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Calculation of the strength of reinforced concrete beams strengthened with composite materials

Keywords: reinforced concrete beam, testing of strength, composite materials, fiber-reinforced polymer (FRP)

Introduction

Nowadays, the question of improving the reliability and durability of structures and buildings remains urgent. During the operation of constructions, structures and buildings, there is a need to strengthen the load-bearing elements. This may occur due to the expiration of its service, changes in the structural design of the element, damage and non-operable or emergency state, increased operational loads, errors in design, construction and use of defective materials. There is also a tendency to strengthen buildings and structures during their restoration, especially those that are architectural monuments or have architectural value. This is explained by the fact that the replacement of structures is not always possible and sometimes not economically advantageous compared to their strengthening.

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A popular and effective option to restore "old" buildings and structures in limited conditions of the construction site is the superstructure of one or more floors with the attic and replanning and arranging apartments with superior comfort. But even the use of modern lightweight building materials in the superstructure of buildings still requires strengthening of the bearing elements.

Industrial and civil engineering is characterized by the extensive use of reinforced concrete constructions, which is one of the main bearing elements of buildings and structures (Chen & Teng, 2001; Adhikary & Mutsuyoshi, 2004). Reinforced concrete bending elements (beams, slabs, trusses, etc.) were widely used in the past. Among the methods of strengthening reinforced concrete bending elements, one can choose two ways: strengthening the compressed zone of the concrete and the tensile zone of the section.

Research on the reliability assessment of strengthened concrete structures has been actively pursued since the beginning of the 21st century, primarily taking into account the effectiveness of the strengthening methods themselves, as well as the increase in the amount of work on the restoration and reconstruction of load-bearing elements of buildings and structures around the world (Islam, Mansur & Maalej, 2005; Kim, Ghannoum & Jirsa, 2016).

The strength of reinforced concrete structures deteriorates due to extreme environmental impacts and various types of loads. Effective retrofitting methods can allow elements to regain their structural strength. Different conventional and progressed strengthening methods, external prestressing, section enlargement and bonding of steel plates and modern materials such as externally bonded fiber-reinforced polymers (FRP), have been implemented for the rehabilitation of reinforced concrete structures.

Fiber-reinforced polymer (FRP) has been one of materials most often used for external reinforcement for about 30 years, and external bonding of FRP is currently the most common strengthening method. This type of polymer has the advantages of simplicity of construction, high strength, high resistance to corrosion and fatigue, and high strength-to-weight ratio.

A modern and effective way to strengthen structures of the tensile zone is to use the external reinforcing in the form of composite bands and liners (Trach et al., 2022). Strengthening reinforced concrete structures by glued external composite reinforcement on the basis of carbon fibers (CFRP) is a widely common practice. Strengthening structures in the tensile zone allows both to eliminate defects, acquired by the structure being in operation, and to increase the bearing capacity and rigidity of the element. The externally bonded carbon fiber-reinforced plastic (CFRP) has often been used due to its cost-effectiveness, high corrosion resistance, improved structural performance under critical loading conditions, high strength-to-weight ratio, and flexibility in use and low thermal conductivity. The strengthening of undershear members with CFRP is an effective method to increase the shear capacity (Bourget, El-Saikaly & Chaallal, 2017).

El-Mandouh, Hu, Shim, Abdelazeem and ELsamak (2022) examined various methods of increasing the torsional strength of reinforced concrete beams. An experimental study was conducted using six different strengthening systems: wrapped aluminum strips with anchor bolts, wrapped stainless steel strips with anchor bolts, wrapped glass fiber reinforced polymer (GFRP), one layer of the wrapped steel wire, and two layers of the wrapped steel wire along the beam. The results showed that the ultimate torque of the beam strengthened with aluminum wrapped strips and the beam strengthened with stainless steel wrapped strips was greater than the control beam by about 32% and 40%, respectively. The ultimate torque of beams strengthened with GFRP, single-layer wrapped steel wire mesh, and double-layer wrapped steel wire mesh along the beam exceeds the control beam by about 62%, 118%, and 163%, respectively.

