ZESZYTY PROBLEMOWE POSTĘPÓW NAUK ROLNICZYCH 1989 z. 378

INVESTIGATION OF FORCES ACTING ON ELEMENTS BUILT IN GRAIN STORAGES

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Synopsis. In many cases the instruments, gages, ventilation stubs and other elements are installed in grain crop storages. The forces acting on elements of any shape can be determined theoretically in the labour-consuming manner, or sometimes they cannot be determined by any means.

Conditions are different when the grain crop is moving and after a certain storing period when the material becomes aggregated.

The task can be approached by means of the theory of cohesionless materials as used in the soil mechanics, too. In the course of moving, the grain crop flow can have three types of stress conditions, such as active plastic limiting state in forced movement or elastic state between the limiting states. Force ratio can be 6-8 fold for the two extremes. Theoretical investigation of the limiting states has already been made by many persons with thorough grounding. Most of them, however, delt with plane wall containers.

Besides the theoretical work the measurements often give the simplified results, especially in the case with the form of certain instruments, gages, ventilation stubs. To solve the task the sample-experiments were carried out. In the experiments we built cylindrical elements in dry sand. Suspension forces were measured and recorded for various sand movement conditions. Measurements were carried out with stored and deposed sands, and in the case of discharge. Force meters, tensometers and recorders were used in the measurements. In addition to the numerical measures, the method of measuring was investigated, in the interest on future measurements in situ.

According to the results, the greatest forces appear after longer storage periods.

INTRODUCTION

Gauges, instruments, and ventilation pipes serving different aims are often mounted in the grain storages (Fig. 1). Forces acting on these elements of any shape can be derived either on theoretical basis or on an in situ measurement.

The basis of the theoretical solution is that of the cohesionless materials such as dry sand. It is known that a storage heap has an undetermined stress condition which can be examined precisely by means of the physical relationships (ma-

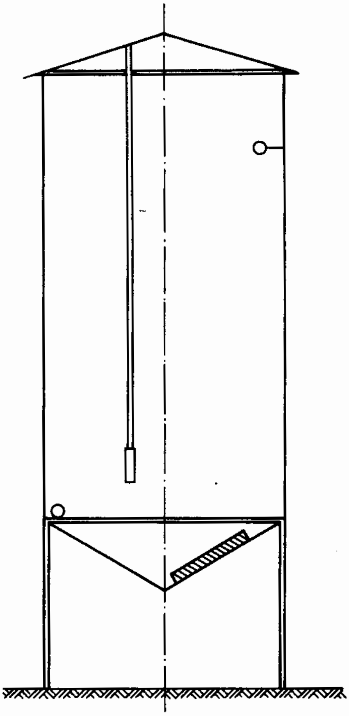

Fig. 1

terial law) of the elasticity. For example the task is solved in [1] making use of the known pressure and shear functions. Since these functions are usually not known, the silo pressure and the material density are derived experimentally and the relationships are further treated mathematically [5].

The measuring results give a good view of the order of magnitude of the pressure conditions. Such investigations have been carried out since the beginning of the decade [2] up to recently [3], on real structures as well [4]. The work resulted in a standard [6], [7] which is being used worldwide for dimensioning silos. Results gained in situ present, however, restricted possibilities because of the great cost. Another problem is the short period of the moisture content changes in open air when high humidity grains being stored are measured. Thus the small scale models are often used in order to transform the test results.

Magnitude of forces - $f(N/m^2, N/m)$ acting on the built-in elements significantly depends on their form, material, and the way of clamping or suspension. Therefore choosing a correct model is not simple at all. It seems practical to choose point-symmetrical elements (cylinder, sphere, cone) first. While choosing the suitable model material the practical aspects and the material properties should be taken into account. That is why we chose dry sand.

Theoretical determination is made as follows. An experienced force increases significantly when the stored material is moving. Grain crop having internal friction comes into an approximately passive platic limiting state in the course of moving.

Although in the classical mechanics a body pushed into the immovable material with a certain velocity corresponds to the limit load, the results give a good approximation.

THEORETICAL DETERMINATION OF PLASTIC LIMIT STATE

Stress state of a cohesionless heap is shown in Fig. 2. Friction coefficient between the element built-in and the heap is considered to be zero first. And stresses of the primary cube are:

$$\sigma_y = \gamma \cdot y$$

$$\sigma_x = \sigma_y \tan^2(45^0 + \varphi/2),$$

where: $\gamma = \rho \cdot g \left[N/m^3\right]$

φ : internal friction of the heap.

