

Investigation of the process of vibrorheology of cement concrete solutions with the external source of dynamic effect

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S u m m a r y . In the article in the frames of the semi-phenomenological approach there have been formulated the main requirements to the mathematical model of movement of the mortar and its interaction with the filler in the planar case, when the external variable force field of the excitation has a turbulent component. It is given the strict interpretation of the influence of the parameters of the model under investigation on the process of vibrorheology of cement concrete solutions and was formulated the criterion of its effectiveness.

Key words: vibrorheology of cement concrete solutions, turbulence, swirling, tensors of the first and second viscosity.

INTRODUCTION

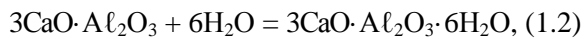
The modern mathematical constructions as according to the theory of vibrorheology of cement concrete solutions [2-5, 7, 8, 11, 15, 17-20] are based on the one and the same unique approach the initial position of which is the consideration of some concrete form with fixed geometry filled with multi component media in the type of visco-plastic fluid with the filler which is under the conditions of periodic vibro excitation within the certain period of time $[0, T]$. The description of the movement of working material is a contact problem taking into account the boundary conditions [12, 13]. It is

easy to see that the exact solution of the problem of compression of cement solution provides its deterministic nature, which is a consequence of the complete task of the behavior of working environment of shuttering form on its boundary. But it is not possible for even the simplest geometric configuration of the border. Thus, the creation of the theory of the vibrorheology in a separately taken shuttering form does not lead to the description of the general laws of interaction of cement solution with the filler which would bring the research process to certain technological solutions and recommendations, including the ability to control the compaction process based on the combination of the values of the constructive parameters of the problem. We cannot count on a clear prospect of application research based on effective technology without a new consequent systematic approach to the theory vibrorheology. This situation underlines the chronic weakness of the previously mentioned concept of the vibrorheology (and in some cases the use of research schemes which unacceptable), improving the design of the shuttering form when done as the result of analysis of numerous experimental data by the method of trials and errors without stringent

performance criteria and in the absence of theoretical results of a general character.

STATEMENT OF THE PROBLEM

Let's consider a plane task associated with the description of the process of the medium of cement solution filling at the initial time $t = 0$ in the state of the rest all Euclidean plane R^2 with Cartesian rectangular coordinate system XOY . At time $t = 0$ in the cement solution starts hydration and hydrolysis of the constituents of cement clinker – tricalcium silicate and tricalcium aluminate with the formation respectively dicalcium hydrosilicate, calcium hydroxide and tricalcium hydroaluminate:



which are gradually moving from one colloidal state to the state of crystallization and, henceforth, to the state of cement stone.

Here and further we define two states of working media: a-state (a priori state of rest) and e-state (state of excitation). It should be marked that the medium of cement solution as visco-plastic fluid transfers from homogeneous and isotropic a-state into e-state («a – e» transfer) with the help of external variable plane-parallel field $\vec{F}(N, t)$, $N(x, y) \in R^2$ which is switched on at the moment of time $t = 0$ and which is a general model of vibro excitation that is the formation of colloidal medium and as the consequence of chemical reactions (1.1) and (1.2), and e-state begins at the moment of time $t = 0$.

The term general model should be understood in the sense that at the special task of vibro field $\vec{F}(N, t)$ can be studied the plane task and in restricted simply connected field $G \subset R^2$. a-state and e-state will be characterized by their sets of the parameters which we'll fix by the indexes correspondingly «a» and «e». For example, the mass surface density of the cement solution (without filler)

let's denote by the symbol $\rho_a = m_{1a}n_{1a} + \rho'_a$, where m_{1a} – mass of a separate cement lobe, n_{1a} – surface density of the particles of cement, ρ'_a – surface density of water in the solution etc.

The statement of the problem «a – e» of transfer of the medium of cement solution is reduced to the following:

- to explore the response of the system under consideration at the model level visco-plastic fluid to an external alternating field $\vec{F}(N, t)$,

- and to consider the process of the interaction of massive flows appearing and filled as the main theoretical and technological working mechanism with the aim of definition of effective viscosity of concrete solution and optimal parameters of the external field of excitation for obtaining maximum density of modified concrete.

It should be marked that full description of the response of working medium to the effect of force vibro field $\vec{F}(N, t)$ should be conducted in the terms of indexes $\{\rho_e(N, t), \bar{\pi}_e(N, t), \varepsilon_e(N, t)\}$, where $\rho_e(N, t)$ – surface density of a mass, $\bar{\pi}_e(N, t)$ – surface density of impulse, $\varepsilon_e(N, t)$ – surface density of an energy. In this case the main parameter is the mass density $\rho_e(N, t)$, through which are easily expressed the other parameters. This is due to the fact that the movement of the cement solution in the presence of an external field $\vec{F}(N, t)$ is compulsory and, therefore, is not the nature of the relaxation. Moreover, dissipative processes in the moving working medium are due only to mechanical sticking of the particles during the two-, three-, and, in general, n-particle sticking with the formation of aggregated particles as the centers of the further relaxation process (crystallization) are already in the absence of an external excitation field. This is so-called – internal friction which is the effect of intermolecular interaction and is associated with the classical transport phenomena in liquids and gases, which can be interpreted by the language of the laws of conservation of mass, impulse and energy, and induced by swirling properties of the force

field $\vec{F}(N, t)$. It should be noted that the presence of turbulence in the working medium leads to a tensor-like distribution of the velocities of two components of cement solution: viscous-plastic liquid of cement particles and liquid vortex flows.

