

## Comparison of Wind Turbine Energy Production Models for Rural Applications

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**Summary:** The paper presents three models used to estimate energy production of a wind turbine. Methodology to model both wind speed probability and turbine power curve is presented. The results include energy production for four selected turbine types suitable for a small farm and accuracy of various models for average values of wind speed typical for most of the territory of Poland.

**Key words:** wind turbine model, Weibull distribution, Rayleigh distribution, wind energy.

### INTRODUCTION

Wind is a source of energy with relatively great potential over many areas of the world. It originates from the atmospheric air pressure differences. The primary source of it is the solar radiation [13]. Although used already in ancient times, the wind energy has recently gained particular attention as an attractive source of renewable energy [15].

Poland has a moderate potential of wind energy. Only small regions located in the northern part of the country (mostly at the Baltic Sea) have yearly energy density over 2 MWhm<sup>-2</sup> at 30 m above ground. Approximately 2/3 of the area has wind energy potential between 750 and 1500 kWhm<sup>-2</sup> (also at 30 m) and is described as quite favorable [16].

In the neighbouring country, Belarus, there are 1840 sites for placing wind energy stations and a potential of 6.5 billion kWh of annual production is estimated [4].

A special attention is paid to renewable sources in the context of agricultural production. Rural areas offer large land availability and farmers can benefit from using renewable energy by using it to supply the energy demand of the farm or selling the excess to the local grid operator. This is an example of the prosumer approach – when the energy producer is also a consumer. Among others, wind turbines are considered as a source of electrical energy suitable for farms and household needs [2][3].

One of the important stages in the investment process is the correct assessment of the energy production and sizing of the turbine for the wind conditions at the planned location. In this context it is essential to use a right model for the turbine energy production.

There are many approaches to the energy production estimation of the wind turbines. Some authors propose sophisticated methods like fuzzy logic and artificial neural networks [12][21], data mining algorithms [14][19] and curve-fitting techniques [5][9]. This paper focuses on the most simple models which can be easily used by farmers to estimate yearly energy production in their location.

### WIND AS A SOURCE OF ENERGY

The wind energy is contained in the kinetic energy of the air particles. The power of the wind stream can be expressed as [9]:

$$P = \frac{1}{2} \rho A V^3, \quad (1)$$

where:

$\rho$  is air density,  $A$  – turbine area and  $V$  – wind speed.

Not all the energy carried by the air can be withdrawn – in such a case the wind speed behind the turbine would have to be zero which is not possible. The maximum power that can be theoretically extracted from the wind is expressed by the Betz law with limit equal to 16/27 of  $P$  defined above [22]. Therefore, the mechanical power delivered by the turbine must meet the following relationship:

$$P_T \leq \frac{8}{27} \rho A V^3. \quad (2)$$

The wind speed data can be obtained from various sources. Some of worth mentioning ones are Nasa Atmospheric Science Data Center [17] and Institute of Meteorology and

Water Management – National Research Institute in Poland [11] which provide a web-based tool to obtain historical wind data for a given location.

The wind speed is highly variable in time. On the contrary, the resources, like those mentioned in the previous paragraph, usually provide an average value within a year or month. The power of the wind stream depends non-linearly on the speed, so in order to estimate energy produced by a given turbine it is necessary to know the distribution of the speed values. Usually, for the wind turbine modelling the Weibull distribution is used.

The Weibull distribution of a variable  $V$  (in our case the wind speed) can be noted as [18]:

$$\begin{cases} f(V) = \frac{k}{c} \left(\frac{V}{c}\right)^{k-1} e^{-\left(\frac{V}{c}\right)^k}, & V > 0, \quad k, c > 0, \\ f(V) = 0, & V \leq 0 \end{cases} \quad (3)$$

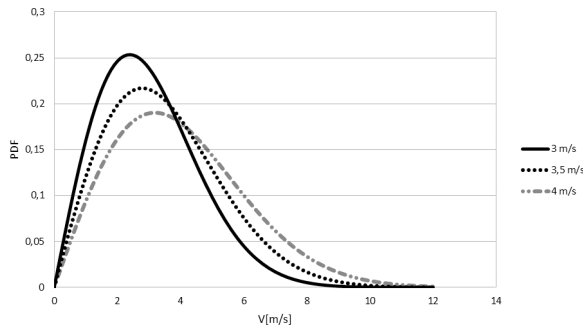
where:

$k$  is a dimensionless shape parameter and  $c$  – scale parameter (in  $\text{ms}^{-1}$  in this case).

