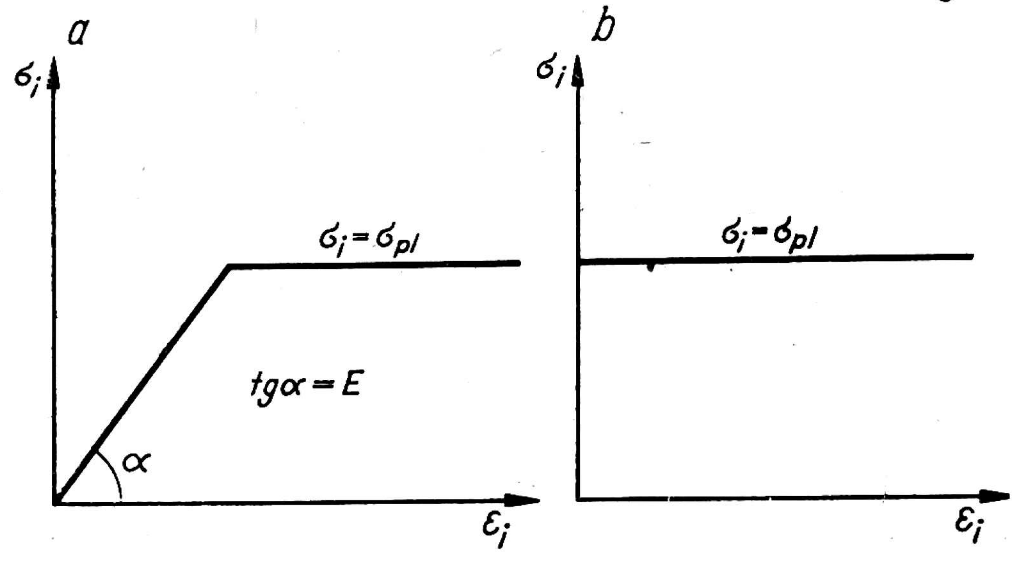


Fig. 13. The simplest elastic models: *a* — elastic-plastic model, *b* — rigid-elastic model

that during the prop's effect it is possible to disinguish three zones (Fig. 14a) in deformable soil, and in each one of them the another kind of phenomenons is dominating.

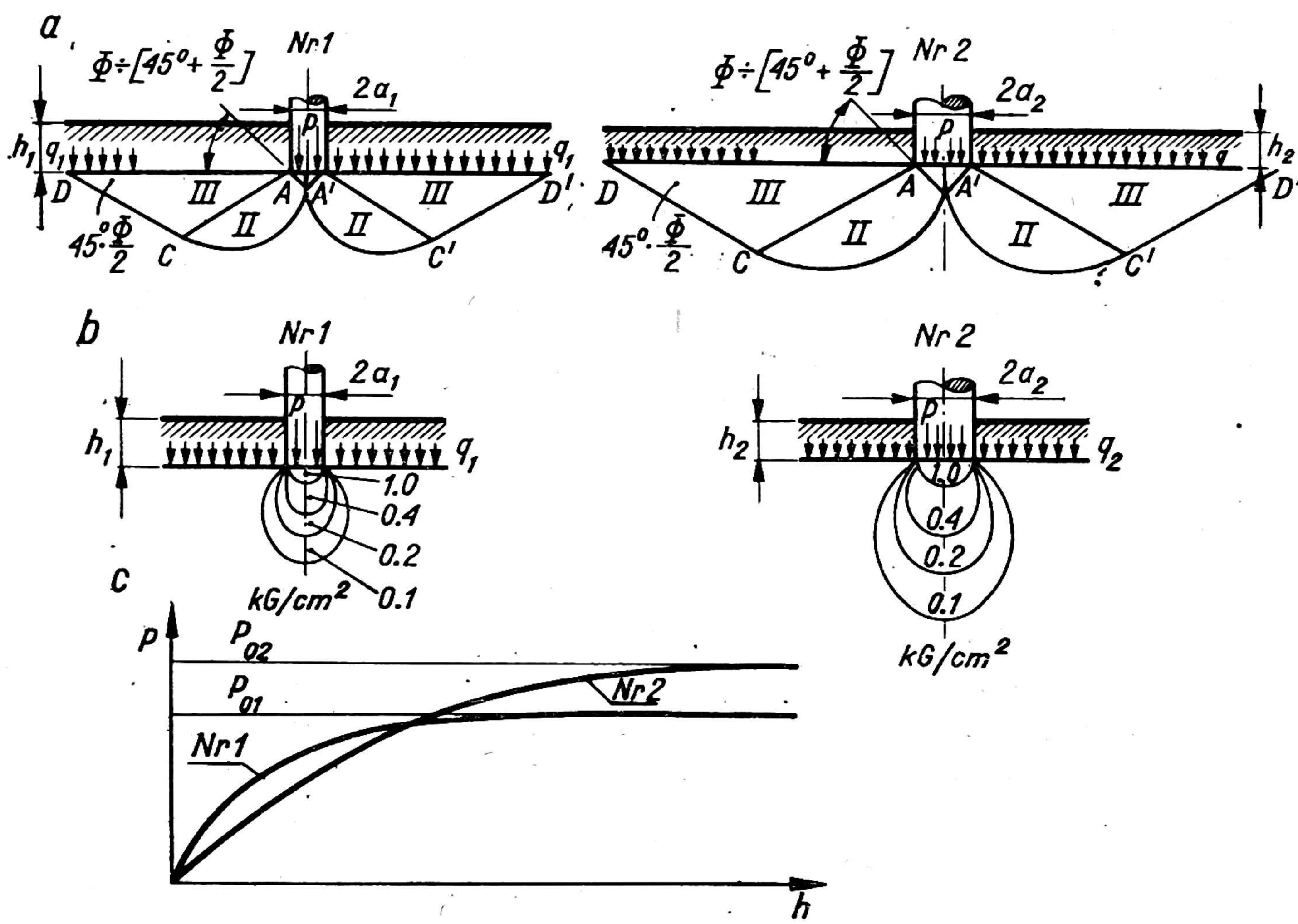


Fig. 14. Zones in deformable soil

Forming of shapes of these zones is also differentiated in time.