Examining the Mohr-circle it can be seen that in a material having greater internal friction the horizontal stress characterizing the passive plastic limit state can have so great value that it can lead to the failure of the built-in material.

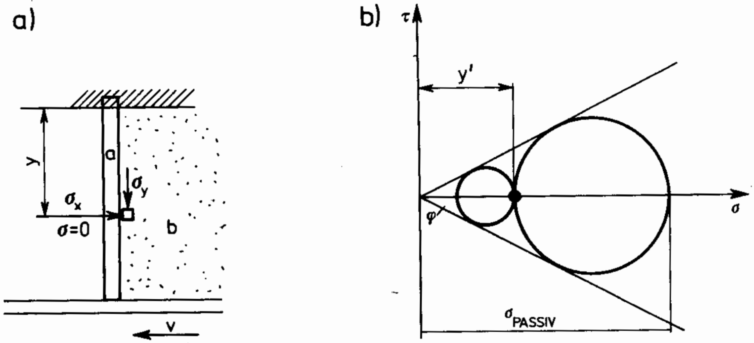

Fig. 2

For example, in the case of wheat heap of a 30 degree internal friction angle, this is three times the vertical component.

The theoretical investigations were carried out by W. J. Rankine [8]. He stated that the sliding planes of an angle 45- $\varphi/2$ rise. The trouble with the theory is that the calculation is difficult if the surface is at an angle to the heap. And the changes in the heap density cause difficulties too.

GENERAL PRINCIPAL EQUATIONS

We can write the physical and geometrical equilibrium equations.

The equilibrium equations are in rectangular system of coordinates (Fig. 3):

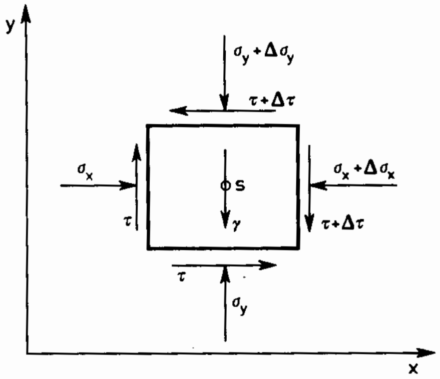

Fig. 3

$$\text{div } F + \bar{\gamma} = \bar{0},$$

which means scalar equations as follows:

$$F = \begin{pmatrix} \sigma_x & \tau & 0 \\ \tau & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}$$

$$-\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial y} = 0$$

$$\frac{\partial \tau}{\partial x} - \frac{\partial \sigma_y}{\partial y} = 0$$

$$\frac{\partial \sigma_z}{\partial z} = 0.$$

And solutions, using, e.g. Airy's function, are as follows:

$$\sigma_x = \frac{\partial^2 A}{\partial y^2}; \quad \sigma_y = \frac{\partial^2 A}{\partial x^2}; \quad \tau = \frac{\partial^2 A}{\partial x \partial y} - \gamma \cdot y \ ,$$

Equilibrium equations in polar coordinates: (Fig. 4).

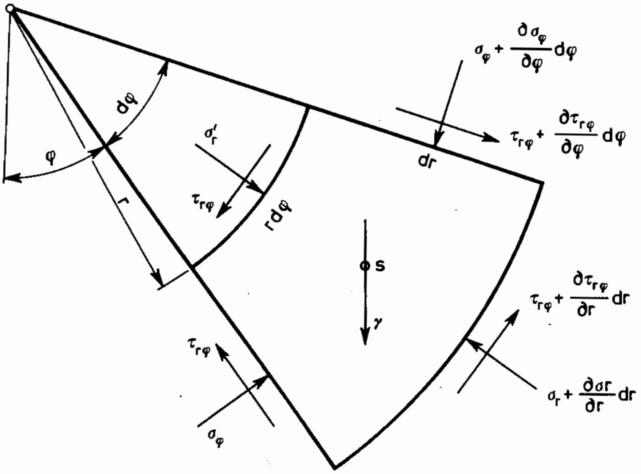

Fig. 4

$$\frac{\sigma_r - \sigma_\varphi}{r} + \frac{\partial\sigma_r}{\partial r} - \frac{\partial\tau_{r\varphi}}{r\,\partial\varphi} - \gamma\cos\varphi = 0,$$

$$\frac{2\,\tau_{r\varphi}}{r} + \frac{\partial\tau_{r\varphi}}{\partial r} - \frac{\partial\sigma_t}{r\,\partial\varphi} - \gamma\sin\varphi = 0.$$

