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The method of 3D reconstruction of apple shape. Part 3. Geometric 3D model of an apple using the interpolation function

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Abstract: *The method of 3D reconstruction of apple shape. Part 3. Geometric 3D model of an apple using the interpolation function.* The study consists of presentation of a Jonagored variety apple. The apple contours have been described by connected Bézier curves, after which the interpolation process was conducted. To model the seeds the chamber and the seed nest of shape similar to rotational solids, the method of rotation of the generating line, consisting of connected Bézier curves, has been developed. The results presented in the second part of the article were used to develop the apple shape model.

Key words: apple, seeds the chamber, seed nest, shape, Bézier curves, mathematic models

INTRODUCTION

Interpolation algorithms are used in order to determine the approximate values of functions within the established range of independent variables of this function and overlapping with the values established by a different function within this range. The interpolation method is based on running of a curve through a series of points. Interpolation procedures include: interpolation by polynomials (interpolation formulas of Newton, Lagrange, Aitken-Neville), harmonic analysis using Fourier sum (Fourier transform), cubic spline interpolation [Bronsztejn et al. 2009]. Approximation algorithms are used to replace the mathematic values expressed as numbers or functions with other, simpler functions, to simplify the issue being described. The approximation procedures known include square standard approximation and Chebyshev approximation. Approximation and interpolation procedures are used to smooth data [Kuhl and Giardinia 1989, Paleczek 2005, Hâruta 2011]. Many research studies are focused on automatic analysis of images generated by microscopes, computer-assisted tomographs, magnetic resonances and digital cameras and camcorders [Goñi et al. 2008, Sekulska-Nalewajko and Gocławski 2009, Mieszkalski 2014a, 2014b, Rogge et al. 2014]. Fourier's elliptic descriptors are used to describe the shape of contours of biological objects [Eguchi and Ninomiya 2008, Marti-Puig et al. 2011, Mebatsion et al. 2011, Dalitz et al. 2013, Visa 2014]. For modeling of physical processes taking place in fruit cells and tissues, Abera et al. [2014] have designed a 3D generator to model fruit tissue. Ambawa et al. [2013] have developed the gas flow model through a pile consisting of 3D models of fruit. Siswantoro et al. [2014] used the Monte Carlo method to model the shape of 3D solids of biological objects.

The aim of the study is to improve smoothness of the apple shape modeled by using interpolation and determining the function values at intermediate points.

MATERIAL AND METHODS

The object of modeling is a Jonagored variety apple of the basic dimensions as follows: apple length h = 77.8 mm, width φa = 84.1 mm, thickness φb = 82.6 mm, dimensions h_1 = 15.1 mm, h_2 = 13.9 mm [Mieszkalski 2017a]. The data obtained from the 3D apple model [Mieszkalski 2017b] generated on the basis of a description of its contour was subjected to interpolation. For this purpose, the procedure of separation from the matrix of coordinates of nodal and control points of Bézier curves of the matrix of squares needed to apply the lspline procedure to smooth the data with spline functions. Using the interpolation procedure with spline functions, additional points were obtained, comprising the apple model surface, making it smooth. For the purpose of verification, a comparison was conducted of matching of the contours described by Bézier curves with the apple contours, as well as a comparison of the projection of the smoothed 3D model of the apple with its actual shape, determining the relative approximation error.

THE MODEL OF INTERPOLATION SPLINES IN MODELING OF THE APPLE SURFACE SHAPE

Separation from matrix $X1$, $Y1$, $Z1$ [Mieszkalski 2017b] of the matrix of squares $Xx1$, $Xx2$, …, $Xx7$, $Yy1$, $Yy2$, …, $Yy7$, $Zz1$, $Zz2$, … , $Zz7$ with 11 rows and 11 columns:

$$\begin{bmatrix} Xx1 \\ Xx2 \\ \vdots \\ Xx7 \end{bmatrix} = \begin{bmatrix} \text{submatrix}(X1,0,10,0,10) \\ \text{submatrix}(X1,11,21,0,10) \\ \vdots \\ \text{submatrix}(X1,60,70,0,10) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} Yy1 \\ Yy2 \\ \vdots \\ Yy7 \end{bmatrix} = \begin{bmatrix} \text{submatrix}(Y1,0,10,0,10) \\ \text{submatrix}(Y1,11,21,0,10) \\ \vdots \\ \text{submatrix}(Y1,60,70,0,10) \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} Zz1 \\ Zz2 \\ \vdots \\ Zz7 \end{bmatrix} = \begin{bmatrix} \text{submatrix}(Z1,0,10,0,10) \\ \text{submatrix}(Z1,11,21,0,10) \\ \vdots \\ \text{submatrix}(Z1,60,70,0,10) \end{bmatrix} \quad (3)$$