MATERIALS AND RESULTS OF RESEARCH

1. Now we formulate the basic terms and principles (system of axioms) of construction of model of the process under research.

A1. Cement solution with the filler in a-state fills the entire Euclidean plane R2: cement solution and k-component filler have surface density of a mass correspondingly:

$$\rho_a = \rho_{1a} + \rho'_a, \quad \rho_{1a} = m_{1a} n_{1a} \quad (1.3)$$

and

$$\tilde{\rho}_a = \sum_{i=2}^{k+1} m_{ia} n_{ia}, \quad (1.4)$$

where: m_{ia}, n_{ia} – mean mass of a particle and surface density of i-components correspondingly, $i = 2, 3, \dots$

A2. At the moment of time $t = 0$ switches on the external variable plane parallel force field providing «a – e» transfer and having surface density of type

$$\vec{F}(N, t) = \vec{F}_{s.p.}(N, t) + \vec{F}_{r.d.}(N, t), \quad (1.5)$$

where: $\vec{F}_{s.p.}(N, t)$ – solenoidal potential field, for which, with the exception of not more than the estimated number of points, $\text{div } \vec{F}_{s.p.}(N, t) = 0$ and $\text{rot } \vec{F}_{s.p.}(N, t) = 0$, and $\vec{F}_{r.d.}(N, t)$ is a component of the turbulent with $\text{rot } \vec{F}_{r.d.}(N, t) \neq 0$ and $\text{div } \vec{F}_{r.d.}(N, t) \neq 0$.

It is easy to see that the surface density of forces $\vec{F}_{r.d.}(N, t)$ – is the vector random variable defined on some initial probable space $\{\Omega, U, P\}$, where $\Omega = \{\omega\}$ space of elementary events ω , $U = \{A\}$ – algebra (or σ -algebra) of some event A и $P = P(A)$ – probability function. Choosing the probability space is associated with the features of the mechanism of vibro excitation and will not be explored in

this work. The action of vibro field (1.5) defines only dynamic properties of cement concrete solution and does not affect the physical chemical processes of its modification.

A3. The presence of external force field in the structure (1.5) of turbulent component $\vec{F}_{r.d.}(N, t)$ contributes to the emergence of vortex flows in the cement mass $\bar{m}_e(t)$ and surface density $\bar{n}_e(N, t)$, which is random function. Surface density of a mass of vortex flows and the density of flow of mass of vortex flows are respectively:

$$\bar{\rho}_e(N, t) = \bar{m}_e(t) \bar{n}_e(N, t), \quad (1.6)$$

$$\bar{\pi}_e = \bar{m}_e(t) \bar{n}_e(N, t) \bar{w}_e(N, t), \quad (1.7)$$

where: $\bar{n}_e(N, t)$ – density of the number of vortex flows in the point $N = N(x, y)$,

$\bar{w}_e(N, t)$ – the rate of the flow of vortex flows in the point $N = N(x, y)$.

Let's define the average index of the density $\bar{n}_e(N, t)$ using dynamic limiting transition:

$$\bar{n}_e(t) \equiv \langle \bar{n}_e(N, t) \rangle \stackrel{\text{def}}{=} \lim_{S \rightarrow \infty} \frac{1}{\mu(S)} \iint_S \bar{n}_e(N, t) d\sigma_N, \quad (1.8)$$

where: $d\sigma_N = dx dy$,

$\mu(S)$ – the square of simply connected field $S \subset R^2$.

Thus, the dynamic density of vortex flows $\bar{n}_e(t)$ can be considered as a random process with the discrete or continuous parameter $t, t \geq 0$. Let's note that here the meaning of the transition boundary (1.8) is to eliminate the influence of the boundary and thereby meet the requirements of the basic problem.

A4. The vortex flow is the unification at one point $M_0(x_0, y_0) \in R^2$ of the vortex flow with the intensity $E(t)$ and virtual flow with the intensity $H(t)$, so that:

$$H(t) = \lambda E(t), \quad t \geq 0, \quad 0 < \lambda < 1, \quad (1.9)$$

where: λ – coefficient of a mass dispersion in the ordinary liquid vortex flow.

The ratio (1.9) defines the specifics of the vortex flow: if $\text{rot } \vec{F}_{r,d}(M_0, t) \neq 0$, then the circulation of the vector field $\vec{F}_{r,d}(N, t)$ leads with the effect of inner friction in the cement solution to the dispersion of the mass in the ordinary liquid with the intensity $H(t)$, which is equivalent to the action of the flow at the point $M_0(x_0, y_0)$. The point $M_0(x_0, y_0)$ moves in the cement solution along the line of the flow of the function $\vec{F}_{s,p}(N, t)$, coinciding with the line of the break of the function $\vec{F}_{r,d}(N, t)$, on the pass:

$$x_0 = x_0(t), \quad y_0 = y_0(t), \quad t \geq 0. \quad (1.10)$$

A5. In e-state the cement solution is considered as two liquid medium with the qualitatively different components.

