Calculation of Deflection of One-Layer and Two-Layer Slabs Supported on Four Sides

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Summary. The article presents the method and results of experimental research of deflection of one-layer and two-layer slabs influenced by short-term lateral load. The proposed method of calculation is based on the limit equilibrium method, the calculation of slab deflections values by LIRA-CAD bundled software. The comparison is carried out of experimental and theoretical results of slabs deflection calculation.
Key words: slab, deflection, steel fiber concrete.

INTRODUCTION

Layered constructions have been increasingly applied in building recently. When appropriate composition of separate layers is selected, multi-layer constructions with perfect construction properties may be created [9]. Layers are mostly composed of heavy concrete and effective steel fiber concrete. Such constructive decisions have a widespread use in road and airfield pavements, logistics areas, heavy-weight industrial floors, etc. [13, 15, 16]. Among the most promising trends of reasonable usage of steel fiber concrete is its application in composite structures, as a rule, in combination with concrete or ferroconcrete, with distinct partition of functions of every material. In particular, the steel fiber concrete that is rather thriftily applied along the construction outline, in a thin layer, provides high crack resistance of constructions, as well as its high durability due to high indices of tensile strength, frost resistance, corrosion resistance and high rates of other types of resistance of the steel fiber concrete [14]. At the same time, this solution provides necessary preconditions for significant reduction of strength and value of the primary concrete and reduction of the number of reinforcement rod. Thus, there are preconditions for obtaining high indices created while their cost drops.

PURPOSE OF WORK

The main purpose of proposed work is the comparison of proposed method of deflection calculation for layered slabs under lateral load with the results of experimental research.

EXPERIMENTAL RESEARCH

According to the set purpose of research, slabs of 4 series were produced, two slabs for each series. The scope and outline of experimental studies is set in Table 1 [5].

The total size of single-layer slabs amounts to 800×800×60 mm; the thickness of each layer of reinforced concrete (concrete and steel fiber concrete) in two-layer slabs is 30 mm.

Series I (ПФ) represents a slab made of steel fiber concrete.

Series II (П3) represents a single-layer reinforced concrete slab with single-layer reinforcement Ø4 Bp-I laid at the bottom of the slab with protective concrete layer 10 mm thick with 75 mm pitch

Series III (ПБФ) represents a two-layer concrete slab, the top layer of which is made of un-reinforced heavy concrete and the below one – of the steel fiber concrete.

Series IV (П3Ф) of the studied samples represents slabs consisting of an upper layer of steel fiber concrete and a heavy concrete layer reinforced with metal reinforcement mesh Ø4 Bp-I set with 75 mm pitch.

Slab concreting was carried out in two stages. Slabs of П3 grade and concrete layers of П3Ф and ПБФ slabs were concreted at the first stage. Siliceous sand and giant gravel of 5…10 mm fraction; Portland cement of M400 grade was used as binding material; water-to-cement ratio

amounted to W/C = 0,4. Plates of ПЗ and ПЗФ grade were reinforced with binding wire mesh of Bp-I grade, 4 mm in diameter and 75 mm pitch in two directions. Concrete protective layer was 10 mm. The surface is bush-hammered while the concrete is immature.

At the second stage, after 4 days, slabs of ПФ grade were concreted and steel fiber concrete layer slabs of ПЗФ grade were additionally concreted.

Steel fiber concrete contained steel fiber with diameter d_f = 1.0 mm and length l_f = 36 mm; volume percent of reinforcement was μ_f = 1%. Fine concrete without coarse aggregate was used as a concrete matrix; water-to-cement ratio equaled to W/C = 0,4.

Table 1. Scope of experimental studies

Series	Slab grade	Section	Composition
I	ПФ-1 ПФ-2	*1* 800 / 60	1–steel fiber concrete
II	ПЗ-1 ПЗ-2	*1* 800 / 60	1–reinforced concrete
III	ПБФ-1 ПБФ-2	*1* *2* 800 / 30, 30	1–concrete; 2–steel fiber concrete
IV	ПЗФ-1 ПЗФ-2	*1* *2* 800 / 30, 30	1–steel fiber concrete; 2–reinforced concrete

General appearance of testing bench is showed in Fig. 1.