The most early is zone I that covers the soil situated directly under prop. At small relative loads the elastic deformations predominate in this zone, however, the phenomenon of soil compensation appears also stronger and stronger.

In the result of activity of both mentioned phenomenons under prop it comes into being the condensated, partially elastic core, shaped like a cone. The cone generating lines are the lines of slip of the soil situated to the horizon at an angle

$$\Phi \text{ up to } 45° + \Phi/2,$$

where Φ angle of internal friction of soil.

The first segments of curves 1 and 2, shown by way of example in Fig. 10, conform to described period of formation of the zone I and they run away from tangents traced from the beginning of co-ordinates sy-

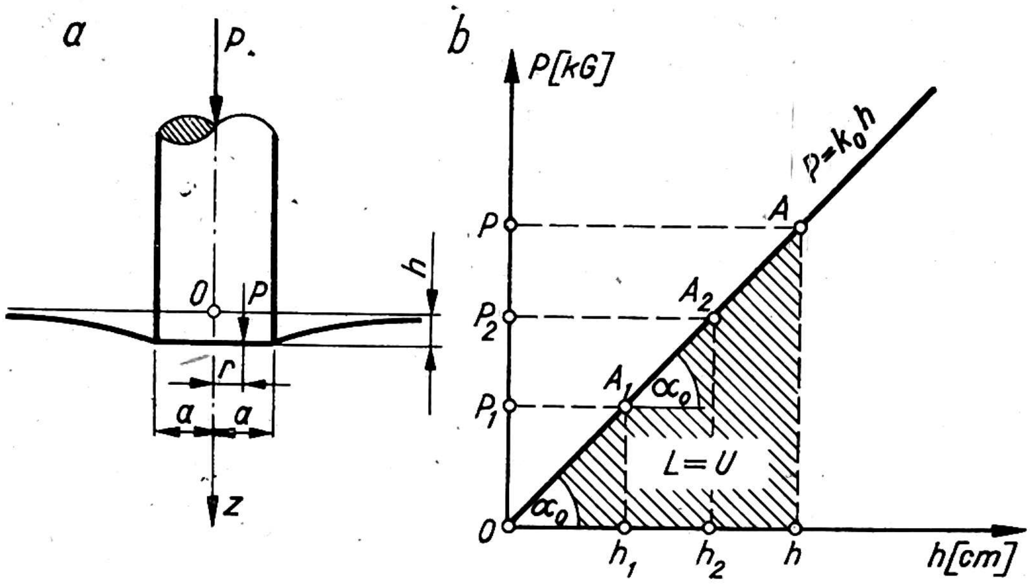


Fig. 15. Half-space loaded by a prop

stem only a little. The further increase of prop load results on both, the intensification of soil condensation and dislocation down of the earlier formated, condensated core. The dislocating core condensates then the adjacent veins of soil and this phenomenon is accompanying by increasing process of radial soil stress (initiated on the side of cone).

In this way in the soil originates zone II, known as the zone of radial stress, limitated on sides by conical planes with generating lines *AB* and *AC*, deviated from horizon at an angle $[\Phi \div (45° + \Phi/2)]$ and $(45° - \Phi/2)$ respectively.

From the bottom the zone is bounded by a curvilinear plane with generating line *BS* shaped as a segment of logarithmic spiral.

The phenomenon of stress dominates in this zone, concomitant with the condensation phenomenon, and which decreases while approaching towards generating line *AC*. The stage of formating of the zone II is assisted by the progressive increase of sinking of the prop, appearing in the diagram by decrease of fine curvature radius (fig. 10 and 14 c). The further increase of prop load results that the soil after the process of condensation is finished, must be displaced anywhere.

By this means there originates the zone III, colled Rankine's zone, in which only the displacement of soil veins happens. This displacement reveals on the surface, among others, by forcing out the soil on side of the pressed prop. The described phenomenom leads to further progress of sinking of the prop, which with time changes into so-called plastic streaming without need of the external load increase. The initary pressure P_0, in the presence of which the plastic streaming happens, is called the soil carrying capacity interval or, the admissible unitary limitary load.

Of course, on the course of the soil deformation process that comes into being under the pressure of the rigid prop results also the dimension and head-plane of the prop. Generally, in the case of flat circular plane the effect is of dual kind. On the one hand, the bigger is prop plane, the bigger is the expansion tension depth (Fig. 14 *b*) and correspondingly bigger is the volume of soil under-going condensation. On the other hand, the bigger is the plane of prop, the bigger is the soil displacement resistance and smaller displacement, respectively. That is why at the rise in prop radius the intensity of sinking of the prop grows larger within the interval of small loads and at the same time the interval of limitary capacity grows larger, so, the intensity of sinking decreases within the big load interval. This oppositing effect of radius of the prop on condensation and displecement is reflected by the course of diagrams, as shown in Fig. 14 *c*.

