A POSSIBILITY OF SPATIAL MODEL UTILIZATION FOR DESCRIPTION OF SOIL DEFORMATION PROCESSES

Stefan Szwaj

Institut of Building Mechanization, Warsaw

INTRODUCTION

A cognitive activity of a man is often based on modelling some different objects or some economical, sociological, biological, technical, etc. processes. This paper, of course, refers to modelling soil media.

The number of soil models is very great [10]. Here, we shall mention Terzaghy's consolidation model which consists of a vessel with a perforated piston inside it. Some elements representing soil skeleton properties are put inside this vessel. Biot (who had properly adapted the Darcy filtration law), made the use of Terzaghy's model with an additional elasticity for description of a two-dimensional and spatial states of deformation. This additional elasticity represents a volumetric soil skeleton deformation [6].

The Terzaghy's idea was utilized, among others, by Taylor, Goldstejn and Tan too (clay models). These authors have additionally inserted into the Terzyaghy's vessel the elements representing viscosity and elasticity (Tan). Other authors (Wiałow, Skibicki [20], Straganow [12], Tan and others) have included to soil models an element of plasticity too. Some models are built to a very great extent, e.g. Dmitruk's [1], Florin's, Tan's, Kawakami's, Ogawa's [6], Goldstejn's [2] models.

It should be noted here that Rachmatulin [9] has considered a soil, subjected to very fast deformations as so-called „plastic gas", which has variable compressibility of particles.

In 1968 Kisiel [3] proposed his *M/V* soil model consisting of two (Maxwell's and St-Venant's) elements connected in parallel. In his papers [4, 5, 6, 7, 8] he gave various examples how to apply his model in civil engineering.

In this paper the *M/V* model was used as a base to build a new model representing the deformation processes of homogeneous and nonhomogeneous (orthotropic) soils. Next some simplifications were introduced to the model adapting it for the description of soil (loes, sand and clay) subjected to the compression process in triaxial apparatus at high rate of deformation [13, 14] and without filtration.

SOIL MODEL FOR TWO-DIMENSIONAL STATE OF DEFORMATION DESCRIPTION AND DESIGN

The soil considered here is treated as a continuous elasto-plastic-viscous medium. The model for a such soil for two-dimensional state of deformation consists of two elements connected in parallel: Maxwell's $M = G^M - \lambda^M$ and St. Venant's $V = \Theta^v - G^v$ elements. These elements are put into the inner vessel S_W possessing two walls inclined at an α angle (Fig. 1). When the force acting on a piston N_f would attain the value adequate to the limit equilibrium state of the soil, the walls of inner vessel S_w could slide over guides S_z.

The forces acting on perforated piston N_f (representing air and water filtration) and on the outer vessel walls are not shown in Fig. 1. Instead of these the main stresses σ_1 and σ_2 are schematically given in the Figure in order to explain better the analogy between this model and considered soil element.

The motion of inner vessel S_w mentioned above, due to vertical force P_f, displaces to the side the guides S_z, that are pressed by two side elements K_b composed of Maxwell's and St-Venant's elements too (Fig. 1).