The textile reinforced concrete (TRC) system is actively used to repair and strengthen worn reinforced concrete structures. Kim, You and Ryu (2021) proposed an enhanced method of installing the TRC system by cementing to strengthen the worn reinforced concrete structures. Four reinforced concrete slabs were strengthened with one layer of carbon textile mesh and 20-milimeter thick cement mortar. Specimens of the strengthened TRC plates were tested for bending and then the test results were compared with the results of the non-strengthened specimen and the theoretical solutions.

Park, Park and Hong (2021) explored the method of strengthening using the textile-reinforced mortar (TRM) – a structural reinforcing material in which a textile consists of fibers with high level of strength and chemical resistance. It is attached to the masonry surface and reinforced concrete structures using a cement mortar matrix. The strength limit of the TRM beam was determined taking into account the possibility of premature failure and the experimental results of four other studies on various samples. The authors of this study proposed a method to evaluate the flexural behavior of TRM beams considering a premature failure due to bond uncertainty and attempts to determine under the working load.

Also, Park, You, Park and Hong (2022) researched the flexural behavior of TRM-strengthened beams and their fatigue performance using carbon and alkali resistant (AR) glass textiles after 200 000 loading cycles. The TRM-reinforced beams were subjected to an optimization strengthening method that checked whether the textile was straightened. According to the test results, the strengthening efficiency of TRM-strengthened beams under cyclic loading was lower than under monotonic loading, except for the straightened carbon textile sample.

Clearly, carbon fiber reinforced polymers (CFRP) have shown significant potential in the repair and rehabilitation of damaged reinforced concrete structures. Currently, several schemes for strengthening reinforced concrete structures with carbon fiber have been studied and applied in a practical manner.

Haroon, Moon and Kim (2021) designed and executed a test program to evaluate the potential use and shear performance of bidirectional CFRP-strengthened rectangular RC beams. A total of 18 beams were designed and tested for the purpose of evaluating various CFRP strengthening characteristics such as strengthening time, presence of CFRP anchors, CFRP layout, etc. The bidirectional CFRP scheme allowed for a more uniform distribution of stirrup deformation compared to the unidirectional CFRP layout at the same load level, which increased the effectiveness of the transverse reinforcement. In addition, the contribution of the CFRP material to the shear was tested according to the CFRP hardening time.

The research of Wang, Ellingwood and Zureick (2010) is more advanced than the previous study, from the point of view of taking into account the joint work of the elements of the reinforced concrete structure and the approach to assessing its reliability. They proposed a method for assessing the reliability of bending reinforced concrete elements strengthened by external fastening of FRP composites. The most important advantage of this technique is that it is applied to strengthened reinforced concrete elements with insufficient bending strength. This study clearly demonstrated the possibility of developing a method for determining the reliability of bending reinforced concrete elements strengthened under the action of load with FRP plates, based on reaching the boundary state in accordance with the American building codes ACI 318-05 (American Concrete Institute [ACI], 2017). In turn, FRP-plates are included in the structure work at a certain stage and increase its strength and deformability. Moreover, in order to assess the reliability of strengthened structures, the scientists in their research used not only reinforced concrete beams as bending elements, but also slabs.

However, along with the progressive solutions developed in all of the works above, the problem of assessing the reliability of structures strengthened under the action of the load remaining open. The authors (Koutas & Triantafillou, 2013; Borysiuk, Ziatiuk, Lysyuk & Yevtushenko, 2018; Karavan, Borysiuk & Filipchuk, 2022a) conducted a number of experimental studies that allow for a more realistic assessment of the performance of strengthened reinforced concrete bending elements.

Material and methods

Stress and strain state of normal sections of the bending reinforced concrete element with single reinforcing, which is strengthened by the layer of any type of concrete h_{cf} in thickness in the compressed zone, and in the tensile zone it is strengthened by the carbon band in dimension of $b_{sf} \times h_{sf}$ is given at Figure 1.

