Poisson's ratio of anisotropic biological media

Bożena Gładyszewska

Department of Physics, University of Life Sciences, ul. Akademicka 13, 20-950 Lublin, Poland
e-mail: bozena.gladyszewska@up.lublin.pl

S u m m a r y . A model of a honeycomb structure is presented showing that in particular conditions Poisson's ratio determined for bodies revealing such structure may well exceed not only 0.5 limit but even may reach values above 1. The model is based on a real cell structure observed in scanning electron microscopy images. Experimental results obtained for fresh tomato skin using a uniaxial strength test are presented and interpreted as an exemplary case. A model of a honeycomb cell is used to compare experimental and calculated results.

K e y w o r d s : Poisson's ratio, tomato skin, mechanical properties.

INTRODUCTION

Studies performed on biological materials often lead to mechanical parameters that are beyond the limits of the elasticity theory. Particularly, Poisson's ratio (ν) measured basing on uniaxial strength tests reaches values that exceed 0.5 or even 1.0. Poisson's ratio is a negative ratio of transverse dimension change to longitudinal dimension change of a body under a stress applied along the longitudinal direction [1, 12]. According to the elasticity theory [12, 18] for isotropic 3D media $-1 < \nu < 0.5$ and this limit is enhanced for 2D isotropic samples where $-1 < \nu < 1$. However, biological objects are rarely isotropic. Most of these materials have quite complicated structure and reveal different properties depending on geometrical directions chosen as the main axis in the uniaxial strength test. It is therefore necessary to stress that for such anisotropic 3D and particularly 2D materials the limits mentioned above become non valid. There are many examples in the literature showing, that Poisson's ratio reaches values "beyond the limits" [2, 13, 14, 17]. Among others Tilleman et al. [16] found that for studied biological materials Poisson's ratio ranged from 0.25 up to 0.85. Hamza et al. [10] measured Poisson's ratio for maize roots (whole roots, stele with cortex combined) and obtained $\nu=0.63$. Elliott et al. [3] studied human patellar articular cartilage and found that Poisson's ratio in the surface zone is about 1.9 and in the middle zone about 0.6. Hepworth and Bruce [11] reported Poisson's ratio of the onion epidermal tissue that reaches 2.0.

In this work, a model of a honeycomb structure is presented showing that in particular conditions Poisson's ratio determined for bodies revealing such structure may well exceed not only 0.5 limit but even may reach values above 1. A honeycomb model can also be used to explain negative Poisson's ratio values. In that case so called re-entrant honeycomb structure is proposed [15]. As an exemplary case, experimental results obtained for fresh tomato skin are presented.

EXPERIMENT

Poisson's ratio of the tomato skin was determined on the base of uniaxial strength tests. The method of random marking was used [5-8]. This method is based on the analysis of the image and the distance of mutual position of the points on the surface of the studied sample being under the tests of uniaxial stretching. After starting the test, the image of the stretched specimen together with graphite markers randomly sprayed on its surface and the current value of the tensile force corresponding to each image are transmitted to a computer memory. The signal from the tensometer is transmitted with the use of an analogue-to-digital converter to the computer memory and the registered image of the stretched specimen to the input of a video card. With the use of a CCD camera equipped with microscope lens which enabled to view the specimen at 240x320 pixel resolution and 5x magnification, it is possible to observe stretched specimen. The specimen is placed in clamping grips connected to a Megaton Electronic AG&Co. KT-1400 tensometer with a force measurement range of 0-100 N.

The main advantage of the used random marking method is that the obtained results are independent from the effects occurring at the edge areas of the specimen. It is also possible to perform measurements in the chosen place of the studied material. An additional advantage is a possibility of monitoring the force instead of the total increase of the strain as takes place in majority of strength testing machines.