The first component is the cement liquid consisting of cement particles with the mass $m_{1e}(t)$ and the ordinary liquid and has the surface density of a mass $\rho_e(N, t)$ and the density of the flow of a mass $\vec{\pi}_e(N, t)$ as:

$$\rho_e(N, t) = \rho_{1e}(N, t) + \rho'_e(N, t), \quad (1.11)$$

$$\vec{\pi}_e(N, t) = [\rho_{1e}(N, t) + \rho'_e(N, t)] \vec{v}_{1e}(N, t), \quad (1.12)$$

where: $\rho_{1a}(N, t) = m_{1e}(t) n_{1e}(N, t)$ – the surface density of a mass of cement particles,

$n_{1e}(N, t)$ – the surface density of the number of cement particles,

$\rho'_e(N, t)$ – the surface density of the ordinary liquid,

$\vec{v}_{1e}(N, t)$ – the rate of cement liquid at the point $N = N(x, y)$.

And the second component is the liquid of vortex flows with the surface density of a mass (1.6) and surface density of the flow of a mass (1.7). It should be noted that the presence of the turbulence in the working medium leads to the tensor type of the distribution of the rates of two component cement solution: viscous plastic liquid of cement particles and the liquid of vortex flows. The dependence

between the rates $\vec{v}_{1e}(N, t)$, $\vec{w}_e(N, t)$ and the external force field $\vec{F}(N, t)$ will be defined below.

A6. The action of solenoidal potential field $\vec{F}_{r,d}(N, t)$ promotes the emergence of two virtual force solenoidal potential vector fields in this model $\vec{G}_{s,p}^{(1)}(N, t)$ и $\vec{G}_{s,p}^{(2)}(N, t)$, which accordingly define the dynamics of the first and the second components correspondingly that is viscous plastic liquid and the liquid of vortex flows.

A7. The transfer of the mass of two components cement liquid in the presence of the external force field $\vec{F}(N, t)$ (1.5) with turbulent constituent $\vec{F}_{r,d}(N, t)$ takes place along the lines of the flow of force solenoidal potential field $\vec{F}_{s,p}(N, t)$ with the rate:

for the first component with the rate:

$$v_{1ei}(N, t) = \int_0^t \iint_{0R^2} \varepsilon_{ik}^{(1)}(N - P, t - \tau) F_{s,p,k}(P, \tau) d\tau d\sigma_P, \quad (1.13)$$

for the second component moving along the line of the flow of the field $\vec{G}_{s,p}^{(2)}(N, t)$ or $\vec{F}_{s,p}(N, t)$ on the pass (1.10) of the line of turbulence for which $\text{rot } \vec{F}_{r,d}(N, t) \neq 0$, with the rate:

$$w_{ei}(N, t) = \int_0^t \iint_{0R^2} \varepsilon_{ik}(N - P, t - \tau) F_{s,p,k}(P, \tau) d\tau d\sigma_P. \quad (1.14)$$

Bivalent tensors $\varepsilon_{ik}^{(1)}(N, t)$ и $\varepsilon_{ik}(N, t)$, which are in the formulas (1.13) and (1.14), should be considered as the coefficients of effective viscosity in two component cement solution.

A8. Let's consider that the radius (r) of vortex flow is distributed according to normal law with expectation function $\alpha = 0$ and root-mean-square deviation $\sigma = \sigma(t)$, which is the function of the parameter (t), that is the density of the probability of random variable (r) is equal:

$$p_{0,\sigma}(r) = \frac{r}{\sigma^2(t)} \cdot e^{-\frac{r^2}{2\sigma^2(t)}}. \quad (1.15)$$

The action of vibro field $\bar{F}(N, t)$ takes place in the time interval $[0, T]$, where T – the time of technological functionality of colloidal medium when the dispersion of the mass of ordinary liquid stops and starts the process of crystallization of gel with dicalcium hydrosilicate, calcium hydroxide and tricalcium aluminate hydrate. If the critical mass of colloidal cement particle is equal to $m_{l_{kp}} = \beta_1 m_{1a}$, $\beta_1 > 1$, then the index of the parameter τ_{tec} – technological time of modification of cement solution in the presence of vibro field $\bar{F}(N, t)$ is in equations

$$m_{l_{kp}} \equiv m_{1e}(\tau_{tec}) = \beta_1 m_{1a}. \quad (1.16)$$

Then the criterion of the effectiveness of the process of the modification of cement solution in the presence of vibro field $\bar{F}(N, t)$, is:

$$\tau_{tec} \leq T, \quad (1.17)$$

where: τ_{tec} – solution of equations (1.16).

A9. The presence of the filler does not affect the mechanical, physical and chemical technological parameters of vibrorheologic process of conversion of cement solution into the state of cement stone.

2. Let's consider further the semi-phenomenological model of vibrorheology of cement concrete solution.

We'll form based on the axioms A1-A9 formulated in the previous item, the mathematical mechanism of the research of the contact task of mutual movement of the medium of cement solution and the filler in the presence of the external vibro field $\bar{F}(N, t)$. First of all, let's find the actual expression of the surface density of a mass of cement solution (1.3) in a-state. For a mass m_{1a} of a separate cement particle we'll have:

$$m_{1a} = \frac{4}{3} \pi r_{1a}^3 \gamma_{1a}, \quad (2.1)$$

where: r_{1a} – average radius of cement particle,

γ_{1a} – volume mass density of cement lobe.