When no information about the wind variability is available, the shape parameter  $k$  is assumed to be equal to 2 [20] which is usually a good approximation for most of the offshore locations. In such a case the Weibull distribution becomes a Rayleigh distribution [18]:

$$f(V) = \frac{2V}{c^2} e^{-\left(\frac{V}{c}\right)^2}, \quad (4)$$

Figure 1 presents the probability density function for three mean values of the wind speed assuming the Rayleigh distribution. As can be clearly seen, the maximum probability has the speed values lower than the mean value, which means that for most of the time the wind speed is lower than the mean value.



**Fig. 1.** Probability density function for selected values of the mean wind speed assuming Rayleigh distribution.

**Table 1.** The main parameters of the turbines used in the analysis.

Model	Rated power [kW]	Turbine diameter [m]	Cut-in speed [ $\text{ms}^{-1}$ ]	Nominal power speed [ $\text{ms}^{-1}$ ]
Evance Wind R9000	5.0	5.5	3.0	12.0
Bergey BWC Excel-S	10.0	7.0	3.0	14.0
Zaber ZEFIR D7-P5-T10	5.0	7.0	3.0	8.8
Zaber ZEFIR D10-P12-T12	12.0	10.0	3.0	9.0

## WIND TURBINE POWER CURVE MODELS

The turbine manufacturers or research centres usually provide the turbine model in a form of power versus wind speed curve. The spreadsheet files for major international manufacturers are available from the Idaho National Laboratory [10]. Curves for turbines chosen for analysis in this paper are presented in Fig. 2. All the turbines are of Horizontal Axis Wind Turbine (HAWT) type. The presented data are based on resources published by the manufacturers [6] [7][8] or research institute [10].

Another way to simulate the power curve of a given turbine is to approximate it by a polynomial equation, which can be written in many various ways. The turbine power curve can be divided into four sections: zero power (when the wind speed is lower than cut-in speed), wind-dependent (approximated by the polynomial), constant (nominal power (above the nominal power speed) and cut-off section (above the cut-off speed). From the formulas presented in [1] the one combining high accuracy with simplicity and ease of determining the parameters from the power curve is the following quadratic equation:

$$P_T = P_R (c_1 V^2 + c_2 V + c_3), \quad (5)$$

in which  $c_1$ ,  $c_2$  and  $c_3$  are coefficients which can be determined from the power curve by fitting the equation to at least three points of data, for example: cut-in speed (power assumed to be 0), nominal power and the corresponding speed and one of the points preferably near the middle of the second section. The fitting can be easily done with help of the LINEST function in the most popular spreadsheet programs.

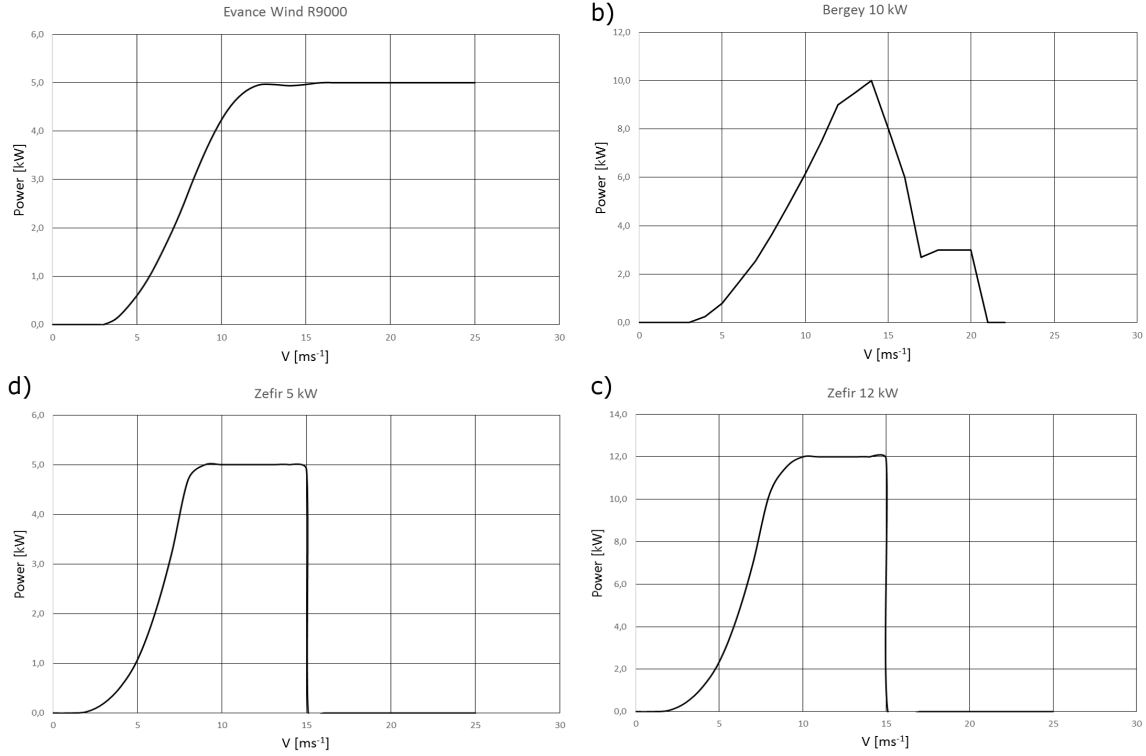
In order to estimate the energy output of a given turbine it is necessary to calculate the average power and then multiply it by the time  $T$  for which the energy needs to be calculated:

$$E = T \int_{V=0}^{V_{\max}} P_T(V) f(V) dV, \quad (6)$$

where:

$f(V)$  is the wind speed distribution function (probability density function). In practice, this equation can be replaced with a sum:

$$E = \sum_{i=1}^M P_T(V_i) T(V_i), \quad (7)$$



**Fig. 2.** Power curves of sample turbines chosen for analysis: a) Bergey 7.5 kW, b) Bergey 10 kW, c) Zefir 5 kW, d) Zefir 12 kW.

In this formula, the wind speed is divided into  $M$  bins  $T(V_i)$  in which the wind can be assumed to be constant and equal to  $V_i$ . For each value of  $V_i$  a corresponding power  $P_T(V_i)$  and time for which such a speed value occurs in a given period (week, month, year) can be obtained.

The third way to model the energy output is to use a fraction (efficiency) of a total wind energy available. As mentioned earlier, the efficiency cannot be higher than  $16/27$  but in practice the values of  $0.25$  or  $0.3$  can be assumed. In this model, the equation (7) will be changed into:

$$\begin{cases} E = \frac{\eta}{2} \rho A \sum_{i=1}^M V_i^3 T(V_i), & V_i > v_{cut-h} \\ E = 0, & V \leq v_{cut-h} \end{cases} \quad (8)$$

where:

$\eta$  is the assumed efficiency, other variables are defined earlier.

## SIMULATION RESULTS

All the three models were implemented in the spreadsheet software. For each model energy production in a 30-day month was calculated for three values of the mean wind speed. The wind speed values were chosen as representative for most of the locations in the Central and Eastern Poland. The results are presented in Table 1, 2 and 3.

The relative errors were calculated with assumption that the energy calculated using the manufacturer's power curve represents the true value:

$$\delta_2 = \frac{E_2 - E_1}{E_1}, \quad (9)$$

$$\delta_3 = \frac{E_3 - E_1}{E_1}. \quad (10)$$

**Table 1.** Energy production (in kWh) in a 30-day month according to different models.

	$v_{mean} = 3 \text{ ms}^{-1}$				$v_{mean} = 3.5 \text{ ms}^{-1}$				$v_{mean} = 4 \text{ ms}^{-1}$			
	$E_1$	$E_2$	$E_3$	$E_4$	$E_1$	$E_2$	$E_3$	$E_4$	$E_1$	$E_2$	$E_3$	$E_4$
R9000	208	185	212	270	347	298	340	398	512	441	504	534
Bergey 10 kW	292	408	466	340	468	625	714	699	722	906	1035	952
Zefir 5 kW	438	408	466	457	656	625	714	668	899	905	1035	914
Zefir 12 kW	951	832	951	1003	1433	1276	1458	1438	1976	1848	2112	1927

$E_1$  – Energy calculated using the manufacturer's power curve,  $E_2 - E_3$  Energy calculated using eq. 8 (for  $E_2 h=0.35$ , for  $E_3 h=0.4$ ),  $E_4$  – energy calculated using the polynomial approximation of the manufacturer's power curve (eq. 5).

