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Application of Weibull distribution for predicting the distribution of cereal extrudate fractions

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Abstract: *Application of Weibull distribution for predicting the distribution of cereal extrudate fractions*. The study discusses the possibility to apply Weibull distribution to describe the granulometric composition of fragmented corn extrudate. The independent variables considered included fragmentation time and temperature profile of the production process settings (temperature profile of the extruder cylinder). The investigations were performed on extrudate obtained from the production line, before the sifting process, whose fragmentation was performed in laboratory conditions. It was determined that the results of empirical investigations and results approximated using Weibull distribution were good for all fragmentation times set. Moreover, the Weibull distribution function correctly described the granulometric composition of extruded corn and wheat crumbs, irrespective of the fragmentation time. Changes in the parameters describing Weibull distribution were estimated depending on the fragmentation time. It was concluded that analyses of this kind may be used for forecasting the fragmentation time and, moreover, granulometric composition obtained on the sieves of a sifting classifier.

Key words: extrusion, granulometric composition, Weibull distribution

INTRODUCTION

Extruded cereal products account for a significant share of the breakfast snack market. Technological capabilities of the extrusion process allow manufacturing of products in various shapes and textural characteristics, shapes and flavours. Advance in the development of extruded cereal products may be divided into three phases. First generation extruded products were obtained from roughly ground grains. The technological process involved the supply of water to the mixture and extrusion of the finished product through a matrix heated up to over 90°C, which allowed achieving increased starch digestibility [Yuan-Yuan et al. 2013]. The process substituted the cooking process. The second generation of extruded product manufacture takes advantage of the expansion capabilities of starch extrudates [Alishahi et al. 2015]. This allows creation of products characterised with various expansion degrees and shapes. Third generation starch products are semi-finished products for further processing [Maya and Rajalakshmi 1999, Martínez et al. 2015]. A vast majority of them is characterised with a generally low degree of expansion. Lack of expansion allows forming the products in various, often sophisticated shapes. In this case, the process of extrusion is used to obtain a material which is further processed after leaving the extruder.

At present, both second and third generation extrudates are commonly produced. One of the groups from the border of the second and third generations are fragmented expanded extrudates [Sciarini et al. 2010]. This product group includes extruded, ready to eat baby cereals, crumbs and toppings [Xue and Ngadi 2007, Altunakar et al. 2008]. The technology used to manufacture this kind of product involves obtaining of a semi-finished product characterised with the right degree of expansion, which is subsequently fragmented in mills or grinders [Chen et al. 2008]. The granulometric composition of the finished product is achieved thanks to the use of sieves at the grinders. Growing requirements of recipients who expected products characterised with defined usable characteristics, connected with appropriate differentiation in terms of product fragmentation, bring the focus to configuring the granulometric product composition based on the time of extrudate fragmentation [Yogesh et al. 2015]. Utilisation of extrudates' natural characteristics to influence the granulometric composition allows reduction of the unit costs of fragmented products. The grain fragmentation process is thoroughly discussed in the literature which, however, does not contain information on the modelling of fragmented extruded grain products.

The model describing the likelihood of occurrence of particles whose size differs from the normal distribution is presented in Weibull's study [1951]. A generalised description related to the likelihood of occurrence of a particle with the diameter d is contained in the following formula:

$$f(\lambda,k,d)=\begin{Bmatrix} \left(\frac{d}{\lambda}\right)^k e^{-\left(\frac{d}{\lambda}\right)^k} \text{ dla } d\geq 0 \\ 0 \text{ dla } d<0 \end{Bmatrix} \quad (1)$$

where:
λ – parameter of scale;
k – parameter of Weibull curve shape.

The model is widely applied for describing the composition of fragmented mixed vegetable products [Lisowski et al. 2014, Guessasma et al. 2015].

OBJECTIVE AND SCOPE OF INVESTIGATION

The objective of the investigation was to determine the influence of the fragmentation time onto the granulometric composition of the obtained crumbs, and describe it using Weibull distribution.