Thus, the surface density of a mass ρ_{1a} of cement particles in a-state is equal to:

$$\rho_{1a} = m_{1a} n_{1a} = \frac{4}{3} \pi r_{1a}^3 \gamma_{1a} n_{1a}. \quad (2.2)$$

Taking into account (2.2) for the surface density of a mass ρ'_a of ordinary liquid in a-state we get:

$$\rho'_a = \left(\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{1a}^3 \right) \gamma'_a n_{1a}, \quad (2.3)$$

where: γ'_a – volume density of a mass of ordinary liquid in a-state.

On the base of the formulas (2.2) and (2.3) the surface density of a mass of cement solution (1.3) in a-state will be:

$$\begin{aligned} \rho_a &= \rho_{1a} + \rho'_a = \\ &= \frac{4}{3} \pi r_{1a}^3 \gamma_{1a} n_{1a} + \left(\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{1a}^3 \right) \gamma'_a n_{1a}. \end{aligned} \quad (2.4)$$

Further in e-state the surface density of a mass of cement solution $\rho_e(N, t)$, taking into account the formulas (1.6) and (1.11), is defined by the equity:

$$\begin{aligned} \rho_e(N, t) &= \rho_e(N, t) + \bar{\rho}_e(N, t) = \\ &= \bar{m}_e(t) \bar{n}_e(N, t) + m_{1e}(t) n_{1e}(N, t) + \rho'_e(N, t). \end{aligned} \quad (2.5)$$

Making (2.5) dynamic boundary transfer (1.8), we get:

$$\langle \rho_e(N, t) \rangle = \langle \rho_e(N, t) \rangle + \langle \bar{\rho}_e(N, t) \rangle, \quad (2.6)$$

or

$$\begin{aligned} \rho_e(t) &= \rho_e(t) + \bar{\rho}_e(t) = \\ &= m_{1e}(t) n_{1e}(t) + \rho'_e(t) + \bar{m}_e(t) \bar{n}_e(t). \end{aligned} \quad (2.7)$$

If in (1.5) is absent the turbulent component $\bar{F}_{r,d}(N, t)$, that is $\bar{F}_{r,d}(N, t) \equiv 0$, then the formula (2.7) will be:

$$\rho_e(t) \equiv \rho_e(t) = m_{1e}(t) n_{1e}(t) + \rho_e'(t). \quad (2.7')$$

Let's mark that in the formula (2.5) the indexes $m_{1e}(t)$, $n_{1e}(N, t)$, $\rho_e'(N, t)$, have determined character and the index $\bar{n}_e(N, t)$ is the random function. First we find the mass $\Delta\rho'(t)$ of ordinary liquid dispersed by the vortex flow. As the geometric features of the vortex flow do not depend on its state on the plane R^2 , then for the simplicity of subsequent constructions we place the vortex flow into center of coordinates $O(0, 0)$, supposing its radial symmetry. We find with the help of the formula (1.15) the average radius \bar{r} of vortex flow:

$$\begin{aligned} \bar{r} = \bar{r}(t) &= \int_0^\infty r p_{0,\sigma}(r) dr = \\ &= \frac{1}{\sigma^2(t)} = \int_0^\infty r^2 e^{-\frac{r^2}{2\sigma^2(t)}} dr = \sqrt{\frac{\pi}{2}} \sigma(t). \end{aligned} \quad (2.8)$$

We calculate the circulation of vortex point (Fig. 1):

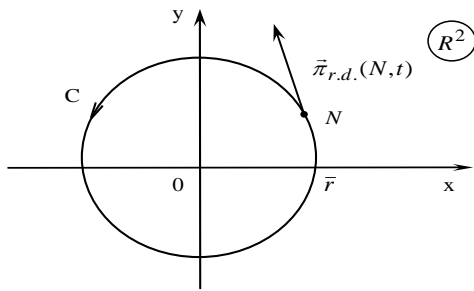


Fig. 1. Vortex flow is in the center of coordinates and average radius \bar{r}

$$E(t) = \oint_C (\bar{\pi}_{r,d}(N, t), d\bar{s}_\tau), \quad C: x^2 + y^2 = \bar{r}^2, \quad (2.9)$$

where: $\bar{\pi}_{r,d}(N, t)$ – the density of the flow of mass of vortex is equal to:

$$\begin{aligned} \bar{\pi}_{r,d}(N, t) &= [m_{1e}(t) n_{1a} + \rho_e''(t)] \bar{v}(N, t), \\ \bar{v}(N, t) &= K(N, t) \bar{F}_{r,d}(N, t), \end{aligned} \quad (2.10)$$

where: $K(N, t)$ – coefficient of turbulence of field $\bar{F}_{r,d}(N, t)$, $\rho_e''(t)$ – surface density of a mass of ordinary liquid supposing that $\bar{n}_e(t)$ is equal to zero.