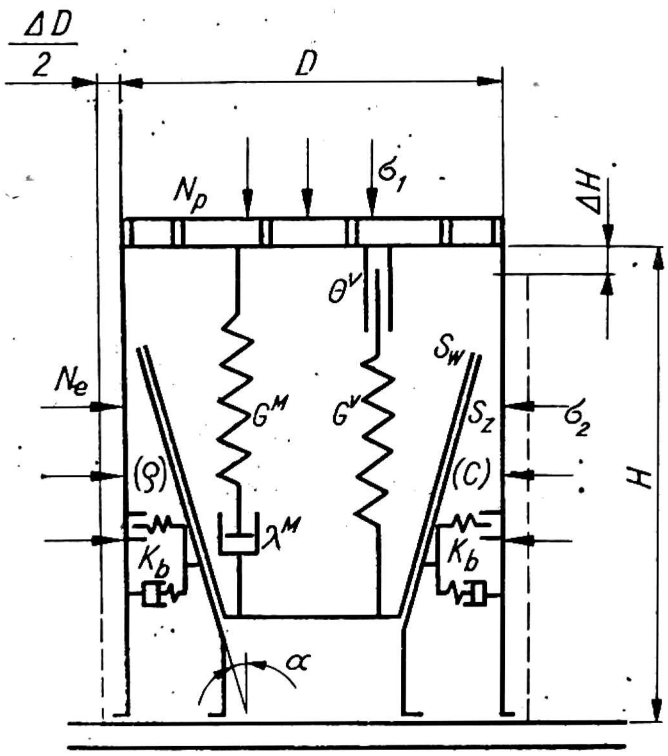

Fig. 1. Model for plane state of soil deformation

As a result of that action, the side walls of outer vessel will be also moved to the side at the moment when model would represent the limit equilibrium state of the soil. M and V indexes shown in Fig. 1 „refer" to Maxwell's and St-Venant's element values respectively.

There are as follows:

G^M, G^V — modulus of rigidity referred to Maxwell's M element and St- Venant's V element,

λ^M — coefficient of resistances depending on speed of non-dilatational strains (viscosity),

Θ^V — a magnitude representing irreversible part of soil strains which depends on many factors such as material, density, lateral pressure σ_2 and other factors e.g. time.

For an example, element Θ^V can be represented by an elastic clasp attached to the piston N_f (Fig. 2) of the model. The clasp side surfaces are

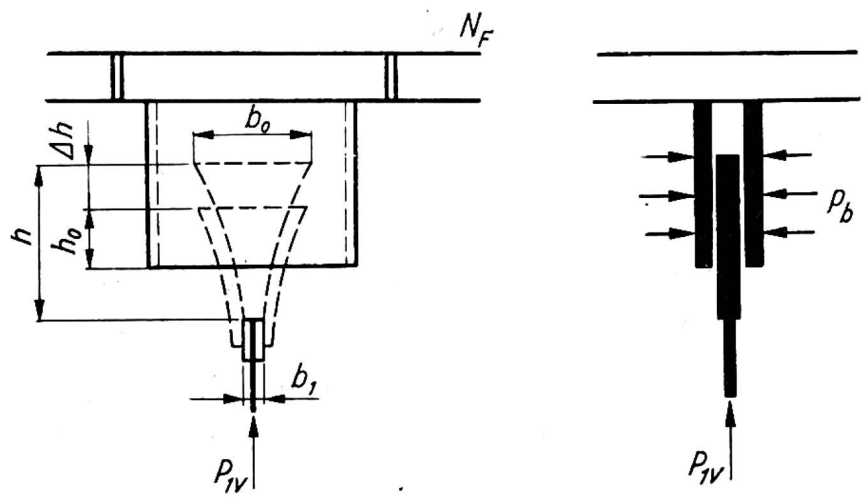

Fig. 2. Element representing the changeable value of Θ

subjected to pressure $p_b = \sigma_2$. During downwards motion of the piston the thin plate of variable width enters into the clasp thus representing irreversible (permanent) soil strains. It is assumed now that the reaction force against the plate movement is proportional to a plate surface area being in contact with clasp that is

$$P_{1V} = 2 f p_b k, \tag{1}$$

where

f — plate surface area in contact with clasp,

k — coefficient of friction.

Calculated force P_{1V} (for St-Venant's element) together with P_{1M} force (loading Maxwell's element) should be referred to piston surface area F_f representing therefore the big main stress σ_1. It should be noted that by assuming trapezoidal plate form of Θ [18] it is possible to illustrate the fact of slower increase of big main stresses when the deformation is going on before the limit equilibrium state is attained. Such phenomena were recorded by author during his investigations of soil using a triaxial apparatus [16, 17].

The model, as shown in Fig. 1, is related to elasto-plastic-viscous body, because there are elements representing elasticity G^M, G^v, plasticity Θ^v and viscosity λ^M.