METHODOLOGY

Two kinds of cereal extrudates, produced in different process conditions, were used in the investigation. The extrudates were taken from the "American" crumbs production line, before delivery to the grinder. The extrudates' composition was homogenous and included 80% cornmeal, 19,5% wheat meal and 0.5% colourant additives whose composition was described by Ekielski [2013]. The temperature profile for each of the processes is presented in Table 1.

After drying to the humidity level of 13%, samples obtained in the process were divided into 3 samples with the weight of m = 120 g each. Subsequently, the produced extrudates were fragmented in a universal BOSCH MKM 6000 grinder equipped with a two-blade knife (power: 180 W, capacity: 75 g), during the time of 2, 3 and 5 seconds, as identified in Table 2. Next, granulometric com-

TABLE 1. Temperature profile of the extrudates used on production lines

Process	Temperature [°C]			
	Section 1	Section 2	Section 3	Section 4
A	120	100	70	50
B	130	130	100	80

position classification was performed on a LP2E-2e (Multiserv, Poland) laboratory sifter equipped with sieves characterised with opening diameters presented in Table 3.

TABLE 2. Identification of the analysed samples of fragmented product depending on process parameters and fragmentation time: *T*(*s*) – operating time of the fragmenting device, *P* – temperature profile code.

P	*T*(*s*)		
	2	3	5
A	A2	A3	A5
B	B2	B3	B5

The investigation was performed in compliance with DIN ISO 3310-1 norm, by applying the vibration amplitude $A = 6$ mm, vibration frequency $f = 2$ Hz and during the time $t = 10$ min. Fractions stopped at subsequent sieves were collected and their weight was checked on a RADWAG WPA 600 laboratory scale with the accuracy of 0.001 g. For each experiment, the measurements were repeated three times.

The data collected were normalised by way of calculation of the weight share w_a of the fraction stopped on the sieve identified with the index a, pursuant to the following formula:

$$w_a = \frac{m_a}{\sum_{i=0}^{k} m_i} \tag{2}$$

where:
w_a – share of weight of the fragmented product, stopped on the sieve with index a (Table 2);
m_a – weight of the product stopped on sieve a;
m_i – weight of the product stopped on sieve i;
$k = 8$.

The dust collected at the bottom of the separator was considered an additional measurement point.

Subsequently, the empirical discreet distribution function $F_e(d_a)$ was built, reflecting the share of material with particle diameters smaller than those stopped on the sieve $a \in < 0; 8 >$ ($d = 0$ presents the share of dust collected below the sifter's sieves), described in the following relationship:

$$F_e(d_a) = \sum_{i=0}^{a-1} w_i \tag{3}$$

TABLE 3. Identification of measurement sieves used in the sifter

Sieve diameter [mm]	<0.1	0.1	0.25	0.5	0.8	1.0	1.6	2	3
Identification of sieve *i*	0	1	2	3	4	5	6	7	8

where: d_a – diameter of the sieve with index a.

Assuming a sufficient number of measurements, one may assume continuity of the distribution function and proceed to the continuous space. Thus, if a match between parameters λ, k of synthetic Weibull distribution model $F(d)$ and empirical distribution $F_e(d)$ is demonstrated in various process conditions, it is possible to control the granulometric composition of the product obtained from fragmented cereal extrudates. Weibull distribution concerning weight of fragmented extrudate is presented by the modified distribution function, described in the following general formula:

$$F(d_a) = 1 - e^{-\left(\frac{d_a}{\lambda}\right)^k} \tag{4}$$

where:

λ, k – Weibull distribution coefficients described in equation (1),

d_a – diameter of a sieve opening in the sifter.