The same calculation model was applied for both the one- and two-layer slabs exposed by lateral load: a slab is hinge-supported on four sides and influenced by evenly distributed load (Fig. 2)
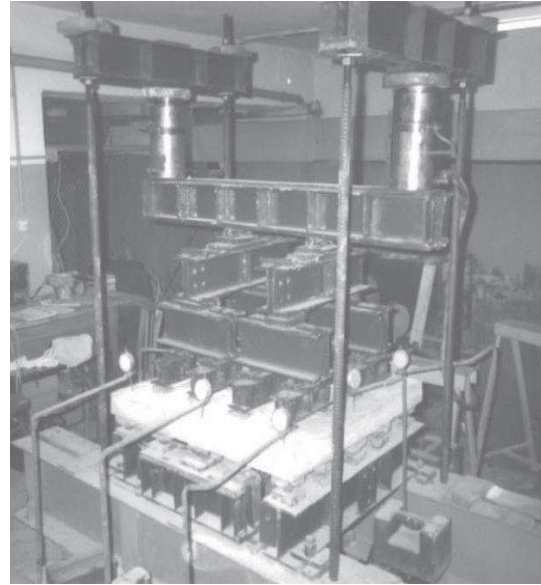
Fig. 1. General appearance of testing bench

a

b
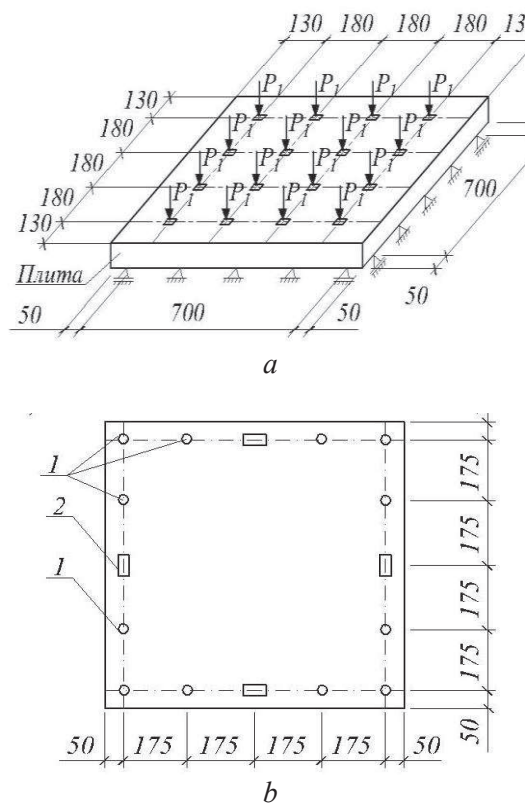
Fig. 2. Distribution of load (*a*), and support (*b*) upon slab: *1* − hinged support, *2* − cylinder support

It is suggested that 16 concentrated forces evenly allocated on the slabs surface show no significant difference during the operation, as compared to evenly distributed load, that is why in further theoretical study of strength, crack resistance, and deformation of studied slabs a design model was adopted for slabs supported by

hinged bearings on four sides and influenced by evenly distributed load. The load was applied by degrees P_i=2,0 kN with 15-minute timing at each stage to take readings from devices. The value of load was fixed by indices of model forcemeter of hydraulic pumping station. Before testing the hydraulic system consisting of a pumping station, jacks, and model forcemeter was calibrated using model forcemeter of Tokarev system.

During the application of load in the center of the slab deflections and deformations over supports were recorded using time indicators with 0.01 mm scale graduation value.

The load was applied by two hydraulic jacks of 250 kN united by common oil circuit connected to a common pump station.

Before testing the slabs, physical and mechanical properties of used materials were determined: those of heavy concrete, steel fiber concrete and reinforcement (Table 2).