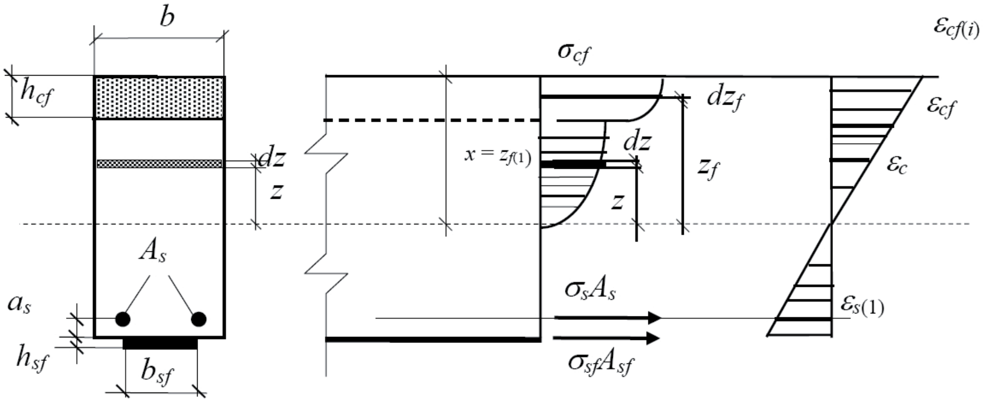

FIGURE 1. Stress and strain state of normal section of the bending reinforced concrete element strengthened in the compressed and tensile zones

Source: own elaboration.

In addition to the established prerequisites (Borysiuk & Ziatiuk, 2020; Karavan, Borysiuk & Filipchuk, 2022b) for the strengthened elements, we assume that: bonding the strengthening materials with concrete of the strengthening structure provides reliable joint work of adjacent fibers, the cross-section is considered as solid; when strengthening the tensile zone by carbon bands, the resultant force in the band is applied at the joint level (Norris, Saadatmanesh & Ehsani, 1997); mechanical condition of concrete in compression is described by dependence.

$$\sigma_c = f_{(ck),(cd)} \sum_{k=1}^{5} a_k \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^k = f_{(ck),(cd)} \left[a_1 \frac{\varepsilon_c}{\varepsilon_{c1}} + a_2 \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^2 + a_3 \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^3 a_4 \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^4 + a_5 \left(\frac{\varepsilon_c}{\varepsilon_{c1}} \right)^5 \right]. \tag{1}$$

The destruction of the element can occur when deformations are achieved in the extreme fibers of the material of strengthening boundary values ε_{cfu1}, and in tensile zone there may be a rupture of the strengthening material as a result of reaching the boundary deformations ε_{fu} therein; when exceeding deformations in the strengthening materials in the tensile zone, the value ε_{f0}($\varepsilon_{f0} = f_{yf}/E_f$) – the stresses in it are taken to be $\sigma_{sf} = f_{yf}$.

At a given deformation of the extreme compressed fiber of the strengthening material $\varepsilon_{cf(1)}$ according to the Bernoulli hypothesis (Fig. 1), the deformations in other materials can be determined thanks to the formulas (Borysiuk, Karavan & Sobczak-Piąstka, 2019):

$$\varepsilon_{s(1)} = \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(h - z_{f(1)} - a_s\right), \tag{2}$$

$$\varepsilon_{sf(1)} = \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(h - z_{f(1)}\right), \tag{3}$$

$$\varepsilon_{cf} = \frac{\varepsilon_{cf(1)}}{z_{f(1)}} z_f ; \varepsilon_c = \frac{\varepsilon_{cf(1)}}{z_{f(1)}} z, \tag{4}$$

$$\varepsilon_{cfc} = \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(z_{f(1)} - h_{cf}\right), \tag{5}$$

where ε_{cfc} is a deformation of the adjacent fiber of strengthening concrete and existing concrete.

Values z and dz when $\varepsilon_{c(1)} = \varepsilon_{cf(1)}$, and then z_f and dz_f are defined by formulas:

$$z_f = \frac{z_{f(1)}}{\varepsilon_{cf(1)}} \varepsilon_{cf} ; dz_f = \frac{z_{f(1)}}{\varepsilon_{cf(1)}} d\varepsilon_{cf}. \tag{6}$$

The z value can also be expressed as follows $z = z_f - h_{cf}$, and when $z_f \leq h_{cf}$, take $z_f = 0$.