MODEL

The simplest model of the body that leads to Poisson's ratio exceeding 1.0 can be elaborated assuming the rhombic structure of tested specimen cells. Figure 1 shows a schematic view of changes occurring in such model structure under applied uniaxial force and defines characteristic geometric parameters of the rhombic cell. Assuming that the rhomb sides are perfectly stiff and only angles between them change, Poisson's ratio can be defined as:

$$v = -\frac{dy}{dx}. \tag{1}$$

This simplification is possible because at the initial stage $x=y$. Taking into account that:

$$a^2 = x^2 + y^2, \tag{2}$$

$$x = \frac{a\sqrt{2}}{2} + dx, \tag{3}$$

$$y = \frac{a\sqrt{2}}{2} + dy, \tag{4}$$

we obtain:

$$v = -\frac{dy}{dx} = -\frac{-\frac{a\sqrt{2}}{2} + \sqrt{a^2 - \left(\frac{a\sqrt{2}}{2} + dx\right)^2}}{dx}. \tag{5}$$
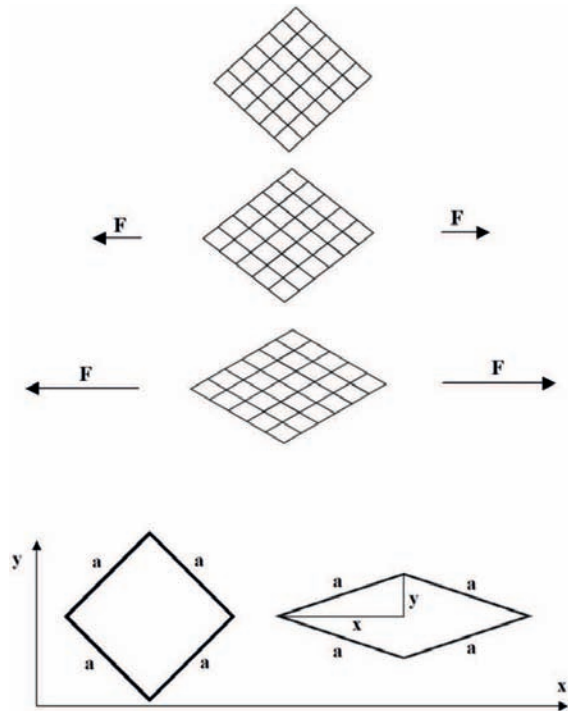

Fig. 1. Hypothetical changes of a rhombic cell under an applied uniaxial force F. Parameters a, x and y determine a rhombic cell dimensions

Figure 2 shows For such structure Poisson's ratio reaches values above 1.0. This simple model has been applied earlier to explain results ($v > 1$) obtained for dried bean covers [4, 6] as their cell structure is similar to this assumed in the model. In the case of dried bean covers that are characterized by relatively stiff cell sides in comparison to a medium inside a cell, this model seems to be adequate. However, in the case of a fresh tomato skin a cross section images reveal rather a honeycomb structure of a cell than a rhombic one. Moreover sides of such honeycomb cells are surely not so stiff, so a more developed model should be used. The model should also
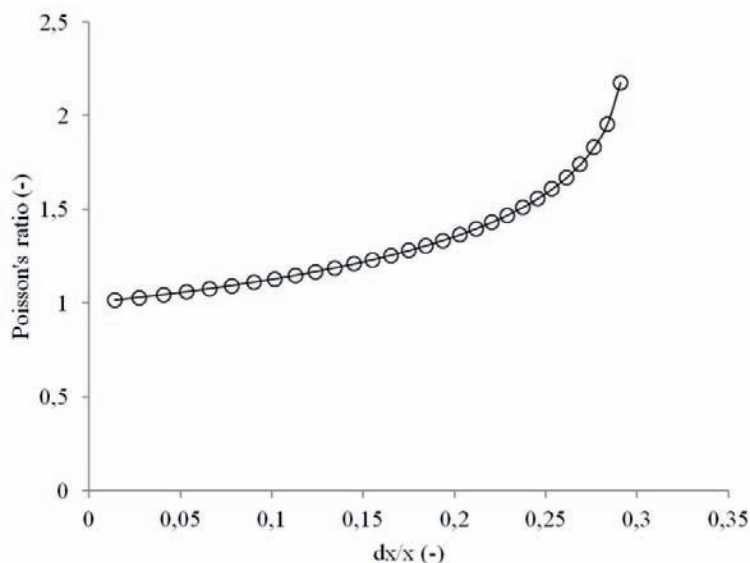

Fig. 2. Dependence of Poisson's ratio on a relative longitudinal dimension change

provide a constant value of Poisson's ratio when a longitudinal dimension of the cell changes.