**Table 2.** Physical and mechanical properties of concretes and reinforcement

Type of concrete or fitting	Strength, MPa		Tensile strength, MPa	Starting elasticity factor, MPa
	cube	prism		
concrete	16,3	11,9	1,5	22500
Steel fiber concrete	19,8	14,5	1,63	24600
Reinforcement Bp-1	–	–	393	192000

According to results of experimental studies with regard to nature of destruction [10], all slabs collapsed according to normal sections.

CALCULATION OF DEFLECTION

The proposed method based on the limit equilibrium method, which may be represented – during the state of limit equilibrium – by the system of disks united along the lines of fracture with plastic hinges.

As proposed by A. A. Gvozdev [4], it is possible to make use of characteristic points 1 and 2 on the diagram (Fig. 3).
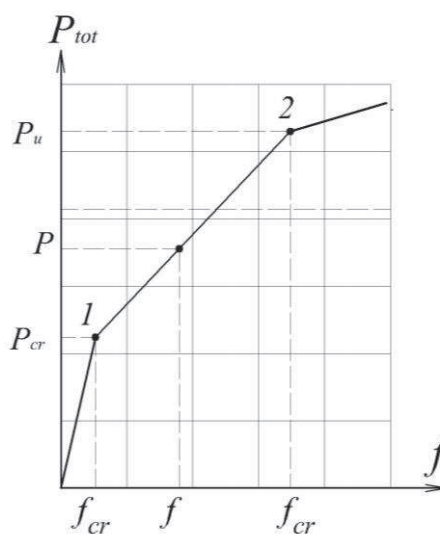
Fig. 3. Design diagram of slab deflection:
1 − cracking, 2 − appearance of the plastic hinge

Deflection values in the areas between f_{cr} and fu are determined by interpolation.

The general view of the formula f of slab deflection of slab supported on four sides and bearing a cracks, may be obtained from the following ratio:

$$\frac{q_u - q_{cr}}{q - q_{cr}} = \frac{f_u - f_{cr}}{f - f_{cr}}, \qquad (1)$$

where:
q_u and q_{cr} – load at destruction and crack formation;
f_u and f_{cr} – slab deflection at the moment of destruction and crack formation correspondingly.

It should look as follows:

$$f = f_{cr} + \frac{q - q_{cr}}{q_u - q_{cr}}(f_u - f_{cr}). \qquad (2)$$

The value f_{cr} is determined based on elastic system calculation according to load q_{cr}, which in turn may be obtained by bending factor, when first crack appeared in a slab area with the highest tension.

The deflections f at the moment of formation of plastic hinges may be determined as follows.

Until the conditional yield point of reinforcement achieved along all the lines of fracture, cracks are formed and significantly increased on a slab. At the same time, areas with cracks will be

especially distorted that will mostly determine the maximum value of slab deflection.

If insignificant curvature of slab areas neglected bearing no cracks, but rigidity is high, the slab calculation model may be represented as stiff disks connected by yielding bracing with width Δ. Bending rigidity of all joints is calculated according to V.M. Murashov theory [7], though the coefficient ψ is taken as a one, as the reinforcements reaches instability the influence of stretched concrete between cracks disappears or becomes insignificant. This assumption substantially simplifies the calculation.

For further simplification the angle fracture between discs equal to $\dfrac{\Delta}{r}$ is assumed as if concentrated along the lines of fracture. The calculated deflection surface prior to exhaustion of the bearing capacity turns out to be similar to the surface used in calculating by limit equilibrium method for the calculation of works on possible movements. Though such likening is not accurate, it allows determining the maximum deflection at a rather decent level.

There is a calculation model of square slab presented on Fig. 3 and a diagram of angle rotation at slab center deflection that is equal to f_u. Owing to symmetry, the fracture diagram of value Δ for all plastic hinges is the same.