DERIVATION OF MATHEMATICAL FORMULAE DESCRIBING MATERIAL CHARACTERIZATIONS OF SOIL

When deriving new equations of soil characteristics we use the following two postulates:

1) values that exist in dependences, are to be the physical values reckonable on the way of experimentation (undirectly or directly);

2) the new dependences to have the character of physical eguations of casual type, expressed with the help of continuous functions and univaluable in the considered interval of determination.

The starting point in the discussion will be the half-space deformation analysis based on the linear theory of elasticity, viz. Boussinesq problem, as presented in Fig. 15 and described by final dependences:

$$P = \frac{P}{2\Pi a\sqrt{a^2 - r^2}}, \tag{1}$$

$$P = \frac{2aE}{(1 - V^2)} \cdot h = k_o \cdot h \tag{2}$$

where

$$k_o = \frac{2aE}{(1 - V^2)}, \tag{3}$$

ν — Poisson's ratio,

E — Young's modulus.

The other denotations as in Fig. 15.

From the last formula it clearly appears that the proportionality factor, k_o, setting the stress P with the vertical displacement h, depends not only on the material filling up the half-space (through the Young modulus and the Poisson's ratio) but, also on the dimensions of the prop (through the radius a). So, this conclusion complies with the physical model of the phenomenon described in the initial part of this work. Denominating, in turn, by

$$m_0 = \frac{k_0}{F_0},$$

it was proved in the paper [12] that the Boussinesq problem could be described univocally by means of the following second order differential equations:

$$\frac{d^2L(h)}{dh^2} = \frac{d^2U(h)}{dh^2} = \frac{2aE}{1-\nu^2} = k_0, \tag{4}$$

$$\frac{d^2L_E(h)}{dh^2} = \frac{d^2U_E(h)}{dh^2} = \frac{2aE}{(1-\nu^2)F} = m_0, \tag{5}$$

which are free from any ratios of physically indefinite coefficients. In the paper [12] it was also proved that these equations are invariable within the mathematical form as reffered to the Galileo's kind of transformation

$$L'_F = L_F - P_c \cdot h = L_F - \left(\frac{dL_F}{dh}\right)_c \cdot h, \tag{6}$$

$$|h| = |h^1|.$$

The schemes necessary for the eduction of the transformation are shown in Fig. 16 — for the Hook's half-space deformability process in relative co-ordinate system and, Fig. 17 — for the geometrical sense of the Galileo type transformation.

Starting from the classic course of material characterization, shown in Fig. 18, it can be proved that for this case the characteristic relative translation of coordinate system is the transformation of Fig. 19, expressed by the equations:

$$\left.\begin{aligned} L^1_F &= \xi\,(L_F - P_c \cdot h), \\ h^1 &= \xi\left(h - \frac{P_c \cdot L_F}{P_0^2}\right), \\ \xi &= \frac{1}{\sqrt{1-\dfrac{P_c^2}{P_0^2}}}, \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} L_F &= \xi\,(L_F^1 + P_c \cdot h^1), \\ h &= \xi\left(h^1 + \frac{P_c \cdot L^1_F}{P_0^2}\right). \end{aligned}\right\} \tag{8}$$