Figure 3 presents a cross section image obtained for a fresh tomato skin (of fruits cv. Admiro) with the use of the BS300 TESLA scanning electron microscope (STM) at accelerating voltage of 15 kV [9]. The image illustrates the cell structure of a specimen. Therefore, in the model a corresponding structure has been assumed. Figure 4 shows a schematic view of the elongated honeycomb cells and also defines characteristic geometric parameters of the cell.

Momentary values of x, y and b under longitudinal strength are as follows:

$$x = x_0 + dx, \tag{6}$$

$$y = y_0 + dy, \tag{7}$$

$$b = b_0 + db, \tag{8}$$

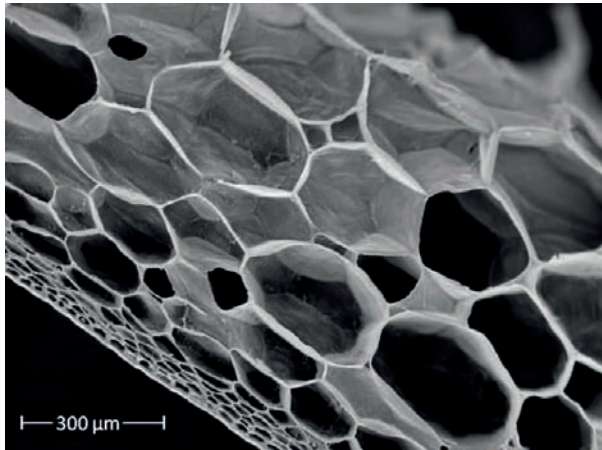where: x_0, y_0 and b_0 indicate cell dimensions at the initial stage.

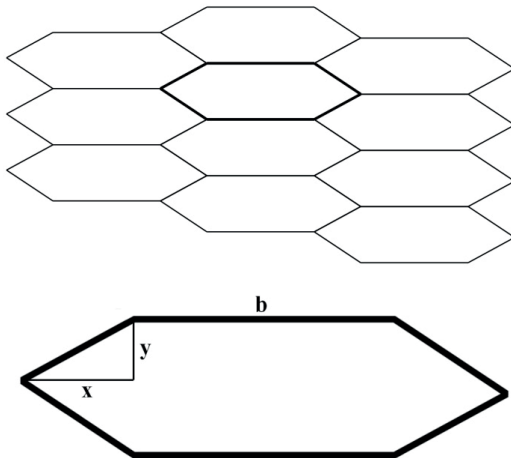**Fig. 3.** STM cross section image obtained for a fresh tomato skin (fruits cv. Admiro) [9]

Fig. 4. Schematic view of the elongated honeycomb cells used in the model. Parameters x, y and b determine a honeycomb cell dimensions

where: ε_y and ε_x are transverse dimension change and longitudinal dimension change of a body, respectively, and taking into account that

$$\varepsilon_x = \frac{dx + db}{x_0 + b_0}, \tag{9}$$

$$\varepsilon_y = \frac{dy}{y_0}, \tag{10}$$

we can calculate the transverse dimension change:

$$y = -\frac{v \cdot y_0 \cdot c \cdot dx}{x_0 + b_0}, \tag{11}$$

where:

$$c = 1 + \frac{b_0}{x_0}. \tag{12}$$

This provides that in the frame of the presented model Poisson's ratio remains constant during the honeycomb cell evolution. It is necessary to mention that from mathematical point of view there is no limitation of Poisson's ratio in the model.