Rigid discs of the slab will turn in relation to supports by angle:

$$\frac{\varphi}{2} = \frac{2f_u}{l}. \qquad (3)$$

Mutual angle adjacent discs

$$\varphi = \frac{2\sqrt{2}f_u}{l} = \Delta\frac{1}{r}, \qquad (4)$$

whereof:

$$f_u = \frac{l\Delta f_{ym}}{2\sqrt{2}E_s(d-x)}. \qquad (5)$$

where:

$\dfrac{1}{r}$ – curvature-multiplication of which by Δ equals to reciprocal angle of rotation of adjacent discs

$$\frac{1}{r} = \frac{f_{ym}}{E_s(d-x)}, \qquad (6)$$

$\dfrac{1}{2\sqrt{2}}$ – factor derived from geometrical considerations, which is transition to the angle of rotation of the disk relative to the support,

For fiber reinforced structures (such as fiber concrete) the value f_y and E_s at the point of critical steel stress is replaced by the value f_{ctf} and elastic modulus E_f suitable to the material. For two-layer slabs the given geometric, strength, deformation properties of two materials are used.

By the combination of Eq. (2)-(6) we can define a final ratio for the calculation of current deflection of the slab supported by four sides influenced by evenly distributed load:

$$f = f_{cr} + \frac{q - q_{cr}}{q_u - q_{cr}}\left(\frac{1}{r}\Delta - f_{cr}\right), \qquad (7)$$

where:
f_{cr} – slab deflection at the moment of of formation of the first cracks in stretched area of the element;
q_{cr} – load, when first cracks were created;
q_u – load corresponding to the limit of the slab bearing capacity;

Thus, the calculated width of deformed area Δ remains unknown in the Eq. (7); as a result it is not possible to calculate the value f.

In order to solve such problem the following method of finding the desired value of deflection f may be proposed. First of all, a boundary value of the slab deflection is set according to the standards for that class of structures $f_u=[f]$ apply it in the Eq. (7) $f=f_u=[f]$. The Eq. (7) is settled with respect to value Δ and then obtained result is applied to the Eq. (5), therefore obtaining the deflection at the time of formation of plastic hinges with regard to real-specified parameters. At the same time, the result of Eq. (7) shows the value of current deflection by introducing the calculated value of width of deformed area Δ.

a

b
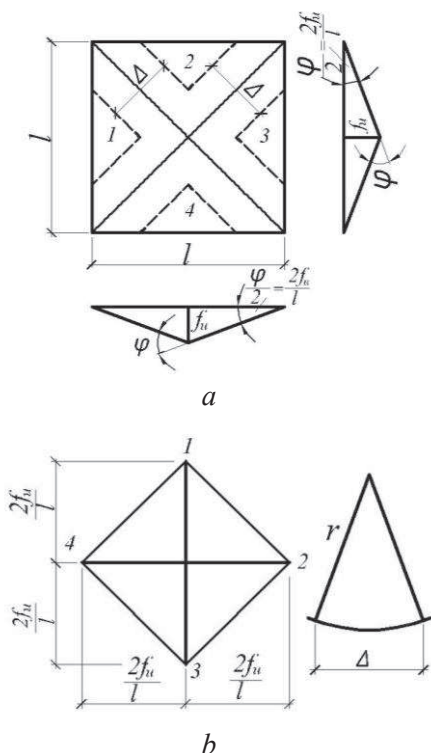
Fig. 4. Calculation diagram of square slab influenced by evenly distributed load:
a – fracture diagram; *b* – diagram of disc rotation angle rate

The value of fcr and qcr can be calculated if the combine Eq. (7) with the formula B.H.Galerkina [2] for square plates, supported on four sides, the deflection at the center of slabs:

$$f = 0{,}04706 \frac{ql^4}{E_i h^3} . \qquad (8)$$

To determine the width of the deformed zone Δ slabs a series of isolated points on several research graphs deflections. Points are usually prescribed in the operational (working) load range: often – is 0,7...0,8 from destructive load. To find the value Δ using Eq. (6) and (7), taking into account the specific slabs construction.

Results of calculation of one- and two-layer slabs by the limit equilibrium method are shown in Fig. 7, 8.

Considering complexity of mathematical calculations for slab deflection due to analytical methods [6] and unsatisfactory precision of results, a decision was made to do calculations on computing machine with a help of LIRA-CAD bundled software [3, 8].

The basis of LIRA-CAD is represented by the calculation of components and structures by finite element method [3].