**Table 2.** Relative error of energy production estimation in a 30-day month according to different models.

	$v_{mean} = 3 \text{ ms}^{-1}$			$v_{mean} = 3.5 \text{ ms}^{-1}$			$v_{mean} = 4 \text{ ms}^{-1}$		
	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_2$	$\delta_3$	$\delta_4$	$\delta_2$	$\delta_3$	$\delta_4$
R9000	-0.108	0.019	0.299	-0.141	-0.019	0.149	-0.138	-0.015	0.043
Bergey 10 kW	0.398	0.567	0.604	0.287	0.471	0.439	0.254	0.434	0.318
Zefir 5 kW	-0.068	0.065	0.044	-0.048	0.088	0.018	0.007	0.151	0.017
Zefir 12 kW	-0.125	0.000	0.054	-0.110	0.017	0.004	-0.065	0.069	-0.025

$$\delta_4 = \frac{E_4 - E_1}{E_1}. \quad (11)$$

**Table 3.** Average power of the turbines (in kW) according to the power curve based model.

	$v_{mean} = 3 \text{ ms}^{-1}$	$v_{mean} = 3.5 \text{ ms}^{-1}$	$v_{mean} = 4 \text{ ms}^{-1}$
R9000	0.29	0.48	0.71
Bergey 10 kW	0.41	0.65	1.00
Zefir 5 kW	0.61	0.91	1.25
Zefir 12 kW	1.32	1.99	2.74

## DISCUSSION AND CONCLUSIONS

As can be seen from the tables, the results and model accuracy differ depending on the turbine, model chosen and mean wind speed value. For most of the turbines, a simple, efficiency-based model gives acceptable results, at least at the preliminary stage of estimating the energy production. Using the efficiency value of 0.4 the error is kept under 10 % for most of the cases, except for the Berger turbine. As it can be seen in Figure 2, the power curve is different from other turbines' curves. Also, despite the rated power of 10 kW, the turbine diameter is 7 m which is equal to the diameter of the 5 kW Zefir turbine. This is why for this type the efficiency-based model is not performing well.

For the polynomial approximation of the power curve, the error is smallest in the case of the Zefir turbines. For the Bergey and Evance Wind the error is high and very high. This is because for the mean wind values simulated, most of the energy comes from a low wind speeds. If the curve is not modelled accurately in this region, the error will be high.

The energy production for the turbines is not very high: the average power (assuming the manufacturers' power curves) is a small fraction of the nominal power. Its values are higher with a higher average wind speed value.

All of the models presented in this paper have the main following limitations:

- 1) Variation of the air density is not taken into account. The air density changes with temperature, atmospheric pressure and humidity. As the density is one of the factors in eq. 1 it will have an effect on the power generated by the turbine.
- 2) The assumed theoretical wind distribution (Rayleigh) does not have to be the same as real distribution in a given place. Since the energy production depends highly on the statistical distribution of the wind speed, if the real distribution is different from the ideal approximation, the energy production will be different than the estimation made by any model.
- 3) The models do not take into account the dynamic states of the turbine.

All the simplified models presented in this paper exhibit considerable errors. It is not possible to choose one of them as one which will perform well for all of the turbines and a wide range of the wind speeds. Therefore, the simple models can be used as a preliminary tool to assess the energy

production when no manufacturer data is available and for a more accurate results the power curve data should be used.

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#### PORÓWNANIE MODELI WYTWARZANIA ENERGII PRZEZ TURBINY WIATROWE W ZASTOSOWANIACH ROLNICZYCH

**Streszczenie.** Artykuł przedstawia trzy modele wykorzystywane do modelowania wytwarzania energii przez turbiny wiatrowe. Przedstawiono metodologię modelowania prawdopodobieństwa występowania prędkości wiatru oraz krzywej mocy turbiny. Wyniki obejmują wartość energii wytworzoną przez wybrane typy turbin możliwych do zastosowania w warunkach rolniczych i dla prędkości wiatru typowych dla większości terytorium Polski.  
**Słowa kluczowe:** model turbiny wiatrowej, dystrybucja Weibulla, dystrybucja Rayleigha, energia wiatru.