Inserting (2.10) into (2.9), we get:

$$\begin{aligned} E(t) &= m_{1e}(t) \oint_C n_{1a} K(N, t) (\bar{F}_{r,d}(N, t), d\bar{s}_\tau) + \\ &+ \oint_C \rho_e''(t) K(N, t) (\bar{F}_{r,d}(N, t), d\bar{s}_\tau) \end{aligned} \quad (2.11)$$

Taking into account that $d\bar{s}_\tau = \bar{s}_0 ds$,

where: \bar{s}_0 – unit vector of tangent line to the wire circuit C in the point N, $\bar{F}_{r,d}(N, t) \parallel \bar{s}_0$, on the base of (2.11) we get:

$$\begin{aligned} E(t) &= m_{1e}(t) \bar{r}(t) \int_0^{2\pi} n_{1a} K(N, t) |\bar{F}_{r,d}(N, t)| d\varphi + \\ &+ \bar{r}(t) \int_0^{2\pi} \rho_e''(t) K(N, t) |\bar{F}_{r,d}(N, t)| d\varphi. \end{aligned} \quad (2.12)$$

Thus, mass $\Delta\rho'(t)$ of ordinary liquid dispersed by the vortex flow is equal to:

$$\begin{aligned} \Delta\rho'(t) &= \lambda \int_0^t E(\tau) d\tau = \\ &= \lambda \int_0^t \int_0^{2\pi} [m_{1e}(\tau) \bar{r}(\tau) n_{1a} + \bar{r}(\tau) \rho_e''(\tau)] K(N, \tau) \times \\ &\times |\bar{F}_{r,d}(N, \tau)| d\tau d\varphi, \end{aligned} \quad (2.13)$$

$$\bar{r}(t) = \sqrt{\frac{\pi}{2}} \sigma(t), \quad t \in [0, T]. \quad (2.14)$$

Let's formulate the analytical scheme for finding indexes $m_{1e}(t)$ and $n_{1e}(t)$. It should be noted that the index $m_{1e}(t)$ has two constituents: colloidal particle and residual (relict) initial cement particle.

Further we give the following definition. *Coefficient of the initial hydration* $k_{i.g.}$ is called the index which is equal to the relative volume of cement lobe which comes into reaction of hydration with the ordinary liquid (water) that is:

$$k_{i.g.} = \frac{\frac{4}{3}\pi r_{1a}^3 - \frac{4}{3}\pi r_{rem}^3}{\frac{4}{3}\pi r_{1a}^3} = 1 - \frac{r_{rem}^3}{r_{1a}^3}, \quad (2.15)$$

where: r_{rem} – radius of relict cement particle.

If the coefficient of initial hydration $k_{i.g.}$ stated *a priori*, the radius r_{rem} of relict cement particle on the base of (2.15) is defined by the equation:

$$r_{rem} = r_{1a} \sqrt[3]{1 - k_{i.g.}}. \quad (2.16)$$

Coming back to the equation (1.16), we make the following refinement regarding the interpretation of the parameter τ_{tec} – technological time of modification of cement solution in the presence of vibro field: the parameter τ_{tec} defines time interval within which takes place the modification of initial cement lobe till relict state with the account of the criterion of the effectiveness (1.17).

In this connection we introduce the index $m_{1e}(t)$ as:

$$m_{1e}(t) = \frac{4}{3}\pi [r_{col}^3(t) - r_{1e}^3(t) \eta(\tau_{tec} - t) - r_{1e}^3(\tau_{tec}) \eta(t - \tau_{tec})] \gamma'_a + \frac{4}{3}\pi r_{1a}^3 \gamma_{1a}, \quad (2.17)$$

where: $r_{col}(t)$ – radius of colloidal cement particle,

$r_{1e}(t)$ – radius of residual primary cement particle at the moment of time t , $t \in [0, T]$,

$\eta(t)$ – Heaviside function.

The obvious expression of the indexes $r_{col}(t)$ and $r_{1e}(t)$ in (2.17) can be found taking into account the following concepts. The formation of colloidal cement particle takes place as the sequence of chemical reactions of hydrolysis (1.1) and (1.2), thus, it is correct the statement that the indexes $r_{col}(t)$ and $r_{1e}(t)$ conform correspondingly the differentiation equations of hydrolysis and hydration:

$$\frac{dr_{col}(t)}{dt} = \alpha r_{col}(t), \quad r_{col}(0) = r_{1a}, \quad \alpha > 0, \quad t \in [0, T] \quad (2.18)$$

and

$$\frac{dr_{1e}(t)}{dt} = -\alpha r_{1e}(t), \quad r_{1e}(0) = r_{1a}, \quad \alpha > 0, \quad t \in [0, T]. \quad (2.19)$$

The solution of equations (2.18) and (2.19) are:

$$r_{col}(t) = r_{1a} \exp(\alpha t), \quad \alpha > 0, \quad t \in [0, T], \quad (2.20)$$

$$r_{1e}(t) = r_{1a} \exp(-\alpha t), \quad \alpha > 0, \quad t \in [0, T_{tec}]. \quad (2.21)$$