The calculation model of a slab (Fig. 5) is built out of tridimensional finite elements (type CE-36). The slab is separated into 1444 finite elements according to plan. The sectional area of the slab is composed of 12 layers 5 mm each. The load is applied in the form of 16 concentrated forces. The distribution of load and supports is accepted as in Fig. 2.

a

b
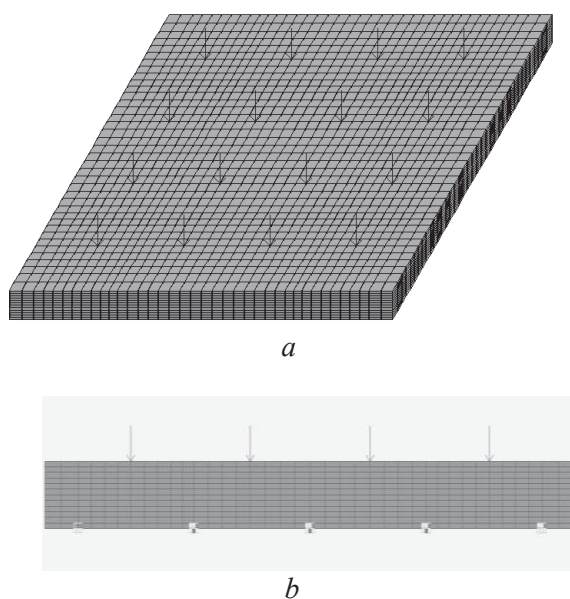
Fig. 5. Calculation model of slab in LIRA-CAD:
a – general view; *b* – side view

The CE-36 finite element is a universal tridimensional eight-node isoparametric finite element, designed for the calculation of tridimensional constructions. There is a diagram represented in Fig. 4. Each of finite element nodes has three degrees of variance *U, V, W* defined with regard to global coordinates *X, Y, Z* and are linear displacements according to axis, whose positive direction coincides with the direction of coordinate axis. As a result of modeling, there are 19,773 nodes and 17,328 elements
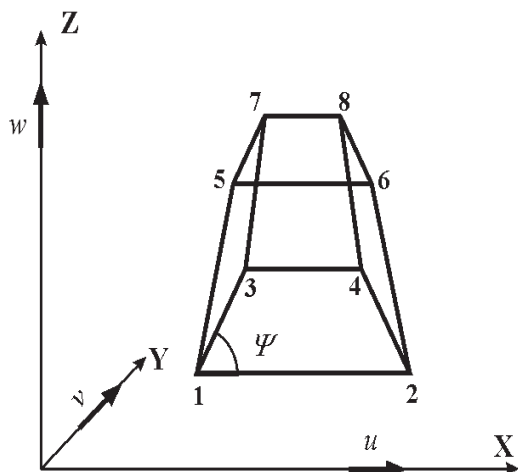
Fig. 6. Diagram of CE-36 finite element

a

b

Fig. 7. Calculation results for deflection of one-layer slabs of series I grade ПФ (*a*) and slabs of series II grade ПЗ (*b*): *1* – experimental*; 2* – calculation by the limit equilibrium method; *3* – calculated in LIRA bundled software

The assumed calculation model makes it possible to change the rigidity of materials for both one-layer and two-layer slabs.

The slab calculation was performed in linear position for loads corresponding to loading pitches at testing. Non-linear physical-mechanical properties were taken into consideration by means of changing the elasticity modulus.

Initial values of elasticity modulus for concrete and steel fiber concrete were assumed according to [1], that is $E_c = 22,5 \cdot 10^3$ MPa, $E_{cf} = 23,8 \cdot 10^3$ MPa.

In accordance with recommendations [11] the nonlinear behavior of construction is recommended to consider by means of introducing decreasing coefficients: 0,2 – in case of any cracks, or 0,3 – in case no cracks revealed.