Similar properties will also represent model which $G^M = \infty$.

When a model is to be designed the following relation between a piston displacement and side walls versus a soil deformation is to be assumed:

$$\frac{\Delta H}{H} + \frac{\Delta D}{D} \approx \varepsilon_1 + \varepsilon_2 = \varepsilon_v = \frac{\Delta V}{V}. \quad (2)$$

Where ε_1 and ε_2 are big and small main strains respectively before the limit equilibrium state of soil is attained,

$$\varepsilon_v = \frac{\Delta_v}{v}$$

is a volumetric strain (V — „slice" volume of the soil of unitary width), ΔH is a piston displacement and ΔD is a side walls displacement.

Assuming that displacements of piston N_f and side walls N_e are proportional to a soil deformation at the time of exceeding the limit equilibrium state, we can write a formula for an inclination angle of guides and slides

$$\operatorname{tg} a = \frac{-\Delta \bar{D}}{2\Delta \bar{H}},$$

where level „dashes" denote the values corresponding to the moment of appearing the limit equilibrium state in the soil.

Furthermore, it is possible to analyse guide inclinations for various cases of soil volumetric deformation during exceeding the limit equilibrium [18].

It should be pointed out that two elements K_b pressing the guides S_z (Fig. 1), at the moment representing the limit equilibrium of soil, are stressed to such an extent that it is sufficient to displace the walls N_e sideways. K_b elements are made of St-Venant's element and Maxwell's elements (symbols are not shown in the figure). Each of element mentioned above takes a part of a load that their sum should represent a small main stress σ_2.

Guides S_z and slides S_w are covered with a material that imitates on one side a cohesion c and on the other side a soil internal friction ϱ, (Fig. 1), A model described above was practically performed by the author introducing some certain simplifications for technical reasons. Some experiments made on the model indicate the analogy to the processes observed in soils.

Total force acting on the inner vessel S_w is equal to the force loading the Maxwell's and St-Venant's elements (compare with Fig. 1), and a displacement of S_w over S_z according to our assumption corresponds to the phenomenon of attaining the limit equilibrium by the soil. Thus we can utilize this fact per analogy to compare the stresses calculated on a base of strength theory with stresses which correspond to the state of the soil limit equilibrium. This method has been used by the author in order to prepare results of soil tests carried out in triaxial apparatus [13].

MODEL OF SOIL FOR SPATIAL DEFORMATION STATE

The model described earlier for two-dimensional states of soil deformations can be extended to spatial states. This can be done by dividing the inner vessel into two parts S_{w2} and S_{w3} having their own separate bottoms B_2 and B_3 (Fig. 3 shows two views of this model). The walls of the vessels S_{w2} and S_{w3} cooperate together with two pairs of guides S_{z2} and S_{z3} which are pressed by K_{b2} and K_{b3} elements respectively. Each of these pairs represents the cohesion (c_3 and c_2) and internal friction of the soil (ϱ_2, ϱ_3).

The bottom B_2 is placed at a certain distance from B_3 that allows for an independent vertical motion of each bottom. This movement is caused by piston load taken on by two systems M/V consisting of the following elements:

$$M_2 = G_2^M - \lambda_2^M, \qquad V_2 = \Theta_2^V - G_2^V,$$

$$M_3 = G_3^M - \lambda_3^M, \qquad V_3 = \Theta_3^V - G_3^V.$$

The forces acting on piston N_f as well as walls of the outer vessel N_{e_2} and N_{e_3} are recognized as forces representing main soil stresses σ_1, σ_2 and σ_3 which coincide with main deformations ε_1 ε_2 and ε_3 according to the previous assumption.

Various values of G, λ and Θ characterize some soil anisotropy phenomena that can be frequently found in the nature. Model in Fig. 3 corresponds to orthotropy medium.