For the strengthened section, the balance conditions in general form can be viewed as:

$$M_{cf(1)} + M_{c(1)} + M_{sf(1)} + M_{s(1)} - M = 0, \tag{7}$$

$$S_{cf(1)} + S_{c(1)} = S_{sf(1)} + S_{s(1)}, \tag{8}$$

where: M; $M_{cf(1)}$; $M_{c(1)}$; $M_{sf(1)}$; $M_{s(1)}$ – values according to the bending moment from the action of the external loads and internal force moments in the compressed strengthening concrete and the primary concrete, in the tension band of strengthening and in the main reinforcement at $\varepsilon_{cf} = \varepsilon_{cf(1)}$; $S_{cf(1)}$; $S_{c(1)}$; $S_{sf(1)}$; $S_{s(1)}$ – internal forces in the compressed concrete of strengthening, in the primary concrete, in the strengthening material and in the tensile reinforcement A_s at $\varepsilon_{cf} = \varepsilon_{cf(1)}$.

Forces in the compressed concrete of strengthening can be found using a formula at a given deformation of the compressed extreme fiber – $\varepsilon_{cf(1)}$

$$S_{cf(1)} = b\int_{z_{f(1)}-h_{cf}}^{z_{f(1)}} \sigma_{cf} dz_f = b\frac{z_{f(1)}}{\varepsilon_{cf(1)}}\int_{\varepsilon_{cfc(1)}}^{\varepsilon_{cf(1)}} \sigma_{cf} d\varepsilon_f. \tag{9}$$

And then, in the concrete of the element which is strengthening

$$S_{c(1)} = b\int_0^{z_{f(1)}-h_{cf}} \sigma_c dz = b\frac{z_{(1)}}{\varepsilon_{cf(1)}}\int_0^{\varepsilon_{cfc(1)}} \sigma_c d\varepsilon_c. \tag{10}$$

After integration and mathematical transformation of Eqs (7) and (8) and taking into account Eq. (1) we get:

$$S_{cf(1)} = \omega_f f_{cfm} b z_{f(1)};\ S_{c(1)} = \omega f_{cm} b z_{f(1)}, \tag{11}$$

where: f_{cfm} – average prism strength of strengthening concrete; ω – coefficient determined by the formula $\omega = \sum_{k=1}^{5} \frac{a_k}{k+1}\left(\frac{\varepsilon_{c(1)}}{\varepsilon_{c1,cm}}\right)^k$ when $\varepsilon_{c(1)} = \varepsilon_{cf(1)}$; ω_f – coefficient determined by the formula

$$\omega_f = \sum_{k=1}^{5} \frac{a_k}{k+1}\left(\frac{\varepsilon_{cf(1)}^{k+1} - \varepsilon_{cfc(1)}^{k+1}}{\varepsilon_{cf1}^{k+1}}\right), \tag{12}$$

where: ε_{cf1} – deformation under maximum stresses in the diagram of mechanical condition of strengthening concrete.

Internal forces in the reinforcement and in the strengthening band are determined by the following formulas:

$$S_{s(1)} = A_s\sigma_{s(1)} = A_s E_s \varepsilon_{s(1)} = A_s E_s \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(h - z_{f(1)} - a_s\right), \tag{13}$$

$$S_{sf(1)} = A_{sf}\sigma_{sf(1)} = A_{sf} E_f \varepsilon_{sf(1)} = A_{sf} E_f \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(h - z_{f(1)}\right). \tag{14}$$

In the same way, to determine the bending moment in the normal section of the element about the zero axis at a given deformation of the extreme compressed fiber, we can obtain the formulas:

$$M_{cf(1)} = b\int_{z_{f(1)}-h_{cf}}^{z_{f(1)}} \sigma_{cf} z_f dz_f = b\left(\frac{z_{f(1)}}{\varepsilon_{cf(1)}}\right)^2 \int_{\varepsilon_{cfc(1)}}^{\varepsilon_{cf(1)}} \sigma_{cf}\varepsilon_{cf} d\varepsilon_f = \beta_f f_{cfm} b z_{f(1)}^2, \tag{15}$$

$$M_{cf(1)} = b\int_0^{z_{f(1)}-h_{cf}} \sigma_c z dz = b\left(\frac{z_{f(1)}}{\varepsilon_{cf(1)}}\right)^2 \int_0^{\varepsilon_{cfc(1)}} \sigma_c \varepsilon_c d\varepsilon_c = \beta f_{cm} b z_{(1)}^2, \tag{16}$$