Let's make some notes regarding the quantitative index of the parameter α in formulas (2.20), (2.21). The magnification of the radius $r_{col}(t)$ (2.20) stops at the moment of time $t = T$, when starts the sticking of adjoining colloidal cement particles (moment of setting). In this connection it is necessary to define the upper boundary r_{sup} of index $r_{col}(t)$. It is easy to see that the parameter r_{sup} should meet the requirement:

$$\frac{4}{3}\pi r_{sup}^3 = \frac{1}{n_{1a} \sqrt{n_{1a}}}. \quad (2.22)$$

On the base of (2.22) we finally find:

$$r_{sup} = \sqrt[3]{\frac{3}{4\pi} \frac{1}{\sqrt{n_{1a}}}}. \quad (2.23)$$

Taking into account the formula (2.23), and the circumstance that the function $r_{col}(t)$ (2.20) is monotonous increasing at the intervale $[0, T]$, we have:

$$r_{col}(T) = r_{1a} \exp(\alpha T) \leq r_{sup} = \sqrt[3]{\frac{3}{4\pi} \frac{1}{\sqrt{n_{1a}}}}. \quad (2.24)$$

The solution of the inequation (2.24) regarding the index α we find as:

$$\alpha \leq \frac{1}{T} \ln \frac{1}{r_{1a}} \sqrt[3]{\frac{3}{4\pi} \frac{1}{\sqrt{n_{1a}}}}. \quad (2.25)$$

Using formulas (2.21), (2.16), we get the ratio:

$$r_{1e}(\tau_{tec}) \equiv r_{1a} \exp(-\alpha\tau_{tec}) = r_{rem} \equiv r_{1a} \sqrt[3]{1 - k_{i.g.}}. \quad (2.26)$$

From the equation (2.26) follows the expression for the index α :

$$\alpha = \frac{1}{\tau_{tec}} \ln \frac{1}{\sqrt[3]{1 - k_{i.g.}}}. \quad (2.27)$$

For the surface density $n_{1e}(t)$ of colloidal cement particles is the expression:

$$n_{1e}(t) = n_{1a} \left[1 - \pi \bar{r}^2(t) \bar{n}_e(t) \right]. \quad (2.28)$$

If $\bar{F}_{r.d.}(N, t) \equiv 0$, then the formula (2.28) is simplified to:

$$n_{1e}(t) \equiv n_{1a}. \quad (2.28')$$

Random function $\bar{n}_e(t)$, which in the formula (2.28), can be introduced as:

$$\bar{n}_e(t) = \bar{n}_e^{(+)}(t) + \bar{n}_e^{(-)}(t), \quad (2.29)$$

where: $\bar{n}_e^{(+)}(t)$ and $\bar{n}_e^{(-)}(t)$ mean accordingly the surface density of vortexes oriented in the opposite directions.

If suppose that at even impactions the vortexes with the equal orientation repel and with the opposite orientation disappear with the formation of the first component then indexes $\bar{n}_e^{(+)}(t)$ and $\bar{n}_e^{(-)}(t)$ will conform the system of differential equations as:

$$\begin{cases} \frac{d\bar{n}_e^{(+)}(t)}{dt} = -\beta\bar{n}_e^{(-)}(t), \\ \frac{d\bar{n}_e^{(-)}(t)}{dt} = -\beta\bar{n}_e^{(+)}(t), \quad \beta > 0, \quad t \in [0, T], \\ \bar{n}_e^{(+)}(0) = \bar{n}^{(+)}, \quad \bar{n}_e^{(-)}(0) = \bar{n}^{(-)}. \end{cases} \quad (2.30)$$

The system of differential equations (2.30) is the normal system which is brought into one linear homogeneous differential equation of the form:

$$\frac{d^2\bar{n}_e^{(+)}(t)}{dt^2} - \beta^2\bar{n}_e^{(+)}(t) = 0. \quad (2.31)$$

As the characteristic equation for differential equation (2.31) is defined by equity:

$$k^2 - \beta^2 = 0, \quad (2.32)$$

then its general solution is:

$$\bar{n}_e^{(+)}(t) = C_1 \exp(\beta t) + C_2 \exp(-\beta t). \quad (2.33)$$

For the function $\bar{n}_e^{(-)}(t)$, taking into account (2.30) and (2.33), we have:

$$\bar{n}_e^{(-)}(t) = -C_1 \exp(\beta t) + C_2 \exp(-\beta t). \quad (2.34)$$

The usage of initial data for the system (2.30) allow us to find easily arbitrary constants in the formulas (2.33) and (2.34):

$$C_1 = \frac{1}{2}(\bar{n}^{(+)} - \bar{n}^{(-)}), \quad C_2 = \frac{1}{2}(\bar{n}^{(+)} + \bar{n}^{(-)}). \quad (2.35)$$

Thus, for the function $\bar{n}_e(t)$ (2.29), taking into account the equities (2.33) - (2.35), we finally get:

$$\bar{n}_e(t) = \bar{n} \exp(-\beta t), \quad \bar{n} = \bar{n}^{(+)} + \bar{n}^{(-)}, \quad t \in [0, T]. \quad (2.36)$$

Let's note that in (2.36) should be followed the terms of coordination:

$$\bar{n}_e(t) \leq \bar{n} \leq \bar{n}^{\max} = \frac{1}{\pi \bar{r}^2(0)}. \quad (2.36')$$