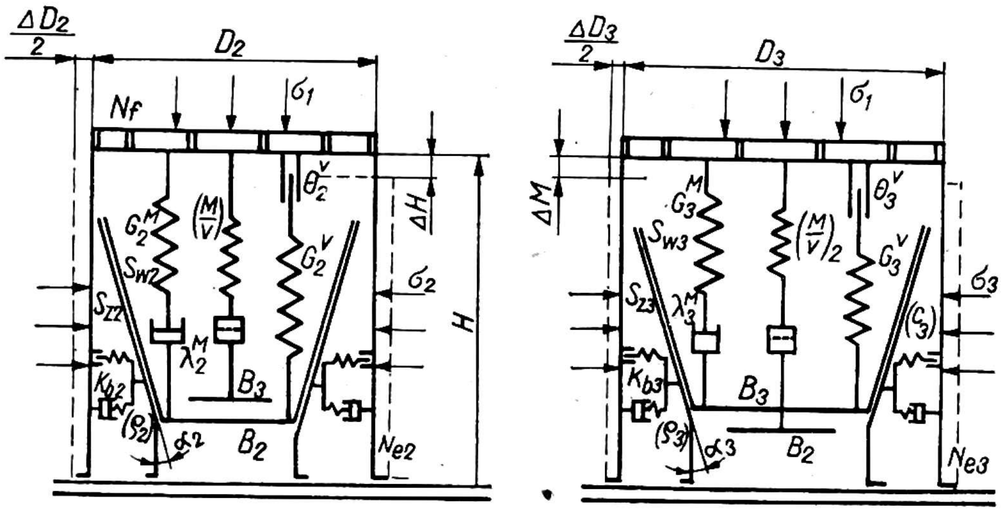

Fig. 3. Spatial model

It was assumed here, similar as before, some relation which is important just before attaining the limit equilibrium state by the soil

$$\varepsilon_v = \frac{\Delta v}{v} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \approx \frac{\Delta H}{H} + \frac{\Delta D_2}{D_2} + \frac{\Delta D_3}{D_3}, \tag{3}$$

where V is a volume of soil in consideration ε_1, ε_2, ε_3 main deformations and remaining values are shown in Fig. 3.

Assuming such a condition that the displacement of piston and side walls will be proportional to the main strains of the soil (being at that time in the limit equilibrium state) we can write a formula for α_2 and α_3 where level dashes above signs refer each value to the state corresponding the limit equilibrium

$$\operatorname{tg}\alpha_2 = -\frac{\Delta \bar{D}_2}{2\Delta \bar{H}}, \tag{4}$$

$$\operatorname{tg}\alpha_3 = -\frac{\Delta \bar{D}_3}{2\Delta \bar{H}}. \tag{5}$$

It is possible to analyse here, like in the case of two-dimensional state of deformation, the inclination of guides for various values of volumetric deformation of soil at the time of exceeding the limit equilibrium.

Material characteristics of medium for 1-2 direction of orthotropy are shown on a given model (Fig. 3) by means of $(M/V)_2$ and for 1-3 direction by $(M/V)_3$. If it is required to present the material characteristics for 2-3 direction, then it is necessary to build a more complex model. Such a model shown as an example in Fig. 4, consists of three times duplicated model of Fig. 3.

Fig. 4 shows three cross-sections of one model which has additionally the same elements on the opposite side of the symmetry axis. Wall corners of outer vessel are connected together by means of an elastic material. Loading forces acting on the model coincide with main stresses σ_1, σ_2, σ_3 that are schematically shown in Fig. 4.