And using Airy's function:

$$\sigma_r = \frac{1}{r^2}\frac{\partial^2 A}{\partial\varphi^2} + \frac{1}{r}\frac{\partial A}{\partial r}\ ;\quad \sigma_t = \frac{\partial^2 A}{\partial r^2} - 2\gamma r\cdot\cos\varphi\ ;$$

$$\tau_{r\varphi} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial A}{\partial\varphi}\right) + \gamma\cdot r\cdot\sin\varphi .$$

The boundary conditions are: stresses at the wall and on the surface (which are known) and the ratio τ_0/σ_{x0} results here in the wall friction angle. The physical relationship is determined from the tangential limit curve of the Mohr circle (Fig. 2).

$$\frac{(\sigma_x - \sigma_y)^2 + 4\tau^2}{(\sigma_x + \sigma_y)^2} = \frac{(\sigma_t - \sigma_r)^2 + 4\tau_{r\varphi}}{(\sigma_t + \sigma_r)^2} = \sin^2\varphi .$$

Let us write the relationships between stresses in the plane at an β angle to the tangent

$$\sigma_\beta = \frac{\sigma_a + \sigma_r}{2} + \frac{\sigma_r - \sigma_a}{2}\cos 2\beta - \tau_{r\varphi}\sin 2\beta ,$$

$$\tau_\beta = \frac{\sigma_a - \sigma_r}{2} \sin 2\beta - \tau_{r\varphi} \cos 2\beta.$$

Substituting φ for β gives y direction stresses:

$$\sigma_y = \frac{\sigma_a + \sigma_r}{2} + \frac{\sigma_r - \sigma_a}{2} \cos 2\varphi - \tau_{r\varphi} \sin 2\varphi .$$

The precise direction of the sliding planes in the passive limit state is:

$$\varkappa_I = 45^0 - \frac{\varphi}{2} + \alpha - \frac{1}{2} \arctan\left(\frac{2\tau_{r\varphi}}{\sigma_r - \sigma_a}\right); \quad \varkappa_{II} = \varkappa_z + (90^0 + \varphi).$$

If the function $\varkappa(\alpha)$ is known the equation of the sliding line can be derived from the differential equation:

$$\frac{dr_G}{d\alpha} = -r_G \tan(\varkappa - \alpha).$$

where r_G is the polar radius belonging to the sliding line.

RESULTS OF THE THEORETICAL EXAMINATIONS

Apart from the approximative solutions (Kötter 1888; Kármán 1926, Reissner 1924, Ritter 1936, Jáky 1937/38, Ohde 1938) Reissner and Duschter [9] found the expression of the sliding planes

$$\varkappa_{I,II} = 90^0 - \left[\frac{1}{2} \operatorname{arct}\left(\frac{2\tau}{\sigma x - \sigma y}\right) \pm \frac{90^0 + \varphi}{2}\right]$$

Stroppel [4] solved the differential equation using an analogue computer and got useful diagrams. He expressed the stress component in the polar coordinate system by means of the stress coefficients, and the Airy's equations can be used

$$\sigma_a = \gamma \cdot r \cdot a(\alpha) = \frac{\partial^2 A}{\partial r^2} - 2\gamma \cdot r \cdot \cos\alpha,$$

$$\tau_{r\varphi} = \gamma \cdot r \cdot b(\alpha) = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial A}{\partial \alpha}\right) + \gamma \cdot r \cdot \sin\alpha,$$

$$\sigma_r = \gamma \cdot r \cdot c(\alpha) = \frac{1}{r}\frac{\partial^2 A}{\partial \alpha^2} + \frac{1}{r}\frac{\partial A}{\partial r}.$$