Thus, function $F(d,\lambda,k)$ presented above describes what part of the fragmented material will pass through a sieve with the opening diameter of d. Determination of the distribution parameters involves presentation of the cumulated distribution of normalised weight (empirical distribution function) $F_e(d_a)$ depending on the diameter d_a of consecutive sieves, in a logarithmic coordinate system. The straight line conducted across the points makes a graph of the linear regression function:

$$y = ax + b \tag{5}$$

If the distribution is described by equation (1), then factors a, b of the straight line (5) allow calculation of coefficients λ and k. In order to demonstrate that, let us transform the formula (4) to the following form:

$$1 - Fe(d_a) = e^{-\left(\frac{d_a}{\lambda}\right)^k} \tag{6}$$

After logarithming the sides of equation (6), one will obtain

$$\ln\left(\frac{1}{1-F(d)}\right) = \left(\frac{d}{\lambda}\right)^k \tag{7}$$

Next, after transformation of equation (7), one will obtain

$$\ln\left\{\ln\left(\frac{1}{1-F(d)}\right)\right\} = k\ln(d) - k\ln(\lambda) \tag{8}$$

New variables are obtained by way of transformation

let $$y = \ln\left\{\ln\left(\frac{1}{1-F(d)}\right)\right\} \tag{9}$$

whereas $x = \ln(d)$, $c = \ln(\lambda)$ (10)

Thus, equation (8) will have the following form:

$$y = kx - kc$$

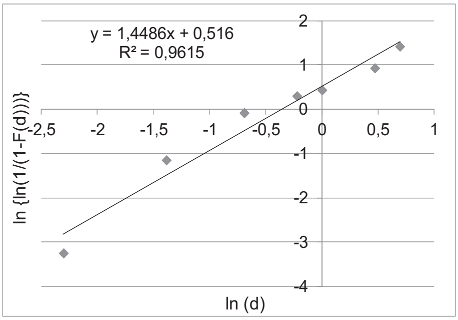
In this equation, the directional coefficient of straight line $a = k$, and free expression $b = -kc$. Thus, coefficient λ is:

$$\lambda = e^{\frac{b}{a}} \tag{11}$$

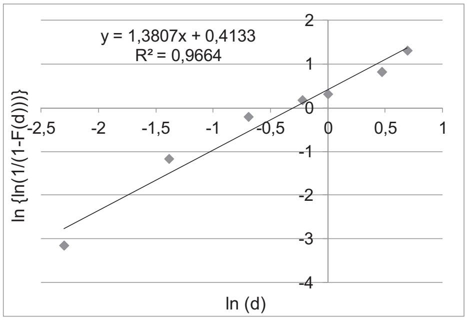
The results of normalised weight w_d and Weibull distribution value were calculated using Matlab v. 2011 software and MicroSoft Office 2010 package.

RESULTS

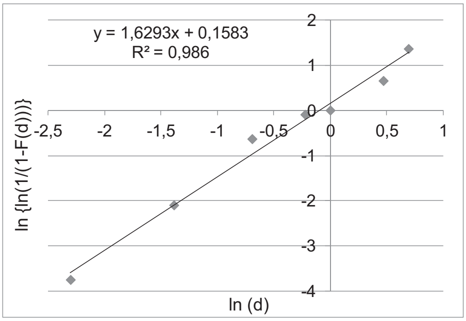
Figure 1 presents granulometric distribution of the fragmented extrudate particles and a matching curve in the logarithmic coordinate system. The graphs (Figs 2–6) present normalised weights (w_d) of the obtained crumb fractions and an approximated Weibull distribution curve. It was determined that the share of particular fractions in all the samples always grew along with increase in the sieve opening size, which was observed on all graphs. Thus, the minimum value of w_d