Thus, on the base of (2.28) and (2.36), the surface density $n_{1e}(t)$ of colloidal cement particles is defined by the equity:

$$n_{1e}(t) = n_{1a} \left[1 - \pi \bar{r}^2(t) \exp(-\beta t) \right], \quad t \in [0, T]. \quad (2.37)$$

Now let's find the surface density of a mass of cement solution (2.7) at the moment of time $t = T$, that is at the moment of time when the reactions of hydrolysis and hydration have been finished. Thus, on the base of (2.7), we have:

$$\begin{aligned} \rho_e(T) &= \rho_e(T) + \bar{\rho}_e(T) = \\ &= m_{1e}(T) n_{1e}(T) + \rho'_e(T) + \bar{m}_e(T) \bar{n}_e(T). \end{aligned} \quad (2.38)$$

In case (2.7'), when $\bar{F}_{r,d}(N, t) \equiv 0$, formula (2.38) will be:

$$\rho_e(T) \equiv \rho_e(T) = m_{1e}(T) n_{1e} + \rho'_e(T). \quad (2.38')$$

In the formula (2.38) the values $\rho'_e(T)$ and $\bar{m}_e(T)$ are undetermined. For the surface density of a mass $\rho'_e(T)$ of residual ordinary liquid we get:

$$\rho'_e(T) = \left[\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{col}^3(T) \right] \gamma'_a n_{1e}(T). \quad (2.39)$$

Further according to equity (2.14), the vortex flow does not contain the residual ordinary liquid and, thus, its mass $\bar{m}_e(t)$ is defined by the formula:

$$\bar{m}_e(t) = 4\pi \bar{r}^2(t) n_{1a} m_{1e}(t), \quad t \in [0, T]. \quad (2.40)$$

For $t = T$, on the base of (2.40), we get:

$$\bar{m}_e(T) = 4\pi \bar{r}^2(T) n_{1a} m_{1e}(T). \quad (2.40')$$

Using the evident expression (2.8) and (2.17) regarding the values $\bar{r}(t)$ and $m_{1e}(t)$ for the moment of time $t = T$, on the base (2.40), we have:

$$\begin{aligned} \bar{m}_e(T) &= \frac{1}{2} \pi^2 \sigma^2(T) n_{1a} \times \\ &\times \left\{ \frac{4}{3} \pi [r_{col}^3(T) - r_{1e}^3(T_{tec})] \gamma'_a + \frac{4}{3} \pi r_{1a}^3 \gamma_{1a} \right\}. \end{aligned} \quad (2.41)$$

In connection with getting the formula (2.41) for the calculation of the mass $\bar{m}_e(T)$ of vortex flow we'll make some details regarding the equity (2.13), which defines the quantity $\Delta\rho'(t)$ of ordinary liquid dispersed by the vortex flow for the interval of time $[0, t], t \in [0, T]$. The value $\Delta\rho'(t)$ at $t = T$ can be also defined on the basis of the following concepts. As the number of colloidal cement particles connected with the system of vortex

flows at $t = T$ is equal to $\pi \bar{r}^2(T) \bar{n}_e(T) n_{1a}$, then can be easily found considering that (2.39), the value $\Delta\rho'(T)$ is defined by the equity:

$$\Delta\rho'(T) = \pi \bar{r}^2(T) \bar{n}_e(T) n_{1a} \left[\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{col}^3(T) \right] \gamma'_a. \quad (2.42)$$

Comparing the formulas (2.13) and (2.42), for the parameter λ we get:

$$\begin{aligned} \lambda &= \pi \bar{r}^2(T) \bar{n}_e(T) n_{1a} \left[\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{col}^3(T) \right] \gamma'_a \times \\ &\times \left[\int_0^T \int_0^{2\pi} [m_{1e}(\tau) \bar{r}(\tau) n_{1a} + \bar{r}(\tau) \rho'_e(\tau)] \times \right. \\ &\left. \times K(N, \tau) \left| \bar{F}_{r,d}(N, \tau) \right| d\tau d\phi \right]^{-1}. \end{aligned} \quad (2.43)$$

In the ratio (2.43) the value $\rho'_e(t)$ is equal to:

$$\rho'_e(t) = \left[\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{col}^3(t) \right] \gamma'_a n_{1a}. \quad (2.44)$$

For the quantitative valuation of the degree of impaction of cement solution we add series of the technological parameters $\lambda_1, \lambda_2, \lambda_3$. The degree of impaction λ_1 of the first level is defined by the formula:

$$\lambda_1 = \frac{\rho_e(T)}{\rho_a}. \quad (2.45)$$

On the base of (2.4), (2.17), (2.38') can be easily written for (2.45):

$$\begin{aligned} \lambda_1 &= \left[\left\{ \frac{4}{3} \pi [r_{col}^3(T) - r_{1e}^3(\tau_{tec})] \gamma'_a + \frac{4}{3} \pi r_{1a}^3 \gamma_{1a} \right\} n_{1a} + \right. \\ &\left. + \left(\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{col}^3(T) \right) \gamma'_a n_{1a} \right] \times \\ &\times \left[\frac{4}{3} \pi r_{1a}^3 \gamma_{1a} n_{1a} + \left(\frac{1}{n_{1a}^{3/2}} - \frac{4}{3} \pi r_{1a}^3 \right) \gamma'_a n_{1a} \right]^{-1}. \end{aligned} \quad (2.46)$$