$$M_{s(1)} = A_s E_s \varepsilon_{s(1)} = A_s E_s \frac{\varepsilon_{cf(1)}}{z(1)}\left(h - z_{f(1)} - a_s\right)^2, \tag{17}$$

$$M_{sf(1)} = A_{sf} E_f \varepsilon_{sf(1)} = A_{sf} E_f \frac{\varepsilon_{cf(1)}}{z_{f(1)}}\left(h - z_{f(1)}\right)^2, \tag{18}$$

where the β coefficient is determined by the formula

$$\beta = \sum\nolimits_{k=1}^{5} \frac{a_k}{k+2}\left(\frac{\varepsilon_{c(1)}}{\varepsilon_{c1,cm}}\right)^k \quad \text{at} \quad \varepsilon_{c(1)} = \varepsilon_{cf(1)}, \tag{19}$$

whereas the β_f coefficient – by the formula

$$\beta_f = \sum\nolimits_{k=1}^{5} \frac{a_k}{k+2}\left(\frac{\varepsilon_{cf(1)}^{k+2} - \varepsilon_{cfc(1)}^{k+2}}{\varepsilon_{cf1}^{k+2}}\right). \tag{20}$$

Maximum (breaking) bending moment (M_u) is found by a joint solution of Eqs (7) and (8) considering values of the internal forces at Eqs (9), (11), (12), and (13)–(18), calculating them at $M = M_u$. The maximum criterion is used for this purpose. In order to do that, the values of the moments are determined when the extreme deformation of strengthening concrete ε_{cf} with certain step, for example at $0.1\varepsilon_{cf1}$, is being changed. At each step, the problem is solved by the successive method of approximation, for which it is possible to take the height of the compressed zone $x = z_{(1)} = 0.5$d at the first step and check the statement of Eq. (7). If the difference between the left and the right parts of Eq. (7) is less than 5%, then we can assume that the $z_{(1)}$ value is correct. If the difference between them exceeds 5%, then it is necessary to make a correction of the z value until the equation statement (8) is satisfied. We find the value of the M moment from the determined z value of Eq. (6). These calculations are repeated at each step.

Based on the performed calculations, a complex of indicators of stress and strain state of the normal section (deformations of the materials and stresses in them, forces, curvature, etc.) is obtained at each stage of the calculations, which can be shown in table form and in the diagram of the mechanical condition of the section. The maximum (breaking) moment is selected from the table or defined as the maximum in the diagram of the mechanical condition of the normal section.

The balance equation presented above can be used in the calculations of the normal section's strength at the boundary states of the first group under the action of the calculated moment from the external loading (M_{Ed}), at that, the above formulas use the calculated values of the strength and deformation characteristics of the materials. In this case, the strength of the normal strengthened section of the bending element will be provided, when the following equations is fulfilled

$$M_{Ed} \leq \beta_f f_{cfd} b z_{f(1)}^2 + \beta f_{cd} b z_{f(1)}^2 + A_{sf} E_f \frac{\varepsilon_{cf(1)}}{z_{f(1)}} \left(h - z_{f(1)}\right)^2 + A_s E_s \frac{\varepsilon_{cf(1)}}{z_{(1)}} \left(h - z_{f(1)} - a_s\right)^2, \quad (21)$$

$$\omega_f f_{cfd} b z_{f(1)} + \omega f_{cd} b z_{(1)} = A_s E_f \frac{\varepsilon_{cf(1)}}{z_{f(1)}} \left(h - z_{f(1)}\right) + A_s E_s \frac{\varepsilon_{cf(1)}}{z_{f(1)}} \left(h - z_{f(1)} - a_s\right). \quad (22)$$

Results and discussion

To carry out the experimental research, eight 2000 mm long beams with a 200 × 100 mm cross-section were produced from concrete, which has the prism strength $f_{cm,prism} = 13.9$ MPa. Beams are reinforced by two rods with a 10 mm diameter of the A500C class. The 6 mm diameter cross roads of the A240C class are placed with the 50 mm spacing (Figs 2 and 3).