Determination of Axx_i, Ayy_i vectors of subsequent numbers of 0 to 10:

$$Axx_i = 0\ldots\text{rows}(Xx1) - 1 \quad (4)$$

$$Ayy_i = 0\ldots\text{rows}(Xx1) - 1 \quad (5)$$

Development of matrix Mxy of subsequent numbers of 0 to 10:

$$Mxy = \text{augment}(\text{sort}(Axx, \text{sort}(Ayy) \quad (6)$$

Determining of k, m vectors of subsequent numbers of 0 to 54:

$$k = 0\ldots p \cdot \text{rows}(Xx1) - 1 \quad (7)$$

$$m = 0 \ldots p \cdot \text{rows}(Xx1) - 1 \tag{8}$$

Development of xx_k, yy_m vectors of numbers of 0 to 10 with step of 0.185 in the following form:

$$xx_k = Mxy_{0,0} + k \cdot \frac{Mxy_{\text{rows}(Xx1)-1,0} - Mxy_{0,0}}{p \cdot \text{rows}(Xx1) - 1} \tag{9}$$

$$yy_m = Mxy_{0,1} + m \cdot \frac{Mxy_{\text{rows}(Xx1)-1,1} - Mxy_{0,1}}{p \cdot \text{rows}(Xx1) - 1} \tag{10}$$

Development of vectors of 124 rows $Wx1$, $Wx2$, … , $Wx7$, $Wy1$, $Wy2$, … , $Wy7$, $Wz1$, $Wz2$, …, $Wz7$ using the lspline procedure, generating a cubic spline:

$$\begin{bmatrix} Wx1 \\ Wx2 \\ \vdots \\ Wx7 \end{bmatrix} = \begin{bmatrix} \text{lspline}(Mxy, Xx1) \\ \text{lspline}(Mxy, Xx2) \\ \vdots \\ \text{lspline}(Mxy, Xx7) \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} Wy1 \\ Wy2 \\ \vdots \\ Wy7 \end{bmatrix} = \begin{bmatrix} \text{lspline}(Mxy, Yy1) \\ \text{lspline}(Mxy, Yy2) \\ \vdots \\ \text{lspline}(Mxy, Yy7) \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} Wz1 \\ Wz2 \\ \vdots \\ Wz7 \end{bmatrix} = \begin{bmatrix} \text{lspline}(Mxy, Zz1) \\ \text{lspline}(Mxy, Zz2) \\ \vdots \\ \text{lspline}(Mxy, Zz7) \end{bmatrix} \tag{13}$$

Interpolation functions $f1x(x,y)$, $f2x(x,y)$, …, $f7x(x,y)$ were recorded as follows:

$$\begin{bmatrix} f1x(x,y) \\ f2x(x,y) \\ \vdots \\ f7x(x,y) \end{bmatrix} = \begin{bmatrix} \text{interp}\left[Wx1, Mxy, Xx1, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \text{interp}\left[Wx2, Mxy, Xx2, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \vdots \\ \text{interp}\left[Wx7, Mxy, Xx7, \begin{pmatrix} x \\ y \end{pmatrix}\right] \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} f1y(x,y) \\ f2y(x,y) \\ \vdots \\ f7y(x,y) \end{bmatrix} = \begin{bmatrix} \text{interp}\left[Wy1, Mxy, Yy1, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \text{interp}\left[Wy2, Mxy, Yy2, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \vdots \\ \text{interp}\left[Wy7, Mxy, Yy7, \begin{pmatrix} x \\ y \end{pmatrix}\right] \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} f1z(x,y) \\ f2z(x,y) \\ \vdots \\ f7z(x,y) \end{bmatrix} = \begin{bmatrix} \text{interp}\left[Wz1, Mxy, Zz1, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \text{interp}\left[Wz2, Mxy, Zz2, \begin{pmatrix} x \\ y \end{pmatrix}\right] \\ \vdots \\ \text{interp}\left[Wz7, Mxy, Zz7, \begin{pmatrix} x \\ y \end{pmatrix}\right] \end{bmatrix} \tag{16}$$

The functions of attributing of interpolated data as square matrices of 55 components $F1x_{k,m}, F2x_{k,m}, \dots, F7x_{k,m}; F1y_{k,m}, F2y_{k,m}, \dots, F7y_{k,m}; F1z_{k,m}, F2z_{k,m}, \dots, F7z_{k,m}$ were recorded in matrix format:

$$\begin{bmatrix} F1x_{k,m} & F1y_{k,m} & F1z_{k,m} \\ F2x_{k,m} & F2y_{k,m} & F2z_{k,m} \\ \vdots & \vdots & \vdots \\ F7x_{k,m} & F7y_{k,m} & F7z_{k,m} \end{bmatrix} = \begin{bmatrix} fx(xx_k, yy_m) & fy(xx_k, yy_m) & fz(xx_k, yy_m) \\ fx(xx_k, yy_m) & fy(xx_k, yy_m) & fz(xx_k, yy_m) \\ \vdots & \vdots & \vdots \\ fx(xx_k, yy_m) & fy(xx_k, yy_m) & fz(xx_k, yy_m) \end{bmatrix} \tag{17}$$

Matrices of 55 columns and 385 rows *Fx, Fy, Fz,* of smoothed data for generation of the 3D model of the apple are as follows:

$$\begin{bmatrix} Fx \\ Fy \\ Fz \end{bmatrix} = \begin{bmatrix} \text{stack}(F1x, F2x, \cdots, F7x \\ \text{stack}(F1y, F2y, \cdots, F7y \\ \text{stack}(F1z, F2z, \cdots, F7z \end{bmatrix} \tag{18}$$

Figure 1 illustrates the chart of the 3D model of the apple contour with the marked contours prior to interpolation (bold contour lines). Figure 2 presents the chart of the 3D model of the apple after interpolation.