That is why during the calculations the decreasing coefficient was 0,3 before appearance of any cracks and 0,2 – after the first cracks appeared. Reduction of slab rigidity as a result of crack was also considered by means of introducing zero rigidity elements in the tensile area within height of the crack. The rigidity of slab supports was not reduced to avoid forcing through the slab thickness

As a result of calculation of one-layer slabs in LIRA bundled software, diagrams of deflection of the slab center *f* of the total pressure $P_{tot}=16P_i$ were obtained, which are presented in Fig. 7.
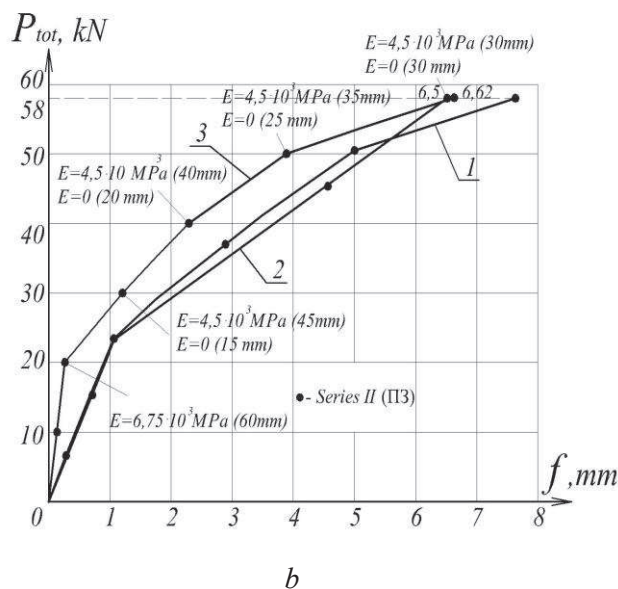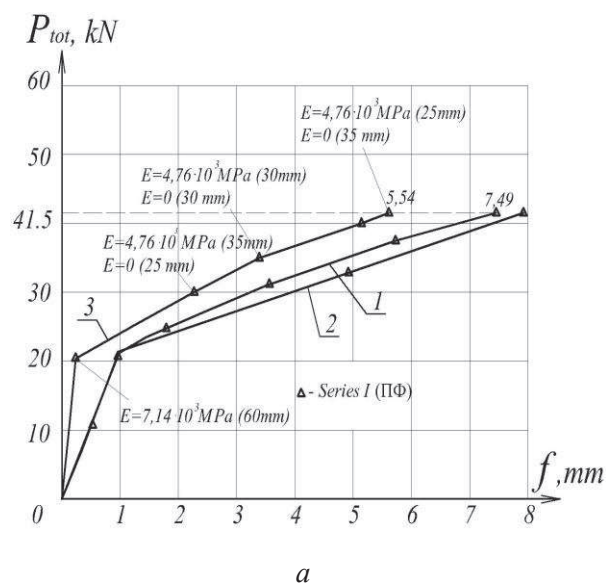
When calculating the two-layer slabs, relevant elasticity modulus of material was applied for each slab layer. The value of decreasing coefficients and zero elasticity elements were applied similar to the calculation of one-layer slabs.

As a result of calculation of two-layer slabs by means of LIRA bundled software, diagrams of deflection of the slab center *f* of the total pressure $P_{tot}=16P_i$ were obtained, which are presented in Fig. 8.
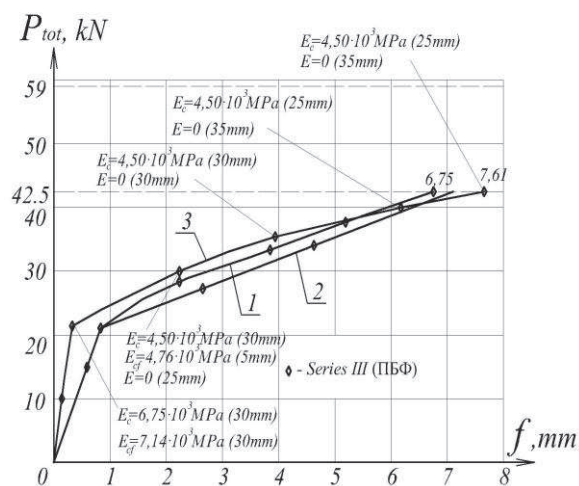
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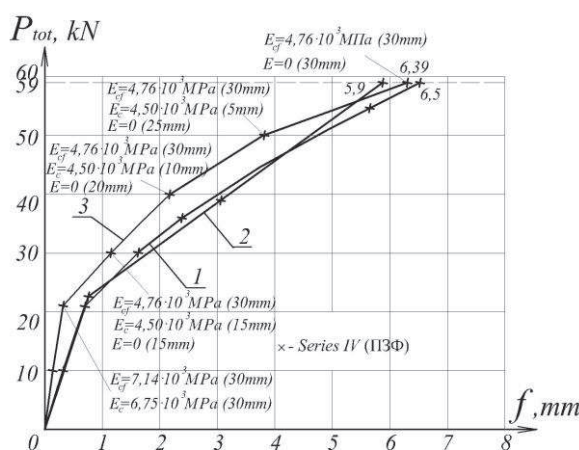
b