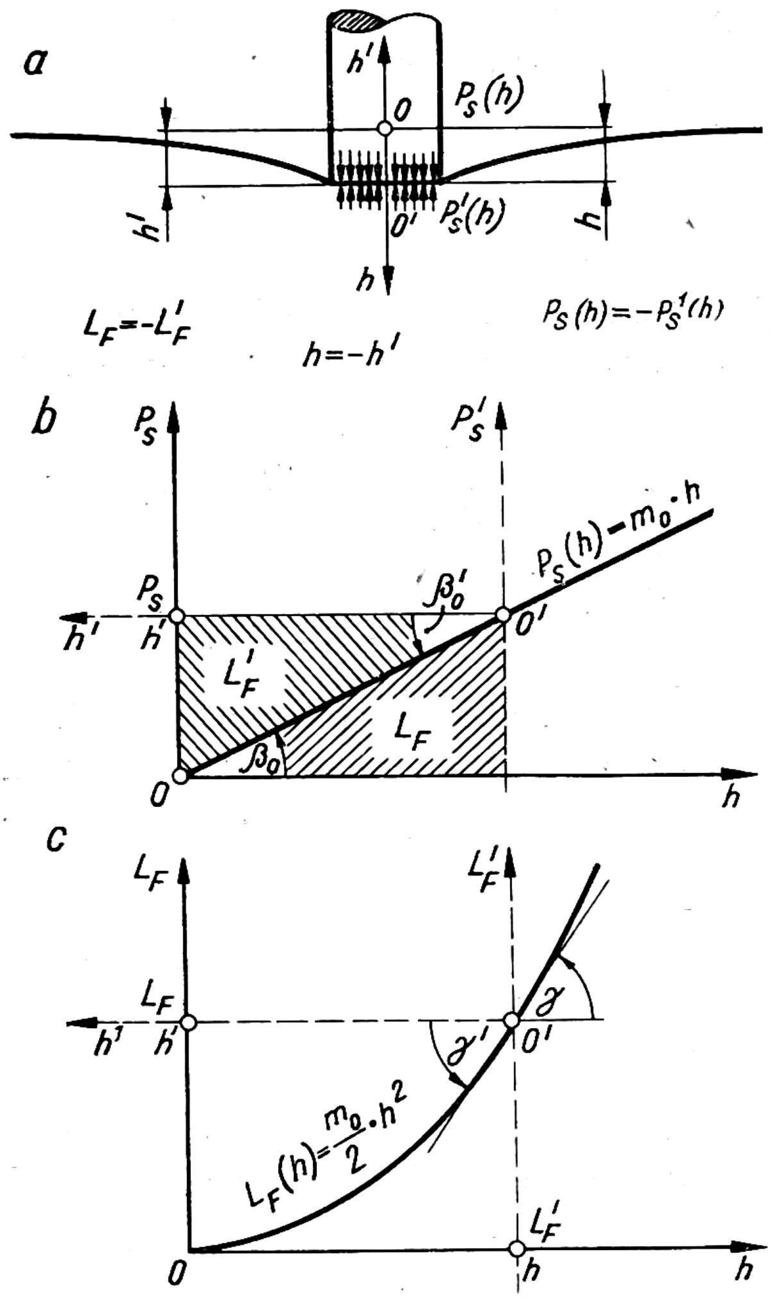


Fig. 16. Hooke's half-space deformability process

From the mathematical point of view this is the Lorenz transformation. It is possible to prove that the equation (5) which describes the Boussinesq problem is not invariable in relation with this transformation.

Further procedure requires to find out a differential equation which would cover the nature of soil deformation process, i.e. a differential equation in an invariable mathematical form in relation to the Lorenz transformation.

When deriving it we can use the following three postulates:

1. The equation searched for must have invariable mathematical shape in relation to the Lorenz transformation.

2. Such aquation for small unitary values of the average loads, P_s of soil must reduce itself to equation (5).

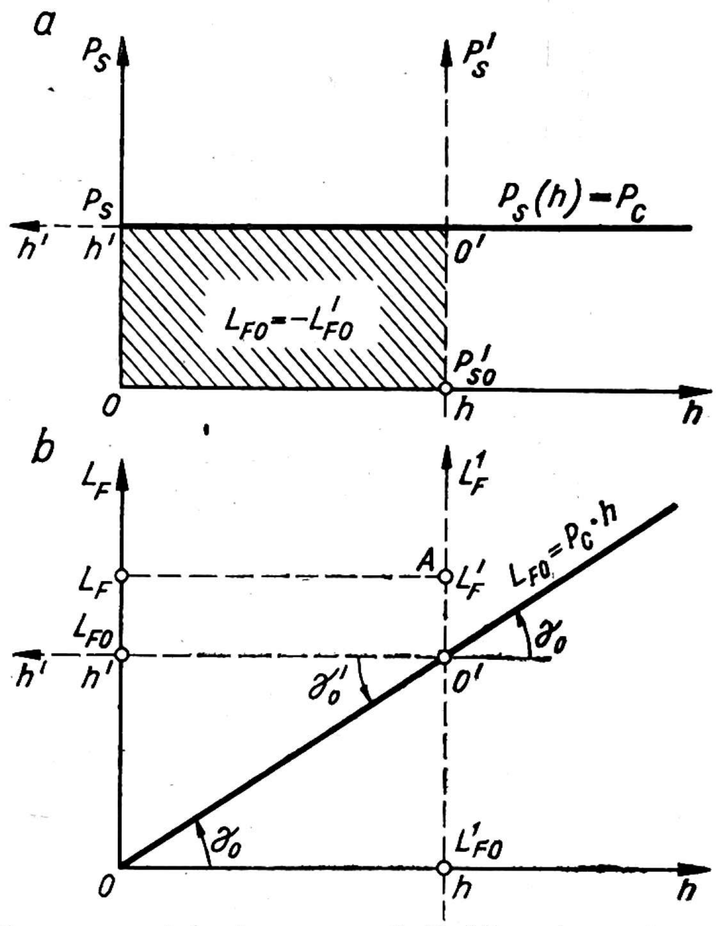
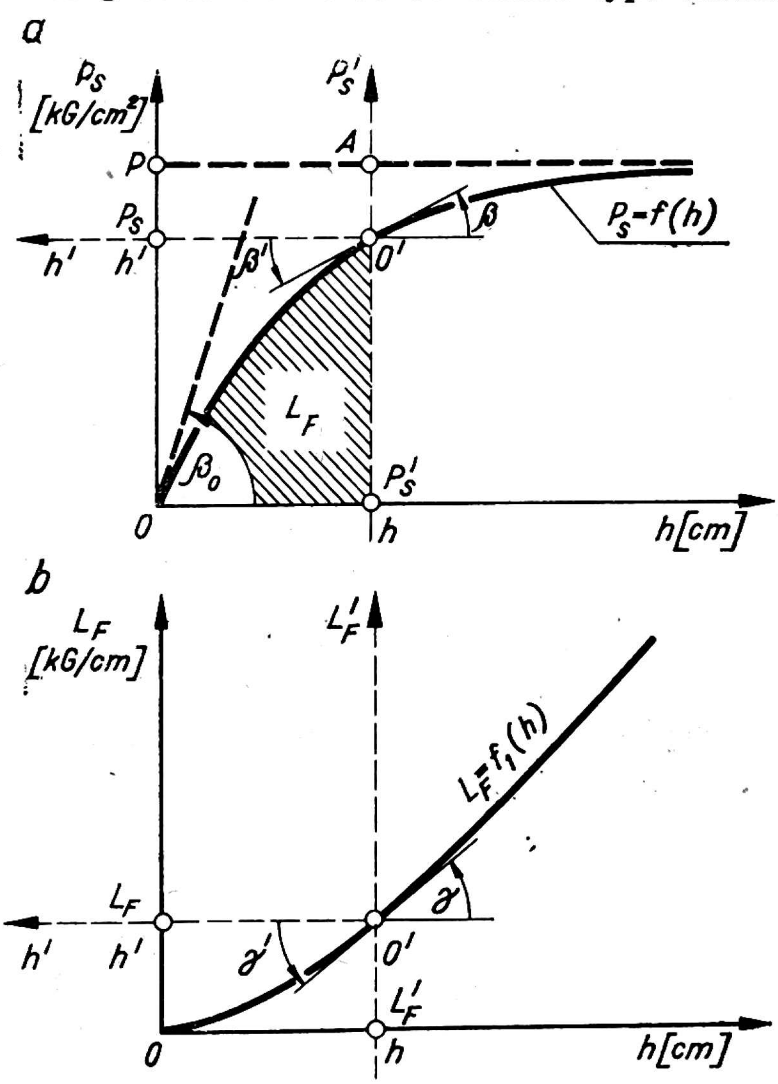