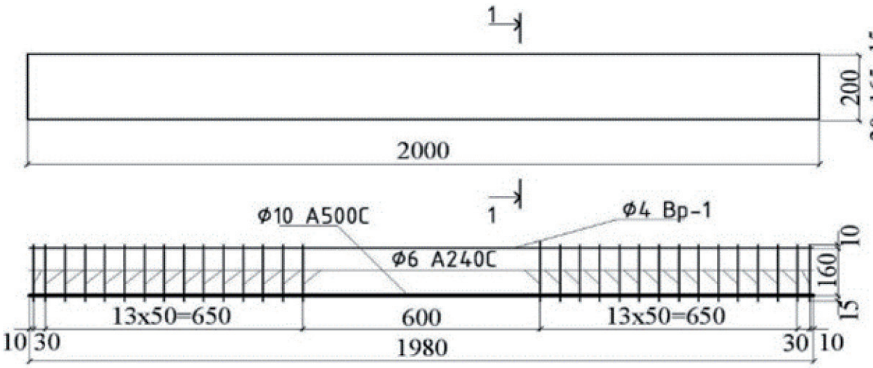

FIGURE 2. Structural scheme of non-strengthened analyzing beams
Source: own elaboration.

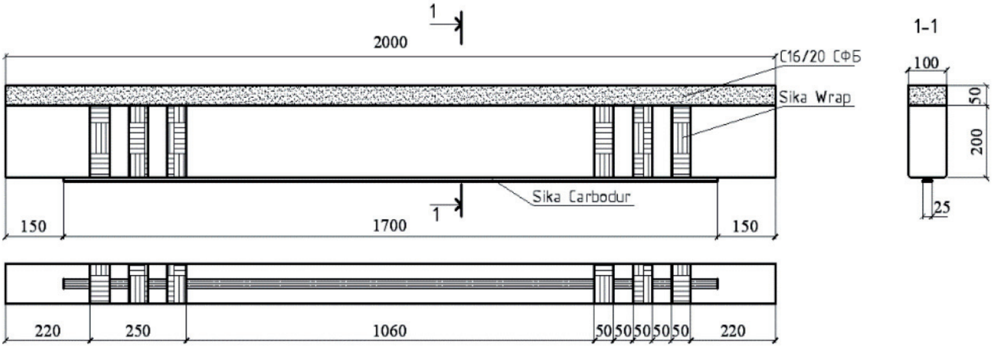

FIGURE 3. Scheme of strengthening analyzing beams by the band and by the concrete
Source: own elaboration.

The calculation of the bearing capacity of the normal section, conducted according to the above-described method (for non-strengthened beams), shows a fine precision of the results. Divergence of the theoretical results of the calculation, compared to the experimental data, accounts for 1.5% and 3%. This approach fully takes into account all factors that affect the bearing capacity of the normal section of the beam.

When determining the theoretical value of the breaking moment according to the above method, the following material characteristics were assumed: the yield point for the reinforcement f_{yk} = 500 MPa; modulus of elasticity $E_s = 21 \cdot 10^4$ MPa;

concrete deformations under maximum stresses in the deformation diagram $\varepsilon_{c1} = 0.00161$; boundary deformations of the concrete compression $\varepsilon_{cu1} = 0.0044$.

Two beams (B-1, B-2) were tested without strengthening (Fig. 2), the rest were strengthened in the tensile zone by gluing the carbon band Sika with the cross section 25 × 5 mm, which was anchored by the cross-section reinforcement in the form of SikaWrap® liner strips. In the compressed zone, two beams (B-3, B-4) were strengthened by a layer of fine-grained concrete ($f_{cfm,prism} = 13.5$ MPa) 50 mm thick, and two beams (B-5, B-6) – by a layer of steel fiber concrete with a fiber content of 3% ($f_{cfm,prism} = 16.4$ MPa), also 50 mm thick (Fig. 3). Testing of the beams were performed by the scheme of the simple bending (Karavan, Borysiuk & Filipchuk, 2022b). According to the calculations, the theoretical breaking moment of the non-strengthened beams was $M_{teor} = 13.6$ kN·m^{-1}, the experimental one – $M_{exp} = 13.4$ kN·m^{-1}, 14.0 kN·m^{-1}. Divergence between the experimental value of the breaking moment in the normal section of the beam and the theoretical value accounted for 3%.