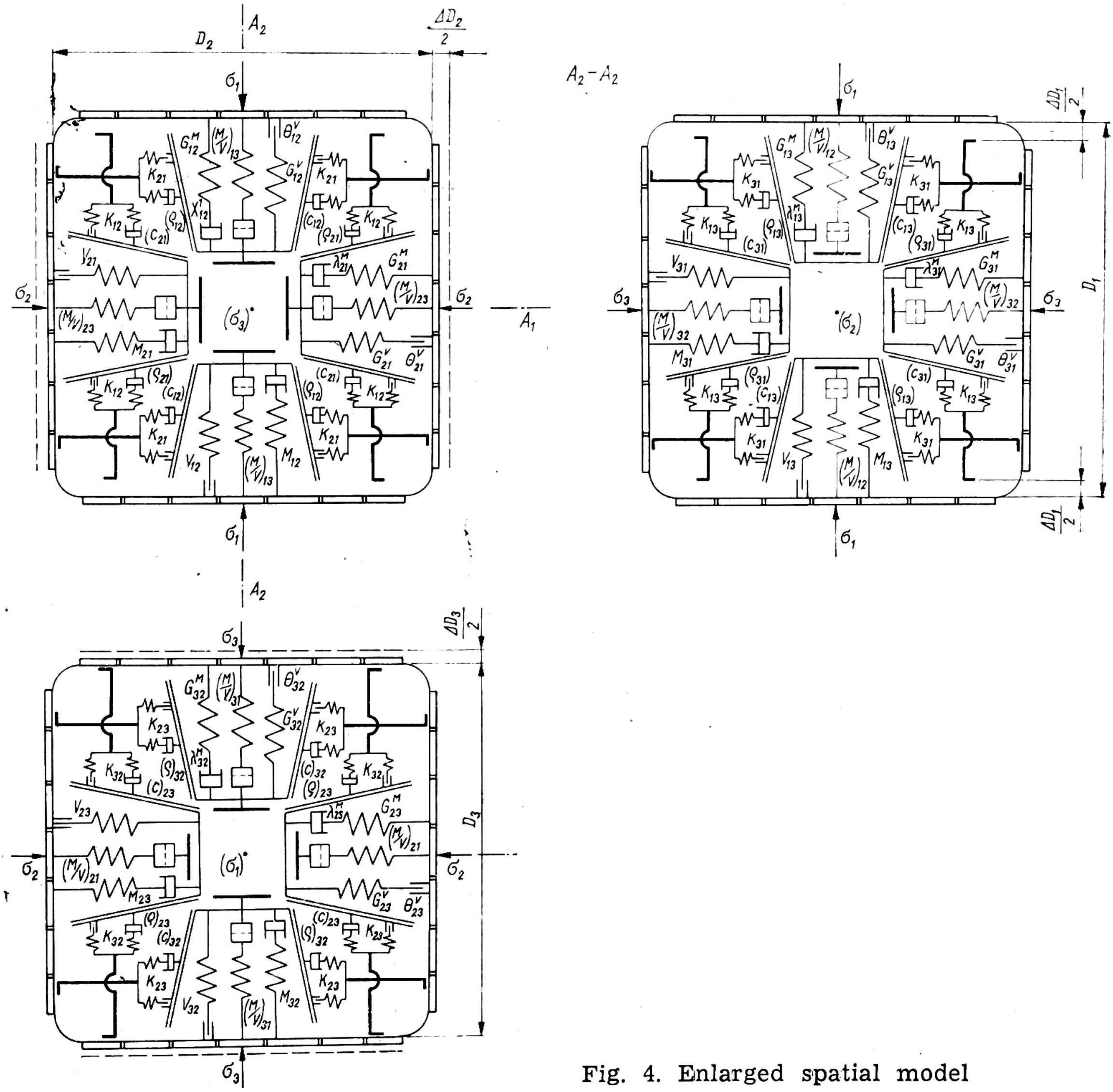

Fig. 4. Enlarged spatial model

The soil properties indicating orthotropy could be demonstrated on the model as follows:
for direction 1-2 of orthotropy by

$$(M/V)_{12} = \frac{G_{12}^M - \lambda_{12}^M}{\Theta_{12}^V - G_{12}^V},$$

for direction 1-3 by

$$(M/V)_{13} = \frac{G_{13}^M - \lambda_{13}^M}{\Theta_{13}^V - G_{13}^V},$$

for direction 2-3 by

$$(M/V)_{23} = \frac{G_{23}^M - \lambda_{23}^M}{\Theta_{23}^V - G_{23}^V}.$$

Change of the index sequence means the change of directions of the orthotropy under the consideration.