The boundary conditions can be written simply

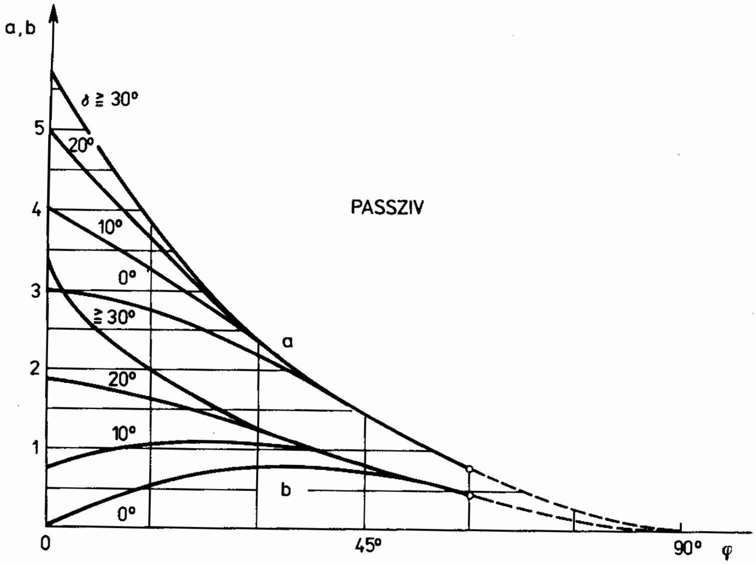

Fig. 5

$$\frac{\tau_o}{\sigma_{xo}} = \frac{b_o}{a_o} = -\tan\delta .$$

Thus the grain heap slides upward along the sliding lines in the passive limit state when the plane plate is moved.

