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STANISŁAW M. ZAGRODZKI jr

MATHEMATICAL MODEL OF A FOUR-EFFECT EVAPORATOR WITH TURBOCOMPRESSOR*

Mazovian Sugar Factories, Płock

Key words: sugar manufacture, heat economy, four-effect evaporator with turbocompressor.

The presented mathematical model of a four-effect evaporator with turbocompressor enables easily to compare the results of four fariants of working conditions and the advantages of proper juice concentration. Minimum steam comsumption is obtained in the sugar factory when taking full advantage of vapours from expansion of condensates, using condensates to heat raw juice and then to supply the extractor with simultaneous steam turbocompressing.

The important task of any sugar factory is to provide an appropriate quantity of heat for sugar production [1-4]. In addition to sugar beet or sugar cane, fuel is the next material, necessary to obtain steam. Besides the transformation of heat into mechanical energy, heat is provided for the technological processes in order to keep an optimum temperature in processes and to concentrate the juice and obtain a maximum of sugar [5].

Since many years combined heat economy based on high pressure boilers has been applied in the sugar industry. In this way it is possible to perform three main tasks using the same amount of steam:

1. to produce the energy required for technological processes,
2. to heat juice and other products up to an optimum temperature,
3. to evaporate water from juice.

In a adequate evaporators system, exhaust steam from the turbine in an amount required to heat juice and other products in all processes, can also evaporate juices to the required concentration. Even in the best evaporators system taking advantage of the heat of condensates and based on an economic consumption of heat for technological purposes — turbocompressors are needed in order to use the disponible steam for an

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optimum thickening of juices without introducing vapours into the condensers [7-13].

The aim of this study is the calculation of the advantages resulting from the application of a four-effect evaporator with turbocompression operating in optimum working conditions as well as the examination of the quantity of vapour compression which enable a reduction of the quantity of steam consumption from the turbogenerators. In order to obiectively examine the influence of the turbocompression of vapour on the sugar factory steam consumption, is was necessary to apply appropriate mathematical models to various variant of work of the four-effect evaporator with turbocompression.

ELABORATION OF THE MATHEMATICAL MODEL OF A FOUR-EFFECT EVAPORATOR WITH TURBOCOMPRESSOR FOR VARIOUS WORKING CONDITIONS

Fig. 1 presents an examplary scheme of the evaporator system using condensate heat and a turbocompressor. The evaporator is supplied with steam from the turbine at a back pressure of about 4 bars. In the first stage Z tons steam are necessary to heat the juice to the boiling point. Vapours from the evaporator used in technological processes to heat juices or other products are marked as follows:

A_1 — vapours from 1-st stage for juice heating before evaporation plant,
B_1 — vapours from 2-nd stage for juice heating before evaporation plant,
B_2 — vapours from 2-nd stage for heating of extractor,
N — vapours from 2-nd stage for heating vacuum pans,
C_1 — vapours from 3-rd stage for heating juice before evaporation plant,
C_2 — vapours from 3-rd stage for heating of extractor,
C_3 — vapours from 3-rd stage for heating juice before filtration,
C_4 — vapours from 3-rd stage for heating of thick juice and syrups,
C_5 — vapours from 3-rd stage for heating of raw juice,
D_5 — vapours from 4-th stage for heating of raw juice,
S — vapours from 4-th stage led into the condenser,
C_5 — vapours from vacuum pans for heating of raw juice,
T — vapours from 1-st stage compressed to a pressure of 4 bars,
E — vapours from expansion of condensates.

Hot condensates after expansion are used to supply steam boilers and the remaining condensates are used to heat raw juice and afterwards to supply the extractor, for sugar affination and for sweetening off precipitates after the main carbonation [14, 15]. In this way the steam requirement for juice heating in the respective technological processes can be reduced by 20% [6]. Steam requirement from the respective stages of the four-effect evaporator are shown in Table 1.