In case of homogeneous isotropic soil, the model will be much simplified; bottom indexes can be omitted. For description of such a soil under loading, Kisiel's equations [3, 4, 5, 6] may be used, referred to the states before attaining the limit equilibrium. In this case a mixture of water and air must be treated as only one agent to be filtrated. Neglecting the weight of the soil we can get equations (6) and (7)

$$\sigma_o = \sigma_m = 3K_o\gamma_o + \sigma_w, \tag{6}$$

$$\sigma_{ij} = 2L_o\gamma_{ij} + [(3K_o - 2L_o)\gamma_o + \sigma_w]\,\delta_{ij}, \tag{7}$$

where

$i, j = 1, 2, 3$ — indexes coinciding with orthogonal coordinates (x_1, x_2, x_3).

σ_{ij} — stress acting in soil (taken on by skeleton, water and air),

$\sigma_o = {}^1/_3\,(\sigma_1 + \sigma_2 + \sigma_3)$ — mean, stress,

σ_w — neutral stress to be taken on by water and air mixture,

γ_{ij} — soil deformation,

$\gamma_o = {}^1/_3\,(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$ — mean soil deformation,

δ_{ij} — Kronecker's symbol, and the remaining values are the operators introduced by Kisiel [3, 5].

$$K_o = \frac{2(1+\nu)}{3(1-2\nu)}\;\frac{G^M G^V + (G^M + G^V)\,\lambda^M \cdot S}{G^M + \lambda^M \cdot S}, \tag{8}$$

$$L_o = \frac{G^M G^V + (G^M + G^V)\,\lambda^M \cdot S}{G^M + \lambda^M \cdot S}, \tag{9}$$

where ν is Poisson's coefficient S is differential operator given by following formula:

$$S\gamma = \frac{\partial \gamma}{\partial t} + \gamma(0). \tag{10}$$

Taking into consideration the initial condition $\gamma(0) = 0$ we obtain

$$S = \frac{\partial}{\partial t}. \tag{11}$$

Other values were already described previously. We shall only mention that the modulus G^V represents here the resistance of the soil against elastoplastic deformation.

MODEL SIMPLIFICATION FOR TESTS TO BE CARRIED OUT IN TRIAXIAL APPARATUS

Kisiel's formulas are rather complicated, therefore for practical use they should be simplified. Assuming that $G^M \to \infty$ we will get the following operators (corresponding to Kelvin's model):

$$K^K = \frac{2(1+\nu)}{3(1-2\nu)}(G^V + \lambda^M \cdot s), \tag{12}$$

$$L^K = G^V + \lambda^M \cdot s. \tag{13}$$

Then we can consider non-filtrational deformation processes of soil not fully irrigated. It is then sufficient to put into formulas (6) and (7) $\sigma_w = 0$; that will give us considerable simplification.

We can consider a case when $\lambda^M = 0$ too (absence of viscosity) that is important for elastoplastic body having bulk modulus

$$K^P = \frac{2(1+\nu)}{3(1-2\nu)} G^P,$$

where G^P transverse strain modulus of elastoplastic medium.

A model fulfilling the last assumptions refers to a dry soil deformed without an initial consolidation. Stopping in turn the action of Θ^v element, we obtain a model of elastic body, at the same time moduli G^P and K^P should be replaced by modulus of transverse elasticity G^e and bulk elasticity modulus K^e. Such a model will represent well a dry soil compacted to a great extent.