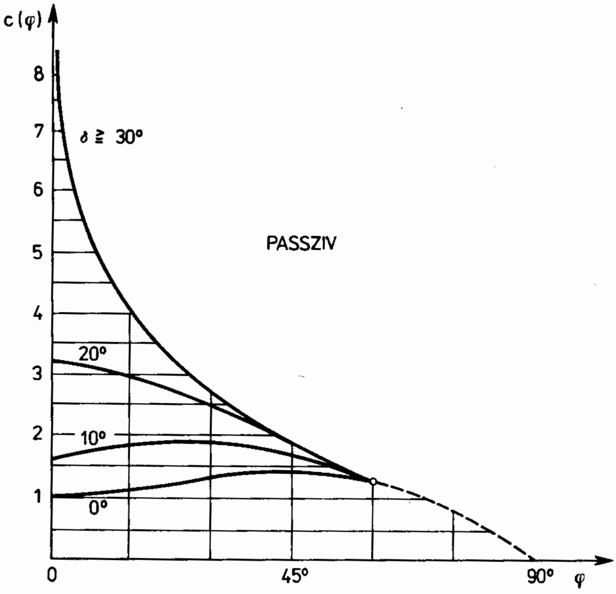

Fig. 6

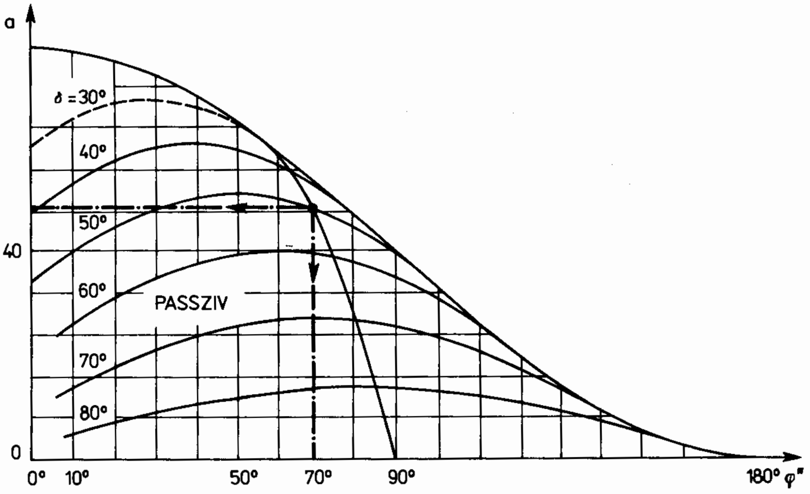

Fig. 7

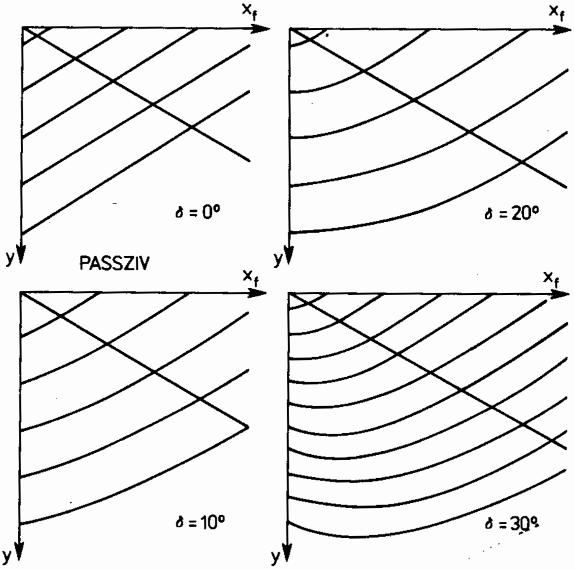

Fig. 8

It should be put on that in the case of wall-material friction angle $\delta > \varphi$ th sliding line cannot be formed near the wall so the relationship

$$\frac{b_o}{a_o} = -\tan\varphi$$

is valid here.

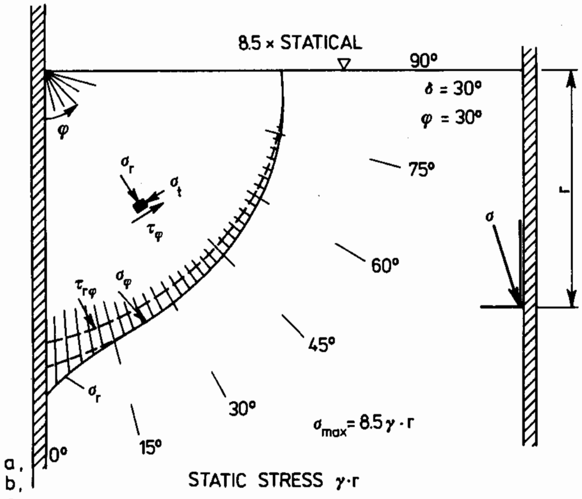

Fig. 9

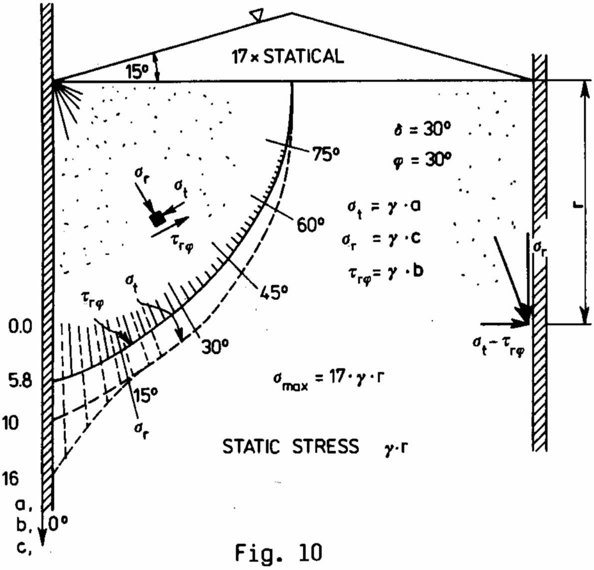

Fig. 10

The results of the examination in the case of $\varphi = 30^0$ are shown in Figs. 5-8. For some cases occurring in practice the further relationships can be considered.

Figures 9 and 10 are made on the basis of the previous ones and show the change of the coefficients versus polar angle and polar radius. It can be seen from these figures that the passive pressure can assume 8.5-17 times higher value if the imperfectness of the internal friction material is taken into account. This behaves as a perfect liquid and it can be seen that significant forces act on the bodies, instruments being in grain heap flows.

LABORATORY INVESTIGATIONS

The goal of the lab investigations

Structures put in a cylindrical silo are loaded by forces of the grain crop being stored. In the foregoings the difficulties in the force determination were discussed. The element most often being built in silos is the strain gauge of different height. The attempts were made to derive the forces acting on a cylindrical strain gauge suspended by a cable.

The other aim of the lab measurements was to prepare the in situ measurements to test and adjust the instrumentation.

EXAMINATION APPARATUS

Small scale bin could be useo only because there is a little space and for the easy handling as well. Internal pressure of the grain crops was very low because of the small dimensions.