RESULTS AND DISCUSSION

Figure 5 presents as an example the experimental dependences $\varepsilon_x(\sigma)$ and $\varepsilon_y(\sigma)$, where σ means stress, obtained for the stretched fresh tomato skin specimen. From the linear fits given in the insets the calculated Poisson's ratio $v=0.825$. Introducing this value into the model it is possible to estimate changes of longitudinal and transverse dimensions of the honeycomb cells. Figure 6 shows the calculated dependences ε_x and ε_y on a relative longitudinal dimension change

$$S = -\frac{2y_0 vc^2}{x_0 + b_0}(dx)^2 + 2y_0 c(1-v)dx + 2y_0(x_0 + b_0). \tag{13}$$

dx/x. The input parameters x_0=100 μm, y_0=100 μm and b_0=400 μm has been set basing on the image shown in Fig. 3. The assumptions taken in both presented models cause that the cross section surface S changes during a cell evolution (see Fig. 7). From this plot we note that the total surface of a cell cross section increases when $dx/x < 0.105$, reaches its maximum (at this point $\Delta S/S = +0.1\%$) and for $dx/x > 0.105$ decreases, because the total cell surface follows equation (13) as calculated for honeycomb cell. It is also worth to note that the dependence of Poisson's ratio on a relative longitudinal dimension shown in Fig. 2 is nearly identical with that obtained by Hepworth and Bruce [11] who studied changes of Poisson's ratio of the onion epidermal tissue as a function of the tensile strain applied parallel to the long axis of the cell. Both experimental and the proposed by the authors unrestricted reorientation model results show that
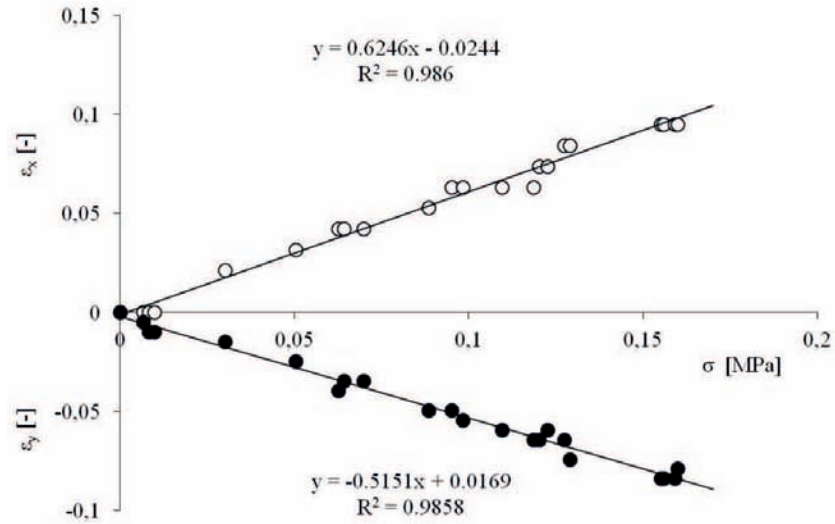
Fig. 5. Experimental dependences $\varepsilon_x(\sigma)$ and $\varepsilon_y(\sigma)$ obtained for the stretched fresh tomato skin specimen
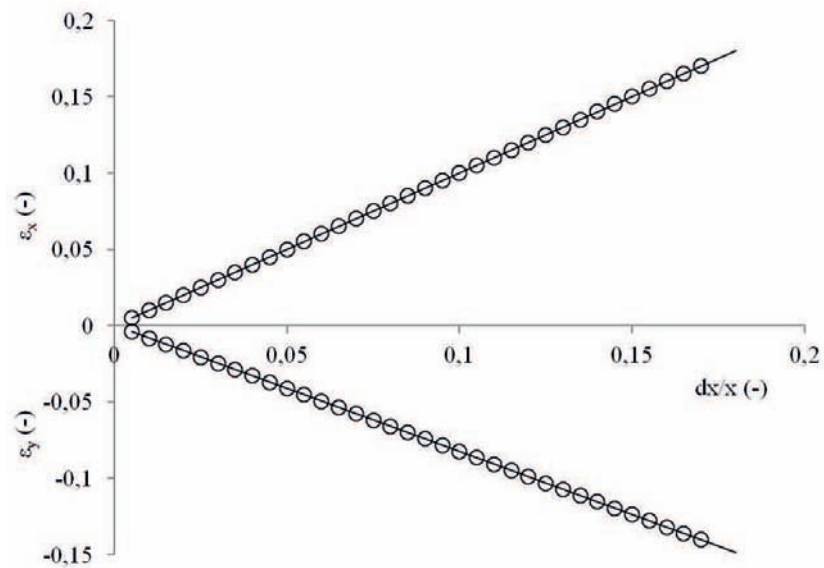