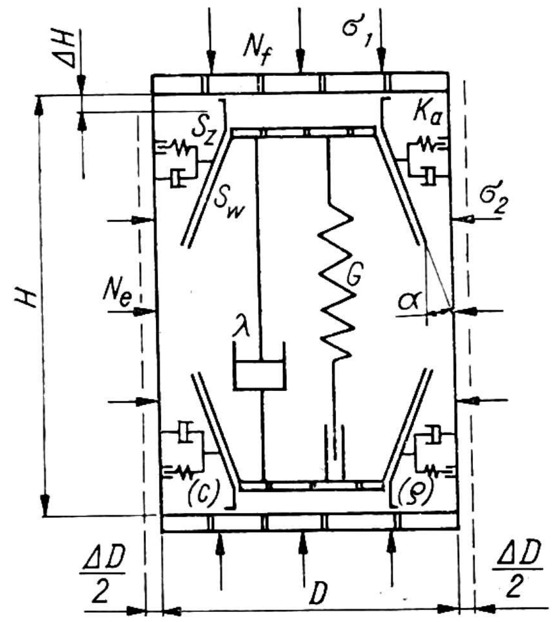

Fig. 5. Simplyfied model of a sample in the triaxial apparatus

Limiting our considerations to axial--symmetrical states of stresses, we find some analogy to these states which are present in a sample tested in the triaxial apparatus. Such a sample can be described by means of Kelvin's model, shown schematically in Fig. 5. This model consists of two vessels in a form the frustrum of cone S_w, and with their bottom ends turned to the outside. G, Θ and λ elements are inserted into cones and these cones are abutting to the guides S_z. These guides can be made of several parts and may alternately represent c and ϱ values. The guides are pressed by K_a elements resting against side walls N_e.

Accordingly to the introduced assumptions, we obtain the following:

$$\sigma_m = 3K^K\gamma_o,$$

$$\sigma_1 - \sigma_2 = 2L^K(\varepsilon_1 - \varepsilon_2),$$

where

σ_1, σ_2 — main vertical stress and main lateral stress,

ε_1, ε_2 — main strains, vertical and lateral, respectively.

Taking denotations for

$$K = \frac{2(1+\nu)}{3(1-2\nu)} G^V,$$

$$\chi = \frac{2(1+\nu)}{3(1-2\nu)} \lambda^M \qquad \text{and}$$

omitting indexes V and M of symbols G, λ — respectively and taking into consideration the formula (11) we shall get the new formulas given in papers [13 to 17].

Basing in worked out formulas and to the results of tests, the author calculated the values that characterize the rheological properties of the soils that are given in the quoted bibliography.

CONCLUSIONS

On the base of the above considerations, we can now state that there is a possibility of modelling the processes of deformation of real soils.

Some models given in this paper (Figs. 3, 4) are more complicated and they correspond to the soils indicating orthotropic characteristics. Basing on the proposed models it is possible to explain many phenomena that take place in soils subjected to deformation. It is especially important because it enables to show, in a drawing, an element representing the discussed phenomena. It is also possible to use a suitable model for description of machine-soil system.

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S. Szwaj

MOŻLIWOŚĆ ZASTOSOWANIA MODELU PRZESTRZENNEGO DO OPISU PROCESÓW DEFORMACJI GLEBY

Streszczenie

Autor traktuje grunt jako ośrodek ciągły, sprężysto-lepko-plastyczny. Wykorzystano model Kisiela M/V rozbudowując go w celu opisu anizotropii. Wprowadzono pewne uproszczenia, tak że ostateczny model jest przystosowany do opisu ściskania gruntów (lessu, piasku i gliny) bez filtracji w aparacie trójosiowym przy szybkich odkształceniach.

С. Швай

ВОЗМОЖНОСТЬ ПРИМЕНЕНИЯ ПРОСТРАНСТВЕННОЙ МОДЕЛИ ДЛЯ ОПИСАНИЯ ПРОЦЕССОВ ДЕФОРМАЦИИ ПОЧВЫ

Резюме

Автор рассматривает почвогрунт как эластично-липко-пластическую сплошную среду. Используется модель M/V Киселя, при ее расширении для описания анизотропии. Вводятся некоторые упрощения, в связи с чем окончательная модель приспособлена для описания сжимания почвогрунтов (лёсса, песка и глины) без фильтрации в трехосевом аппарате в условиях быстрых деформаций.