The degree of impaction λ_2 of the second level is defined by the ratio:

$$\lambda_2 = \frac{\rho_e(T)}{\rho_a} \quad (2.47)$$

Using (2.4), (2.17), (2.38), (2.39), (2.41), we modify the expression (2.47) as:

$$\lambda_2 = \left[\left\{ \frac{4}{3} \pi r_{col}^3(T) - r_{ie}^3(\tau_{tec}) \right\} \gamma'_a + \frac{4}{3} \pi r_{ia}^3 \gamma_{ia} \right] n_{ie}(T) + \left[\frac{1}{n_{ia}^{3/2}} - \frac{4}{3} \pi r_{col}^3(T) \right] \gamma'_a n_{ie}(T) + \bar{m}_e(T) \bar{n}_e(T) \times \left[\frac{4}{3} \pi r_{ia}^3 \gamma_{ia} n_{ia} + \left(\frac{1}{n_{ia}^{3/2}} - \frac{4}{3} \pi r_{ia}^3 \right) \gamma'_a n_{ia} \right]^{-1} \quad (2.48)$$

The degree of impaction λ_3 of the third level is defined by:

$$\lambda_3 = \frac{\lambda_2}{\lambda_1}, \quad (2.49)$$

where: λ_1, λ_2 , are defined by the equities (2.46) and (2.48).

Now let's describe the interaction of the structured two-component cement solution with the filler (1.4), taking into account axiom A9. The optional particle of the filler is introduced as restricted simple connected field $\Sigma \subset R^2$, with the center of masses at the point P, has sectionally smooth border L, mass M, rate $\bar{u}(N, t)$ at the point N and unit vector of external normal line \bar{n} at the point N (Fig. 2).

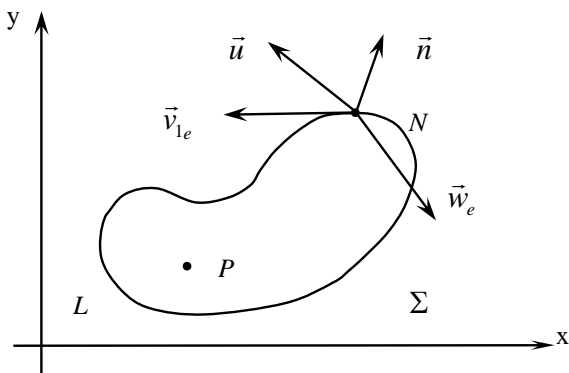


Fig. 2. Scheme of interaction of two-component cement solution with the optional particle Σ of the filler at the point N

The interaction of two-component cement solution with the particle Σ along the border L at the time interval $[0, t], t \in [0, T]$ leads to sticking of colloidal mass ΔM , which is defined by the formula:

$$\Delta M(t) = \chi_1 \int_0^t d\tau \int_L dl [m_{ie}(\tau) n_{ie}(\tau) + \rho'_e(\tau)] \times |(\bar{v}_{1e}(N, \tau) - \bar{u}(N, \tau), \bar{n}(N, \tau))| + \chi_2 \int_0^t d\tau \int_L dl \bar{m}_e(\tau) \bar{n}_e(\tau) \times |(\bar{w}_{2e}(N, \tau) - \bar{u}(N, \tau), \bar{n}(N, \tau))|, \quad \chi_1, \chi_2 > 0. \quad (2.50)$$

In the equity (2.50) χ_1 and χ_2 mean coefficients of sticking of colloidal medium on the optional particle Σ .

CONCLUSIONS

On the ground of the results of experimental theoretic researches:

1. It is shown that mathematical constructions stated above have general character and determine the direction in the largest degree in which the theory of vibrorheology of cement concrete solutions must develop.

2. It is proved that the detailed study of the geometry of the grid vector $\bar{F}(N, t)$ leads to the specific technological recommendations related to the parameters and the structure of shuttering forms which are used in the practice of vibro excitation.

3. It is argued that the results obtained in the article can be also used in the study:

- firstly, the process of corrosion of concrete blocks under the influence both of aggressive media solutions of inorganic substances [1, 9, 10], and biologically active habitat of some bacterial species (particularly sulfur-oxidizing thione bacteria) [6],

- secondly, the technologies of chemical treatment of especially dense concretes by the method of fluosilicate treatment [14].

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ИССЛЕДОВАНИЕ ПРОЦЕССА
ВИБРОРЕОЛОГИИ ЦЕМЕНТНО-БЕТОННЫХ
РАСТВОРОВ С ВНЕШНИМ ИСТОЧНИКОМ
ДИНАМИЧЕСКОГО ВОЗДЕЙСТВИЯ

Владимир Пилипенко

Аннотация. В статье, в рамках полуфеноменологического подхода сформулированы основные требования к математической модели движения среды цементного раствора и его взаимодействия с наполнителем в плоском случае, когда внешнее переменное силовое поле возбуждения имеет турбулентную составляющую. Дана строгая трактовка влияния параметров исследуемой модели на процесс виброреологии цементно-бетонных растворов и сформулирован критерий её эффективности.

Ключевые слова. Виброреология цементно-бетонного раствора, турбулентность, вихросток, тензоры первой и второй вязкости.