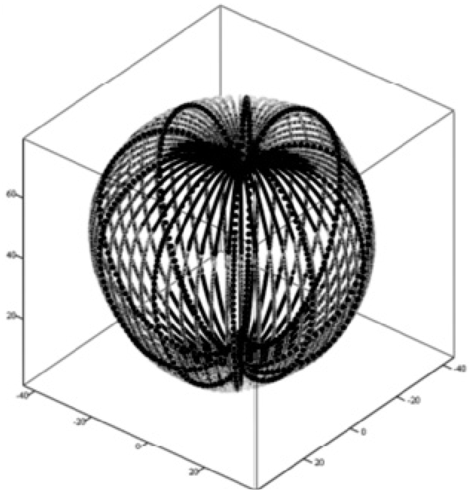

FIGURE 1. The chart of the 3D model of the apple contour with the marked contours prior to interpolation (bold contour lines)

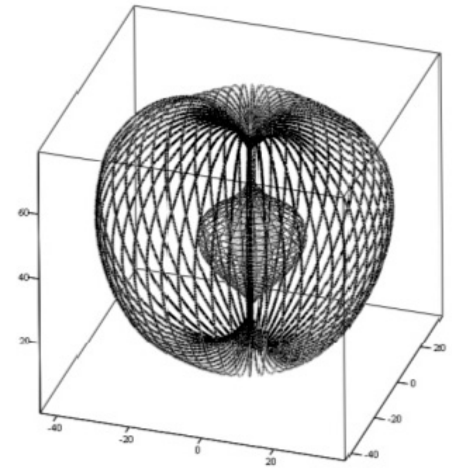

FIGURE 2. A chart, after smoothing of data, of the 3D model of the apple shape with the seeds the chamber and the seed nest

Figure 3 presents the apple rotated by the angle of 0°, 36°, 72°, 108°, 144° for the purpose of visual assessment of accuracy of matching of the contours described by Bézier curves and the apple contours.

FIGURE 3. Contours of the apple rotated by 0°, 36°, 72°, 108°, 144°

At the points, in which the contours described with Bézier curves do not match precisely the contours of the apple, the relative approximation error ranges from 1.59 to 3.32%. Figure 4 presents a comparison of projection of the 3D apple model and the actual apple shape. In the case of the model of the apple generated by interpolation (Fig. 4) at the points, in which the contours of the 3D model projection do not match precisely the contours of the apple in an analogical projection, the relative error is 4.28%.

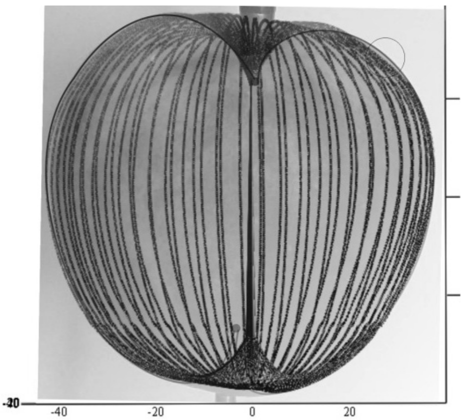

FIGURE 4. Comparison of the 3D apple model projection with the actual apple shape

SUMMARY

The method consisting of description of apple contours using Bézier curves may be applied to describe the shape of apples of other varieties, differing in terms of shape. Use of interpolation and approximation of data of the 3D model of apple shape did not result in exceeding of the relative approximation error by more than 5%. The method proposed can be used for modeling of shape of apples as biological objects with concavo-convex surface. The interpolation method for functions glued is laborious. Using the interpolation procedure with spline functions, additional points were obtained, comprising the apple model surface, making it smooth.

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Streszczenie: *Metoda rekonstrukcji 3D kształtu jabłek. Cz. 3. Geometryczny model 3D bryły jabłka z wykorzystaniem funkcji interpolacji.* Praca obejmuje prezentacje kształtu jabłka odmiany Jonagored. Kontury jabłka opisano łączonymi krzywymi Béziera, a następnie przeprowadzono proces interpolacji. Do modelowania komory nasiennej i gniazda nasiennego zbliżonych kształtem do brył obrotowych opracowano metodę polegającą na obrocie linii tworzącej, którą stanowią dwie gładko połączone krzywe Béziera. Do tworzenia modelu kształtu jabłka wykorzystano wyniki zamieszczone w drugiej części artykułu.

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