Fig. 17. The geometrical sense of Galileo type transformation



18. The classical course of material characterization

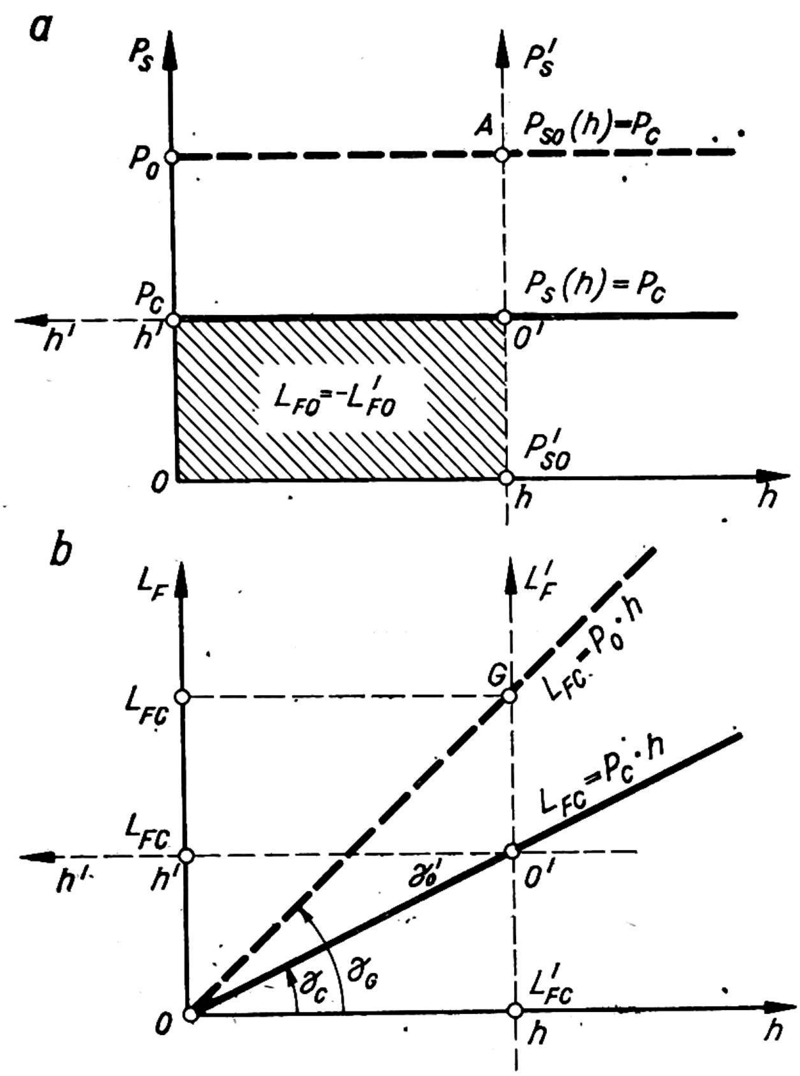


Fig. 19. The characteristic relative translation of coordinate system

3. Average unit pressure P_s which acts on the soil is determined according to the rule of "weighted average".

From the considerations carried out in the paper [12] results that the differential equation searched for has the form of

$$\frac{d[F_o \cdot P_s(h)]}{dh} = \frac{d}{dh} \sqrt{\frac{F_o \cdot P_s(h)}{\sqrt{1-\frac{P_s^2(h)}{P_o^2}}}} = \frac{2aE}{(1-\nu^2)} \cdot \tag{9}$$

Therefrom after carrying out integration and acceptance of appropriate boundary conditions we obtain

$$P_s(h) = \frac{\frac{k_o}{F_o} \cdot h}{\sqrt{1+\frac{k_o^2}{P_o^2 \cdot F_o^2} \cdot h^2}} = \frac{m_o \cdot h}{\sqrt{1+\frac{m_o^2}{P_o^2} \cdot h^2}}, \tag{10}$$

i.e. average stress P_s as a function of deformation h.

CONCLUSION

The nature of natural soils deformation caused by the load-carrying rigid prop is expressed by the differential equation (9) which is invariable in relation to the Lorentz type transformation.