The silo height of 10 m or above it is of an order of magnitude larger than the laboratory container which influences the behaviour of the small grain heap. That is why we used smaller grains. We examined some plastic granulates, but they had low specific gravity and special surface so that they were not suitable. Finally the screened dry sand of grain size below 0.5 mm was chosen. Dry sand is easy to trickle and has similar slope angle as the grain materials. Its specific gravity is greater than that of the grains, so it gives a pressure measurable in the bin. We built a box 700 mm high, 560 mm long and 80 mm wide. In order to observe the loading and discharging processes the sidewalls were made of prespex. Instead of the strain gauge a steel rod 12 mm in diameter and 50 mm long was used with a strong cord fixed in a frame on the top of the bin through a force meter cell. Using this set-up we could measure the force in the string, i.e. the force acting on the strain gauge. It can be seen in the photograph. The signal given by the load cell is led to the analogue meter of type DAFO-04. The current was registered by a mA-recorder, the voltage was measured by a digital voltmeter. Thus this set-up made possible either analogue or digital registration (see Figs. 11, 12).

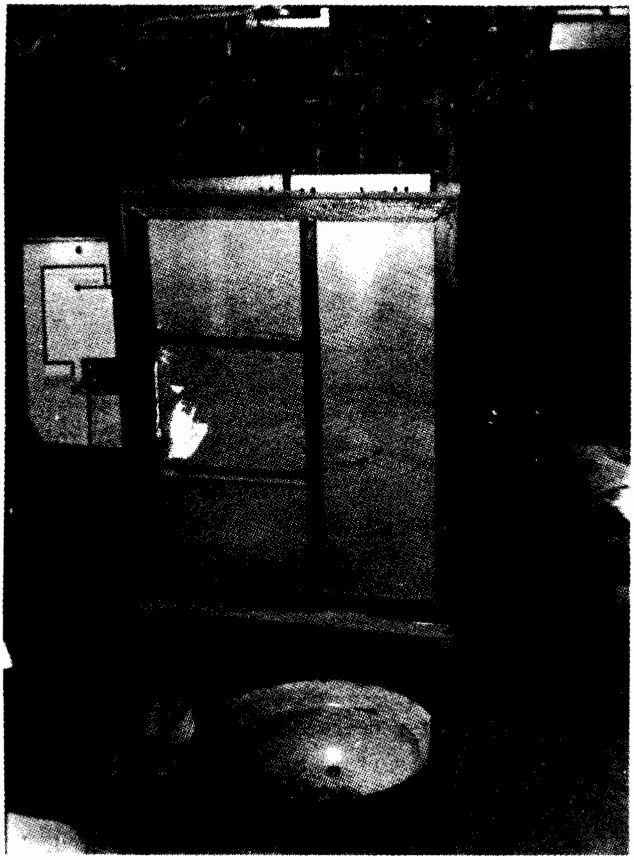

Fig. 11

MEASUREMENTS AND RESULTS

The instruments were calibrated before measurements. The calibrated weights suspended were used for this purpose. For calibration we established a table for measured and fitted data. So we could use the obtained expression to calculate from the mV values the forces acting on the model strain gauge.

Investigations completed in the course of the model measurement are as follows.

The sum of the hydrostatic pressure and the additional one is 90 + 33.7 = = 123.7 g/cm^2, the force calculated has a value 1.4 N (139 g). Force increment as calculated is 37.6% and the measured one is 52.3% .

The loaded model - bin-strain gauge set-up was allowed for 2850 minutes (2 days), while the heap realigned itself.