Fig. 6. Calculated dependences ε_x and ε_y on a relative longitudinal dimension change dx/x calculated for a honeycomb cell where $x_0=100$ μm, $y_0=100$ μm and $b_0=400$ μm
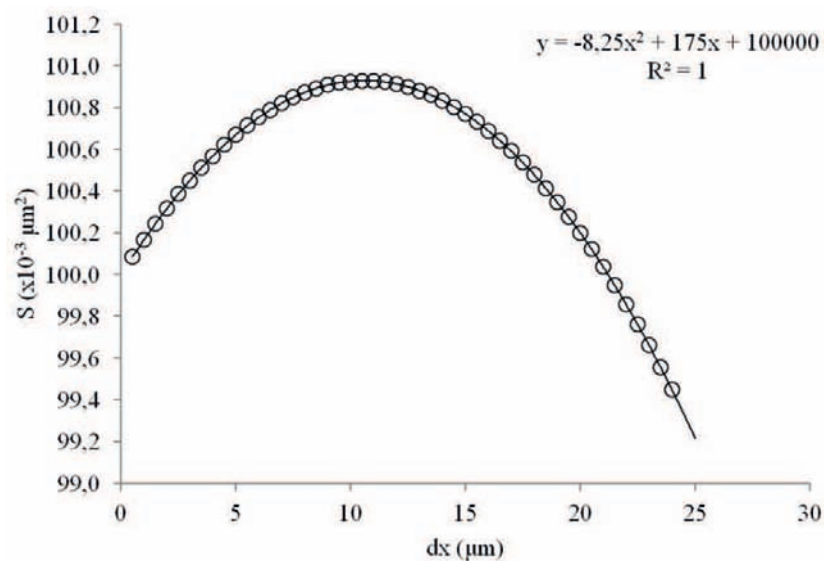

Fig. 7. Evolution of the cross section surface S of a honeycomb cell with the longitudinal dimension change dx

Poisson's ratio measured for biological media may well exceed 1.0 (in the case of results presented in the mentioned work, ν reaches 2.0). The model presented here is based on another cell structure, however also proves that for biological anisotropic media, relatively high values of Poisson's ratio can be explained with models that are created basing on realistic cell structure, for example mimicking observed cross section images obtained with the use of the scanning electron microscopy.

CONCLUSIONS

1. Poisson's ratio exceeding 1.0, frequently measured in biological materials, should not be considered as beyond the limits and therefore – not correct. The $-1 < v < 0.5$ limit for 3D materials and $-1 < v < 1$ for 2D materials is valid for isotropic body. Biological media are often anisotropic and therefore do not follow limits given by the elastic theory.
2. Models based on a real cell structure observed via different microscopy techniques, can be used to interpret high values of Poisson's ratio.
3. Model of a honeycomb cell seems to be adequate for interpreting results of mechanical parameters obtained from uniaxial strength tests of a tomato skin.

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WSPÓŁCZYNNIK POISSONA ANIZOTROPOWYCH
MATERIAŁÓW BIOLOGICZNYCH

S t r e s z c z e n i e . W pracy zaproponowano model oparty na strukturze plastra miodu, umożliwiający wykazanie, że w szczególnych warunkach materiały o takiej strukturze mogą charakteryzować się współczynnikiem Poissona nie tylko przekraczającym wartość 0,5, ale nawet większym od 1. Model oparty jest na obrazach rzeczywistej struktury skórki pomidora obserwowanej przy użyciu skaningowej mikroskopii elektronowej. Wyniki doświadczalne uzyskane w testach jednoosiowego rozciągania skórek świeżego pomidora zostały poddane analizie, jako przykładowe. Model struktury plastra miodu został wykorzystany do porównania wyników doświadczalnych i obliczonych.

S ł o w a   k l u c z o w e : współczynnik Poissona, skórka pomidora, właściwości mechaniczne.