The particular integral of this equation for initial conditions at $h = 0$ gives a useful physician relation (10) which combines the unit average loading of soil with the deformation value h of this soil by means of the rigid prop. Equation (10) shows that deformation value depends not only on the kind of soil and value of average load, but, also on the dimensions of loading element.

Authors are of the opinion that after gathering necessary experimental evidence the derived formula will be more useful for consideration of soil influence on terrain characteristics of cultivating machines and vehicles than the up-to-now applied formulae, due to greater close up to real conditions. This formula is relatively simple feasible for mathematical transformations.

COMPARISION OF CONFORMITY OF THE DERIVED MATHEMATICAL FORMULAE IN DESCRIPTION OF SOILS WITH RESULTS OF EXPERIMENTS

In Fig. 20 there are shown six experimental curves of average prop load as a function of its sinking h in the soil. The values used to draw out the curves were taken from the research of S. S. Korczunow as cited by F. A. Opiejko [19]. Korczunow investigated peat deposits of 30% decomposition, 82—84% moisture, 0.32 Poisson ratio. He used, in his research, round props loaded with the speed of 200—400 kG/min.

The curves 1, 2, 3, 4, 5 and 6 represent the prop diameters of 15, 20, 25, 30, 35 and 40 cm, respectively. In the above figure, on the basis of

Table

Young's moduli calculated from the experiment of Nasietkin

Number of curve	$2a$ [cm]	F_0 [cm²]	m_0 [kG/cm³]	V	E [kG/cm²]
1	15	176	0.5		5.28
2	20	316	0.4		5.44
3	25	490	0.31	0.32	5.46
4	30	720	0.25		5.4
5	35	960	0.22		5.4
6	40	1580	0.18		5.76

Average Young Moduls E_{aver} = 5.46 kG/cm².

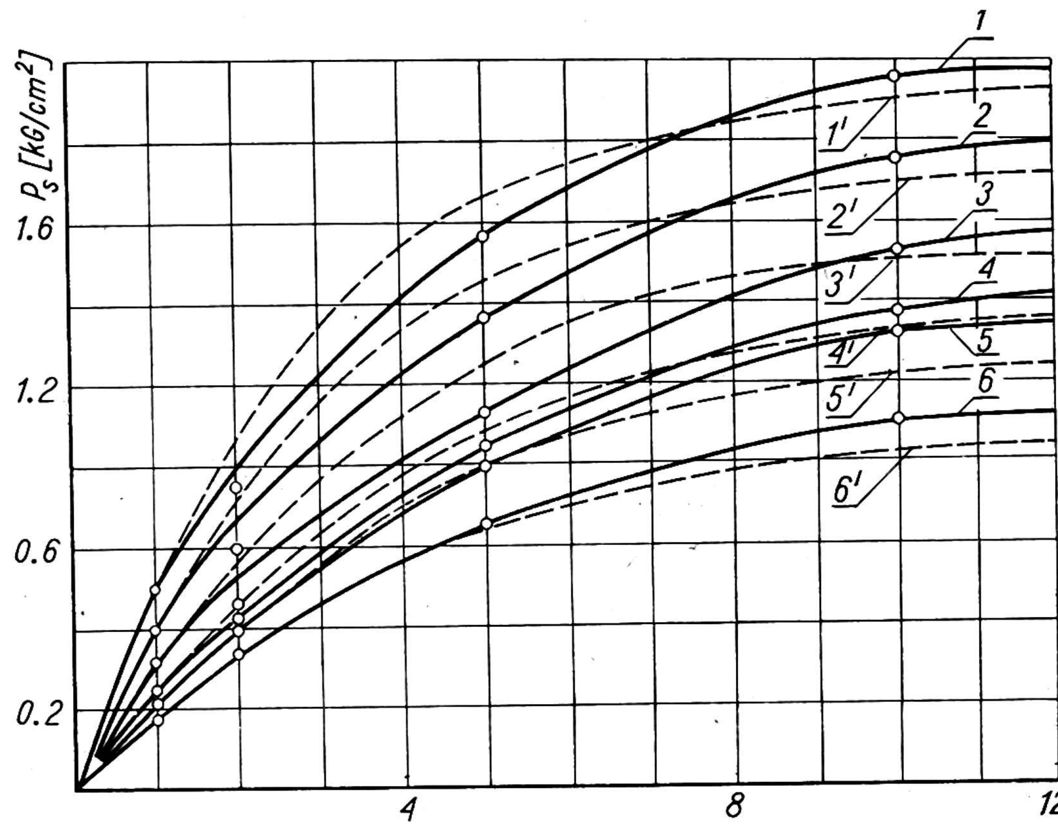


Fig. 20. Comparison of experimental and theoretical curves: 1-6 experimental curves, acc. to Korczunow's research [19], 1'-6' curves derivated from dependence (10)