Fig. 8. Calculation results for deflection of two-layer slabs of series III grade ПБФ (*a*) and slabs of series IV grade ПЗФ (*b*): *1* – experimental; *2* – calculation by the limit equilibrium method; *3* – calculated in LIRA bundled software

CONCLUSIONS

1. Given that today's construction industry is represented by the vigorous process of increasing the strength of construction materials, particularly concrete and reinforcement, due to achievements in chemistry, there is a strengthening of quality indicators of buildings and constructions, in particular bearing construction arrangements. Thus, systems making it possible to work in a multi-axial load state (shell structures, slabs, wall-beams, etc.) are getting more popular. There is an opportunity to significantly reduce the cost by reduction of cross-section oper-

ational overcuts at the expense of increased strength of materials. The new technologies make it possible to perform the most complex design elements made of any materials. Considering the current trends, it should be noted that the issue of performance reliability for buildings and structures is not about the strength requirements, but the rigidity of elements and buildings in general. Therefore, the study of rigidity (most commonly the deflections) of examined slabs seems to be a topical issue.

2. Another question this work bears an answer to is the feasibility of using multi-layer slabs. From the point of view of rigidity (bending) of slabs, the two-layer slabs have an advantage: they show 10% less deflections, than the one-layer ones. Considering the higher level of crack resistance of tow-layer slabs, the use of two-layer slabs is absolutely justified.

3. Calculation by a computer using LIRA bundled software provides wide opportunities to determine rigidity (deflection) for both single-layer and multi-layer slabs owing to computing technologies. However, the obtained calculation results indicate a deficiency in accuracy, compared to the experimental data. The fact is that compliance with regulatory guidelines, implemented to consider nonlinear structure operation by introduction of decreasing coefficients, sometimes does not match the actual conditions of slab deformation. Furthermore, there are some doubts as to the correctness of defining the depth crack distribution along the slab height, which determines its actual rigidity.

4. The comparative analysis of experimental and theoretical diagrams of slab deflection evidences good matching of results. Deviations at maximum loads were as follows: for one-layer slabs of series I (ПФ) – 26%, for slabs of series II (ПЗ) – 13%; for two-layer slabs of series III (ПБФ) – 11%, for slabs of series IV (ПЗФ) – 2%. Such deviation in calculations may be explained by complexity of defining the real rigidity of slabs, i.e. the definition of deformation modulus and the height of crack creation to set the zero rigidity elements.

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РАСЧЕТ ПРОГИБОВ ОДНО- И ДВУХСЛОЙНЫХ ПЛИТ, ОПЕРТЫХ ПО КОНТУРУ

Аннотация. Приведены методика и результаты экспериментальных исследований прогибов однослойных и двухслойных плит под действием поперечного кратковременного нагрузки. Рассчитаны прогибы плит способом, основанном на методе предельного равновесия, и методом конечных элементов, с помощью програмного комплекса ЛИРА. Выполнены сравнения экспериментальных и теоретических результатов расчета прогибов плит.

Ключевые слова: плита, прогиб, сталефибро-бетон