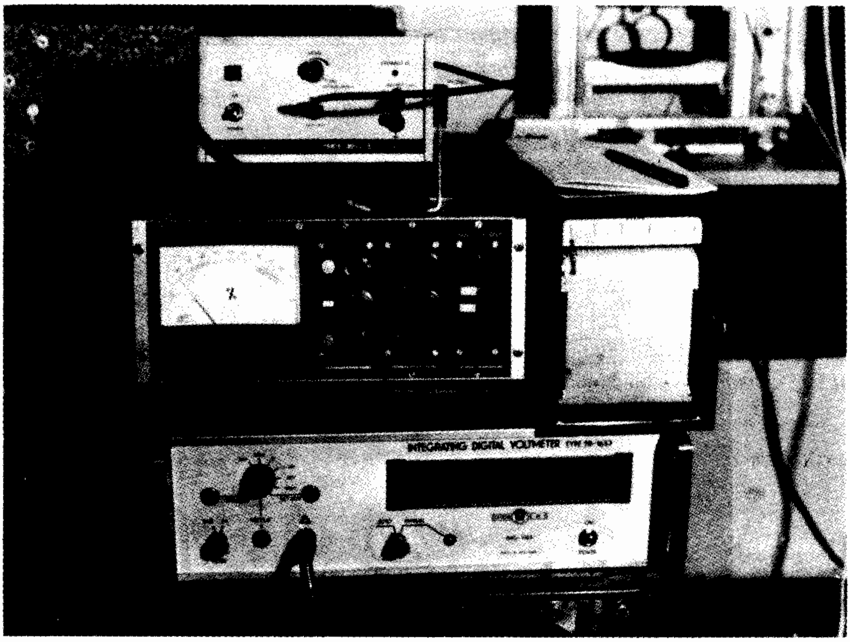

Fig. 12

The value read by the voltmeter decreased to 47 mV which corresponds to 73.5 g tension force. In this period the surface of the sand did not change. To determine the effect of compaction the frame of bin was kept limited by 10 dag hammer while the tension force increased and the sand surface lowered. The force increased to 703 g (196 mV) and the sand surface settled 19 mm down.
The value of compaction on the basis of sand surface heights is:

$$\frac{600-19}{600} \cdot 100 = 96.8\%.$$

During the compaction the tension force increased ten times. After 30 minutes the force value decreased to 673.7 g (189 mV) and this was 665.3 g (187 mV) after 1610 minutes.

At the beginning of the discharging, at opening, the force decreased to a 424.3 g (130 mV) value and during the discharge was continuously decreasing.

To observe the longer time discharge the material was stored back, while the force value was fluctuating between 373,6 g (118 mV) and 458.2 g (138 mV) because of the intermittent loading. If setting the discharging opening larger, the limits were 339.8 g (110 mV) and 434.3 g (130 mV).

In the further experiments the attempts were made to elucidate the relation between forces due to selfweight and statical loading. Full loading of the bin gave 67.7 g (46 mV) tension force and while using 49.1 kPa additional pressure by weights this increased to 109.9 g (56 mV).

The measurement lasting about 8 days proved that after 3-4 days the starting maximum tension force value decreased to the half of its initial value.

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BADANIA SIŁ DZIAŁAJĄCYCH NA ELEMENTY WBUDOWANE W ZBIORNIKACH NA ZIARNO

Streszczenie

Wiele przyrządów (mierników) jest instalowanych w magazynach ziarna. Siły działające na element dowolnego kształtu mogą być oznaczone teoretycznie lub w wyniku pomiarów laboratoryjnych.

Warunki są różne,kiedy ziarno jest w ruchu i po dłuższym czasie przechowywania, kiedy w materiale tworzą się agregaty. Problem jest zbliżony do teorii materiałów spójnych takich jak wykorzystywane w mechanice gleby. W czasie ruchu cząstki ziarna mogą być poddawane trzem typom naprężeń, określanym jako stan plastyczny lub stan sprężysty pomiędzy granicznymi stanami. Teoretyczne badania stanów granicznych były przeprowadzone przez wielu badaczy. W pracy przedstawiono próbę eksperymentu, wbudowano cylindryczny element w suchym piasku. Mierzono siły zawieszenia i rejestrowano je dla różnych warunków ruchu piasku. Pomiary przeprowadzono z zagęszczonym i luźnym piaskiem. Zastosowano mierniki siły, tensometry i rejestratory. Ilościowe pomiary i zastosowane metody dobrano tak, żeby mogły być zastosowane bezpośrednio w polu.

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ИССЛЕДОВАНИЯ СИЛ, ДЕЙСТВУЮЩИХ НА ЭЛЕМЕНТЫ, ВСТРОЕННЫЕ В СКЛАДЫ ЗЕРНА

Резюме

Многие приборы (измерители) помещаются в склады зерна. Силы, действующие на элемент произвольной формы, могут быть определены теоретически либо в результате лабораторных измерений. Условия различны, когда зерно находится в движении, и после длительного хранения, когда в материале образуются агрегаты. Проблема близка к теории связных материалов как используемых в механике почвы. Во время движения частицы зерна могут подвергаться трем типам напряжений, определяемым как пластическое состояние либо упругое состояние между предельными состояниями. Теоретические исследования предельных состояний велись многими исследователями. В работе представили попытку эксперимента, встроили цилиндрический элемент в сухой песок. Измеряли силы завешивания и записывали их для разных условий движения песка; Измерения провели с уплотненным и рыхлым песком. Применили измерители силы, тензиометры и регистраторы. Количественные измерения и примененные методы подобрали так, чтобы они могли быть применены непосредственно в поле.