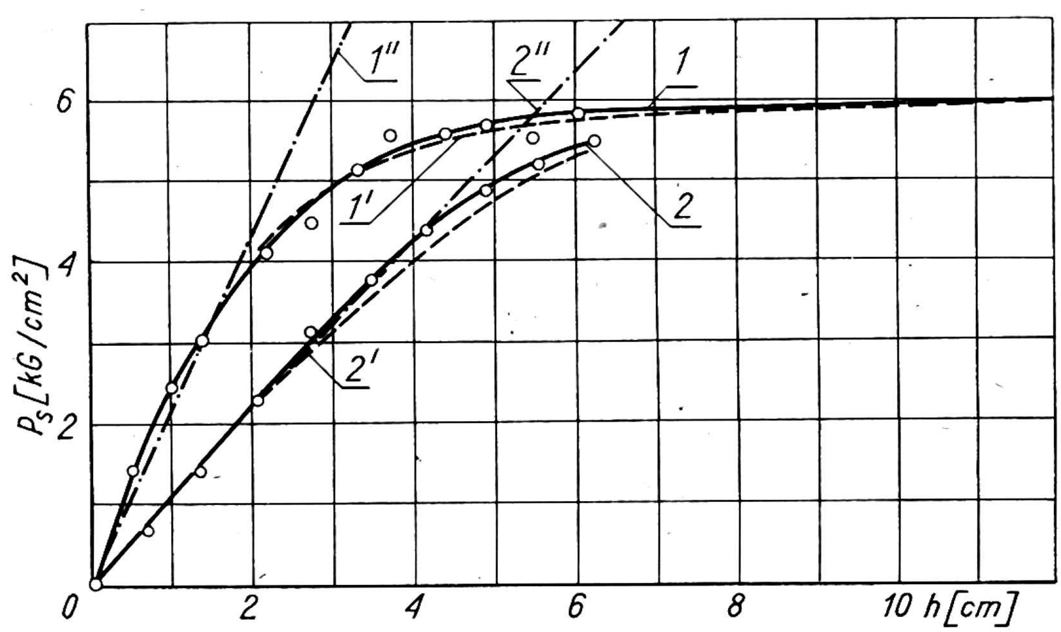


Fig. 21. Comparison of Bekker's formula and formula (10) with experimental results: 1 and 2 — experimental curves, acc. to Saakjan's research [23], 1' and 2' —curves derivated from dependence (10), 1'' and 2'' — straight lines derivated from Bekker's formula

measuring the inclination angle of curves in the origin of co-ordinates, and, average boundary values of loads p_o which appear in particular series of researches there were drown out six theoretical curves according to formula (10). The degree of conformity of experimental and theoretical curves is very high.

Taking into account the existing values m_o of tangents of inclination angles of curves at origin of co-ordinates there were calculated the values of Young modulus.

The results of the calculations make Table. The calculated values of Young modulus are characteristic in that they are highly convergent and almost reach the values given to peats by N. A. Nasiedkin [19], which additionally support the theory laid out in this work.

The other results of the research are shown in Fig. 21. The figures come from S. S. Saakjan's paper [23], who was making researches of black earth with the use of two props with diameters 11 cm (curve 1) and 14 cm (curve 2).

On the figure there were drawn out theoretical curves determined according to formula (10) and Bedder's formula. The figure illustrates convergence of theoretical curves determined in accordance with formula (10), what cannot be said of other approximations, e.g. Bakker's approximation.

The results obtained encourage further generalization. Therefore, it is worth to notice that

$$m_o = \frac{2aE}{(1-\nu^2)F_o} = \frac{2\Pi aE}{\Pi(1-\nu^2)\,F_o} = \frac{S_o \cdot E}{\Pi(1-\nu^2)\,F_o}\,, \tag{11}$$

where

$$S_o = 2\Pi a, \tag{12}$$

is the circumference (perimeter) of the round loading prop. Designating the dependences (11) by

$$K = \frac{E}{\Pi(1-\nu^2)} \tag{13}$$

we obtain finally

$$m_o = \frac{K \cdot S_o}{F_o}\,, \tag{14}$$

and from eguation (10)

$$P_s(h) = \frac{K}{\sqrt{1 + \dfrac{K^2 \cdot S_o^2}{P_o^2 \cdot F_o^2} \cdot h^2}} \cdot \frac{S_o}{F_o} \cdot h. \tag{15}$$

The constant K value present in formulae (14) and (15) depends entirely on the characteristics of soil. Having in mind that this constant value comprises the Young modulus and Poisson ratio we will go on calling it a coefficient of linear deformability of soil.

FINAL RESULTS

1. The equation (10), which describes deformability of nonlinear half--space by means of a rigid prop having the shape of circular cylinder is characterised by high precision of description for various soils.

2. The shape of equation (13) suggests that it is possible to use it also for non-circular surfaces which put load on half-space. For symmetric surfaces (square, rectangle, ellipse) preliminary analysis proves in full such possibility. Opiejko [24], examining a similar problem for linear half-space recommends in the case of rectangular planes to substitude the relation F_o/S_o — taken out from equation (14), for

$$\frac{F_o}{S_o} = \frac{\sqrt[3]{ab^2}}{4}, \tag{16}$$

where a and b are the sides of rectangle, and $b \leqslant a$. This is obviously useful because of change in distribution of tensions on non-circular planes of the external load and half-space joint in relation to circular surfaces.

DENOTATIONS

F — surface of interaction (of the prop.),
I_1 — first invariable of the stress status tensor,
b — with of the surface of interaction (of the prop.),
$k\ k_1\ k_2$ — parameters of equations defined by their authors, respectively; undimensional or dimensional,
n — undimensional exponent (in equations) dependent on nature and kind of soil,
P — soil load,
P_o — boundary soil loading,
z, h — size of the vertical soil deformation,
γ_{okt} — octahedron angle of shape deformation,
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ — main deformations,
ε_i — intensity of deformation,
$\varepsilon_{śr}$ — average deformation,
$\varepsilon_x, \varepsilon_y, \varepsilon_z$ — components of deformation in Cartesian system,
σ_1 — intensity of tension (stress),
$\sigma_{śr}$ — average stress (tension),
τ_{okt} — octahedron tangent tension,
Φ — angle of the internal friction of soil.