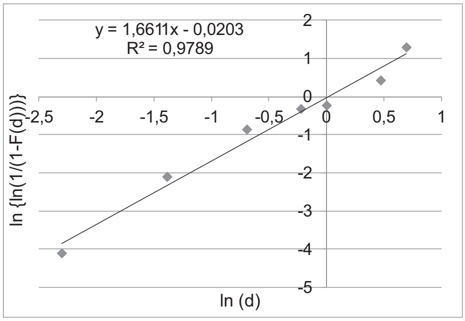
A2



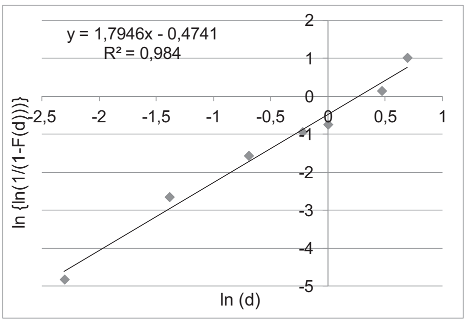
B2



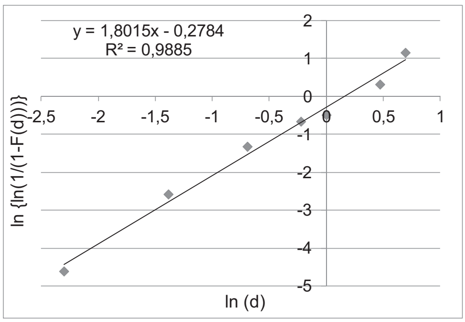
A3



B3



A5



B5

FIGURE 1. Granulometric distribution of the fragmented extrudate particles and a matching curve in the logarithmic coordinate system The coding system is presented in Table 2

parameter was in each case obtained on the sieve with 0.1 mm openings, whereas the maximum value was obtained on the sieve with 3 mm openings. Moreover, comparing the graphs, one may observe significant concurrence of the results, except for graph A4 where, although a growing trend is observed, the results are different. At the initial stage of the graphs from (0.1 to 0.8 mm sieves), very intensive growth in the share of particular fractions is observed, whereas next – as the sieve size grows (from 1 to 3 mm sieves) – weight of particular fractions stabilises, irrespective of the obtained fraction. This intensive value growth followed by slow stabilisation is typical of Weibull distribution. Significant concurrence of the results being compared is, therefore, surprising. The approximated Weibull distribution curve may, in this case, be a sensitive indicator, which may be used for analysing the granulometric composition of crumb products.

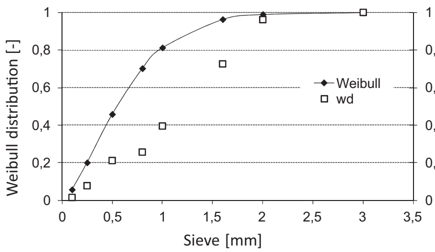


FIGURE 2. Distribution function of the fractions of crumb particles fragmented for 2 s, process A

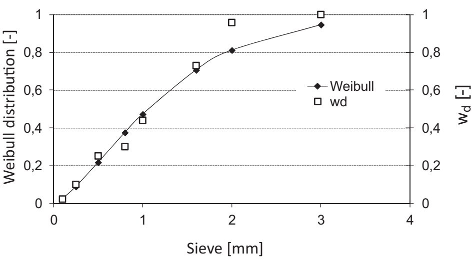


FIGURE 3. Distribution function of the fractions of crumb particles fragmented for 2 s, process B

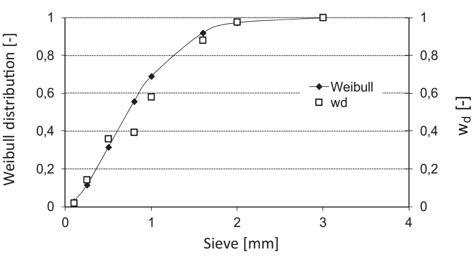


FIGURE 4. Distribution function of the fractions of crumb particles fragmented for 3 s, process A

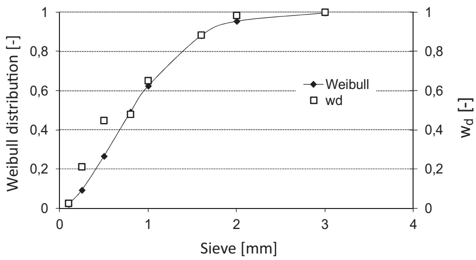


FIGURE 5. Distribution function of the fractions of crumb particles fragmented for 3 s, process B

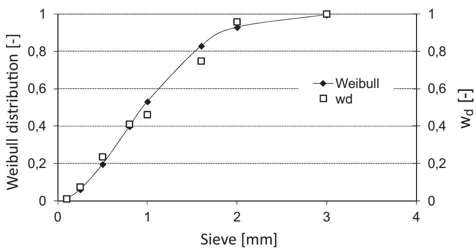


FIGURE 6. Distribution function of the fractions of crumb particles fragmented for 5 s, process (A5)