Two beams strengthened by the fine-grained concrete in the compressed zone (Fig. 4) and by the carbon band in the tensile zone were destructed under the action of the external moment 21.13 kN·m^{-1} and 21.52 kN·m^{-1} accordingly, which on the average is $M_u = 21.32$ kN·m^{-1}.

Two beams, strengthened by the steel fiber concrete in the compressed zone (Fig. 4), and in the tensile zone by the carbon band, were destructed under the action of the external moment 20.91 and 21.96 kN·m^{-1} accordingly, which on the average is $M_{u,BPsfb} = 21.4$ kN·m^{-1}.

When determining the breaking theoretical moment, we took the following strengthening material characteristics into consideration: the yield point of the carbon band $f_f = 3\ 100$ MPa; modulus of elasticity of the band $E_f = 16.5 \cdot 10^4$ MPa; average prism strength of the steel fiber concrete $f_{cfm} = 16.4$ MPa; steel fiber concrete deformations at maximum stresses in the deformation diagram $\varepsilon_{cf1} = 0.00166$; boundary deformations of the concrete in compression $\varepsilon_{cfu1} = 0.00434$.

During the experiment, the deformations in the compressed zone of the concrete and the stretched reinforcement were recorded, the width of the opening of cracks and deflections of the structure were observed. When calculating the load-bearing capacity, the actual values of the prism strength of the concrete were substituted in the formula, and the actual values of the yield points of the reinforcement were taken as the calculated resistance of the reinforcement.

The test of the beams turned out to be 10% of the theoretically calculated bearing capacity. In order to remove the readings of the devices, to fix the development and change the width of the opening of the cracks, after applying each degree of load, exposures of 5–10 min were made.

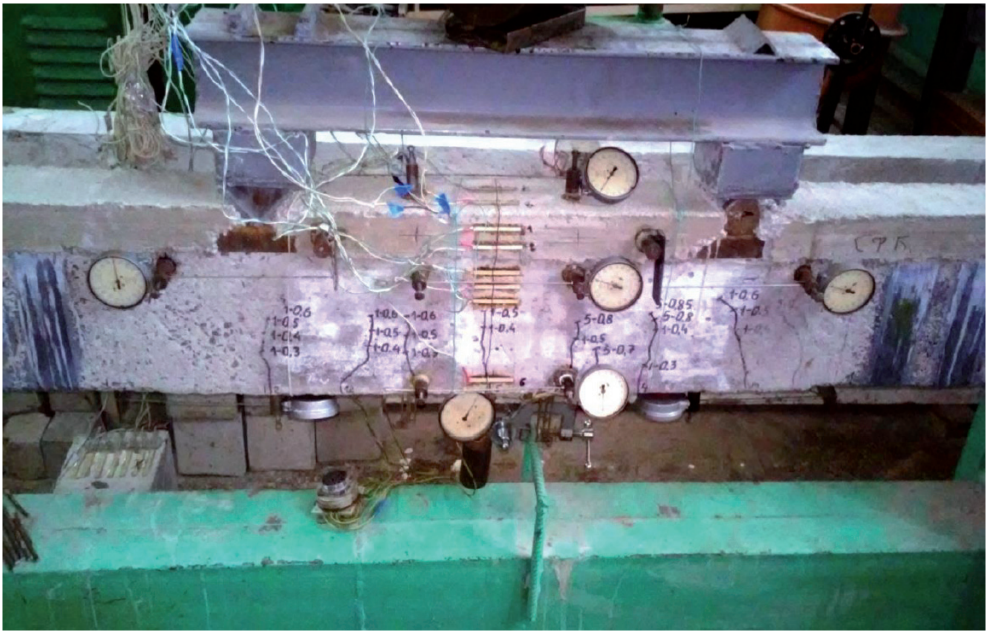

FIGURE 4. Overall view of testing the strengthened beams
Source: own elaboration.

The calculation of the strength of the normal sections of the beams, strengthened in the compressed and tensile zones by the steel fiber concrete and composite materials accordingly, showed a decrease in theoretical results. As the calculation does not consider anchorage of the band, it only takes into account the bearing capacity of the strengthening section. The experimental indicators exceeded theoretical 1.23, …, 1.33.