REFERENCES

1. Agiejkin J. S.: Ocenka deformirujemosti grunta pri rassmatrieniji prochodimosti koliesnych maszyn. W.A.B.T.W. 1968.
2. Agiejkin J. S.: Ocenka deformirujemosti grunta pri rassmotrienii procjodimosti awtomobiliej. Awtomob. prom. nr 6/1970.
3. Andrejew W. A.: Issledowanija scepnych kaczestw makietow koliesnych dewiżytieliej i szyn so gniegom. Trudy NII-21 sb. nr 3, 1964.
4. Azmi M. H. M.: Issledowanije wlijanija parametrow szyn na prochodimost maszyn po sypuszczim gruntam, Moskwa 1968.
5. Bekker M. G.: Theory of Land Locomotion. Ann Arbor. The University of Michigan Press, 1956.
6. Bekker M. G.: Off the Road Locomotion. Ann Arbor. The University of Michigan Press, 1960.
7. Bekker M. G.: Introduction to terrain-vehicle systems. Ann Arbor. The University of Michigan Press, 1969.
8. Bernstein R.: Probleme einer experimentallen Motorpflungmechanik. Der Motorwagen, nr 9, 10, 1913.
9. Cytowicz N. A.: Mechanika gruntów. Wyd. geologiczne, 1958.
10. Gerstner: Zwei Abhandlungen über Frachtwagen und Strassen und über die Frage, ob und in walchen Fallen der Bauschiffbarer Kanale, Eisenwagen oder gemachter Strassen Vorzuziehen Prag. 1813.
11. Jeske T., Przedecki T., Rossiński B.: Mechanika gruntów. PWN 1966.
12. Kołodziej W.: Wpływ budowy mechanizmów jezdnych na właściwości samochodów terenowych. Praca doktorska. Politechnika Warszawska, 1971.
13. Kołodziej W.: Badania deformacji podłoży naturalnych w mechanice układu pojazd-teren. Informator WITPiS, 1972.
14. Kołodziej W., Piróg J.: Badania odkształceń granicznych podłoży naturalnych występujących pod terenowymi oponami niskociśnieniowymi. Sprawozdanie WITPiS nr 104/Z-6/73.
15. Kondner R. L.: Hyperbolic stress-strain response cochesive soils Proc. A.S.C.E, 89, S.M., 1, 1963.
16. Letoszniew M. N.: Wzaimodiestwije konnoj powozki i dorogi. Transpieczt NKPS, 1929.
17. Lo K. Y.: Stress-strain relationships and pore water pressure characteristics of a normally consolidated clay. Proc. of the 5th Int. Conf. on soil Mech. and Found. Eng., 1, Dunad, Paris 1961.
18. Nitzsche W.: Bessere Bodenbearbeitung, Berlin 1939, RKTL — Schift nr 70.
19. Opiejko F. A.: Koliesnyj i gusienicznyj chod. Izd. Akademii sielsk.-choz. Nauk B.S.S.R., Mińsk 1960.
20. Praca zbiorowa pod red. W. W. Kacygina: Woprosy tiechnologii miechanizirowannogo sielsk.-choz. proizwodstwa. Cz. I. Gos. Izd. Sielskich. Lit. B.S.S.R. Mińsk 1963.
21. Praca zbiorowa: Ziemliedielczieskaja mechanika. Sbornik trudow, tom X, Izd. Maszinostrojenije, Moskwa 1968.
22. Saakjan S. S.: Wzaimodiejstwije wiedomogo koliesa po gruntowoj powierchnosti. Erewan 1959.
23. Saakijan S. S.: Wzaimodiejstwije wiedomogo koliesa i poczwy. Erewan 1959.

24. Stroganow A. A.: Plane plastic deformation of soil. Brussel Conf. 58 on Earth Pressure. Proc. 1, 1958.
25. Stroganow A. S.: Niektóre problemy plastyczności i reologii w mechanice gruntów. Jabłonna, maj 1966.

W. Kołodziej, J. Piróg

CHARAKTERYSTYKI MATERIAŁOWE GLEB PODCZAS OBCIĄŻENIA

Streszczenie

Przedstawiona praca zawiera krótki opis zjawisk fizycznych zachodzących w glebie pod wpływem zewnętrznych obciążeń i wyniki badań teoretycznych zależności deformacji pionowych gleby od obciążeń normalnych. Wyprowadzono nowe równanie charakterystyki obciążenia gleby. Wszystkie parametry mają określone znaczenie fizyczne.

Wprowadzona zależność została porównana z wynikami badań różnych eksperymentów i uzyskano zadowalającą zgodność.

В. Колодзей, Ю. Пируг

МАТЕРИАЛЬНЫЕ ХАРАКТЕРИСТИКИ ПОЧВ ВО ВРЕМЯ НАГРУЗКИ

Резюме

В труде приводится краткое описание физических явлений происходящих в почве под влиянием внешних нагрузок, а также обсуждаются результаты теоретических исследований зависимости вертикальных деформаций почвы от нормальных нагрузок. Предложено новое уравнение по характеристике нагрузки почвы. Все параметры имеют определенное физическое значение.

Введенную зависимость сравнивали с результатами разных опытов, причем установлено их удовлетворительное сходство.