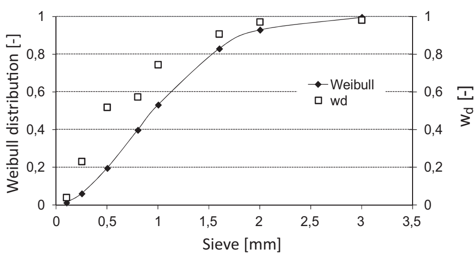


FIGURE 7. Distribution function of the fractions of crumb particles fragmented for 5 s, process B

TABLE 4. Equations of a straight line passing through points of empirical distribution function $F_e(d_a)$

Code	k	λ	Obtained values of empirical distribution in the logarithmic coordinate system	
			straight line equation	R^2
A2	0.700328	1.4486	$y = 1.4486x + 0.516$	0.962
A3	1.6293	0.9074	$y = 1.6293\,x + 0.1583$	0.986
A5	1.7946	–0.4741	$y = 1.7946\,x – 0.4741$	0.984
B2	1.3807	0.7413	$y = 1.3807\,x + 0.4133$	0.9664
B3	1.6611	1.012296	$y = 1.6611\,x – 0.0203$	0.9789
B5	1.815	1.167	$y = 1.8015\,x – 0.2784$	0.9885

The above observations are confirmed by high correlation of empirical results with Weibull distribution approximated values. Table 4 contains equations of linear functions correlated with the results. The parameter determining match of the regression line equation to empirical data was the coefficient of determination (R^2), which – in the case of linear function – is equivalent with the coefficient of correlation.

All the results, except for results presented in graph A4, were characterised with very high values of R^2, above 0.9. Best correlated results of both methods were observed on graph A2. Table 4 presents as well equations of the straight line constituting the line of regression of Weibull distribution results in logarithmic system [ln (a,b,k)], which was used to determine λ and k.

CONCLUSIONS

1. Weibull distribution function successfully describes the granulometric distribution of corn and what crumbs.
2. Descriptions concerning the model of fragmented product granulometric composition may thus be used to forecast the required time of extrudate fragmentation in order to achieve the required granulometric composition on the sieves of the sifting classifier.
3. The results of empirical investigations and the approximated Weibull distribution function were similar in the case of all pre-set fragmentation times, except for crumbs obtained during the fragmentation time of 2 s and the temperature profile A.
4. Short-term grinding in an impact grinder for 2–5 s causes an increased share of crumb fractions on the sieves, from the smallest to the largest one.

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Streszczenie: *Wykorzystanie rozkładu Weibulla do predykcji rozkładu frakcji ekstrudatów zbożowych.* Rosnące wymagania odbiorców, oczekujących produktów o sprecyzowanych cechach użytkowych, związanych z odpowiednim zróżnicowaniem rozdrobnienia produktu powodują, że zwraca się uwagę na konfigurowanie składu granulometrycznego produktu. Poszukuje się dlatego nowych metod, które umożliwią dokładne i szybkie prognozowanie rozkładu frakcji rozdrobnionych produktów. W tym celu w pracy podjęto próbę opisu składu granulometrycznego rozdrobnionego ekstrudatu kukurydziano-pszennego z wykorzystaniem rozkładu Weibulla. Zmiennymi niezależnymi, które brano pod uwagę był czas rozdrabniania oraz profil temperaturowy nastaw procesu produkcji. Stwierdzono, że wyniki badań empirycznych i aproksymowane do nich funkcje rozkładu Weibulla były zbliżone dla wszystkich zadanych czasów rozdrabniania. Ponadto, zastosowanie tej metody umożliwia opis składu granulometrycznego materiału niezależnie od czasu rozdrabniania. Stwierdzono, że opisanie modelu składu granulometrycznego produktu może być wykorzystane do prognozowania niezbędnego czasu rozdrabniania w celu uzyskania zakładanego składu granulometrycznego uzyskiwanego na sitach klasyfikatora.

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