According to the experimental data it was stated that due to anchoring of the bands by the liner, and increased strength characteristics of the steel fiber concrete, the bearing capacity of the normal sections of the reinforced concrete beams increased compared to the non-strengthened ones by 40–60%.

When calculating the strength of the normal strengthened section of the bending element, it is proposed to introduce the coefficient that takes into account an increase of the bearing capacity of the normal sections of the strengthened reinforced concrete beams by anchoring the bands of strengthening, into Eq. (17). Formula (21) will take the following form

$$M_{Ed} \leq \left[\beta_f f_{cfd} b z_{f(1)}^2 + \beta f_{cd} b z_{(1)}^2 + A_{sf} E_f \frac{\varepsilon_{cf(1)}}{z_{f(1)}} \left(h - z_{f(1)} \right)^2 + A_s E_s \frac{\varepsilon_{cf(1)}}{z_{(1)}} \left(h - z_{f(1)} - a_s \right)^2 \right] P_{CFRP}^{SFC}, \tag{23}$$

β – coefficient determined by Eq. (19) when $\varepsilon_{c(1)} = \varepsilon_{cf(1)}$; β_f – coefficient calculated using Eq. (20); P_{CFRP}^{SFC} – coefficient that takes into account an increase of bearing capacity strength of the normal sections of the strengthened reinforced concrete beams when anchoring the strengthening bands, and it is assumed by the experimental data. If the data are absent, the value of this coefficient can equal 1.25.

The results of experimental research and comparison with theoretical calculations are presented in Table 1.

TABLE 1. Results of experimental research

Beam	Strengthening elements cross section area		Internal steel reinforcement area	Experimental bending moment	Calculated bending moment*	Divergence with experimental value	Calculated bending moment according to the proposed method	Divergence with the experimental value
	A_{sf} [cm^2]	A_{cf} [cm^2]	A_s [cm^2]	M^{exp} [kN·m^{-1}]	M^{norm} [kN·m^{-1}]	δ [%]	M × P_{CFRP}^{SFC} [kN·m^{-1}]	δ [%]
B-1	0	0	1.57	13.4	13.6	1.5	13.2	–1.5
B-2	0	0	1.57	14.0	13.6	–2.9	13.2	–5.7
B-3	1.25	50	1.57	21.13	18.3	–14.4	21.25	0.6
B-4	1.25	50	1.57	21.52	18.3	–15.0	21.25	–1.3
B-5	1.25	50	1.57	21.96	19.0	–13.5	21.62	–1.6
B-6	1.25	50	1.57	20.91	19.0	–9.8	21.62	3.3

*According to the Ukrainian standard DSTU-NB V. 1.2-18:2016 (DP UkrNDTS, 2017).

Source: own elaboration.

Conclusions

The use of modern and highly efficient materials, namely fiber concrete and composite materials based on the carbon fibers, to strengthen the compressed and tensile zones of the reinforced concrete bending structures, is a promising trend in view of the advantages of these materials over the traditional ones because they can be easily and quickly set up and be reliable in the further operation. The calculation of the strengthened bending reinforced concrete elements in the compressed and tensile zones of the section based on carbon fiber reinforced plastics and fiber concrete for the first group of boundary states, which we suggested, gives good results from all currently proposed methods.

Acknowledgements

This article is dedicated to the bright memory of the author Oleksandr Borysiuk (1954–2023). Rest in peace.

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Summary

Calculation of the strength of reinforced concrete beams strengthened with composite materials. Currently, in view of the previous theoretical and experimental researches, the regulatory documents for the calculation of reinforced concrete elements strengthened by composite materials and the calculation and design of fiber reinforced concrete structures are in force in Ukraine and in the world. Simultaneous strengthening of the compressed and tensile zones has not been sufficiently studied. Therefore, further research of reinforced concrete elements, strengthened by modern and highly efficient materials, such as steel fiber concrete and composite materials, is of great theoretical and practical importance. The urgency of the study is due to the obvious need to improve the method of calculation of the reinforced concrete bending elements after simultaneously strengthening compressed and tensile zones.