On this basis a certain mathematical model of steam requirement of the four-effect evaporator with turbocompressor can be established. We may assume roughly that one ton of steam fed into the respective stage evaporates 1 ton of water from juice, taking into account slight heat losses, higher evaporation heat as compared to condensation heat under higher pressure and also water evaporation by juice expansion in the next stage with lower pressure.

Table 1. Requirement of vapours from particular effects of evaporator or of vacuum pans

Effects of evaporator	1st	2nd	3rd	4th	Vaccum pans
Heating of juice before evaporation plant	A_1	B_1	C_1		
Heating of extractor		B_2	C_2		
Heating of vacuum pans		N			
Heating of juice before filtration			C_3		
Heating of thick juice and syrups			C_4		
Heating of raw juice			C_5	D_5	G_5
Vapours to condenser				S	
Total:	A	$B+N$	C	$D+S$	G

It has been assumed that in a sugar factory processing 100 t/h sugar beet 125 t/h thin juice fed in the evaporator contains 14% dry matter and that 112 tons of water should be evaporated from juice and massecuite, including water added for sugar affination and fed into the vacuum pans. The amount of steam from second stage fed into the vacuum pans N is directly proportional to the amount of water evaporated in all vacuum pans. Considering the difference between steam condensation heat and water evaporation heat from massecuite as well as heat losses during massecuite boiling processes it can be calculated that N tons of steam fed into the vacuum pans chambers causes evaporation of 0.826 N tons water from massecuite.

On this basis it can be calculated that the amount of water W evaporated from juice in an evaporation plant is

$$W = 112 - 0.826\,N\,\mathrm{t/h} \tag{1}$$

assuming that 100 t of beet are processed per hour. It is known that the quantity of water evaporated in the evaporator depends on the concentration of thick juice, i.e.: for example 70° Bx $W_{70} = 125/1 - \frac{14}{70} = 100$ t/h. Table 2 shows amount W as depending on different concentrations of thick juice from 70° to 50° Bx.

Table 2. Dependence of the amount of water evaporated from juice upon the amount and concentration of juice introduced to the evaporation plant and upon the concentration of thick juice; e.g. for 70° Bx $W_{70} = 125(1-\frac{14}{70}) = 100$ t/h for 100 t sugar beets processed per hour

Concentration of thick juice °Bx	Amount of evaporated water W t/h
70°	100.000
65°	98.078
60°	95.833
55°	93.182
50°	90.000

Consequently the amount of water to be evaporated from massecuite and the quantity of steam necessary for this purpose from second stage N t/h depends on the amount of water evaporated in evaporators.

It is known that the amount of water evaporated in the evaporation plant is

$$W = A+2B+2N+3C+4D+4S \tag{2}$$

While the amount of water evaporated in the four-effect evaporation plant with turbocompressor is

$$W = A+2B+2N+3C+4D+T \tag{3}$$

Comparing equations (2) and (3) we can say that $T = 4S$ what means that the amount of steem compressed in the turbocompressor should be four times greater than the amount of vapours fed into the condenser to prevent vapours from the 4th stage from flowing into the condenser. In these conditions, according to equation (3), the amount of compressed vapours is as follows:

$$T = 112-A-2B-3C-4D = 2.826\,N \tag{4}$$

As it is known, the steam requirement of an evaporation plant without turbocompressor will be:

$$X = Z+A+B+N+C+D+S \tag{5}$$

and with turbocompressor:

$$Y = Z+A+B+N+C+D \tag{6}$$

which means that the use of a turbocompressor brings economies in steam consumption amounting to:

$$X-Y = S = \frac{T}{4} \tag{7}$$

In order to show the above in numbers we have assumed that the beet processing output amounts to 2400 t per day or 100 t/h. The amont of steam required for technological purposes can be established in t/h:

$$Z = 2.5;\ A_1 = 2;\ B_1 = 2;\ B_2 = 2;\ C_1 = 2.5;\ C_2 = 1;$$
$$C_3 = 2.5;\ C_4 = 1;\ C_5 = 3;\ D_5 = 3;$$

that is $A = 2;\ B = 4;\ C = 10;\ D = 3.$

N depends on the concentration of thick juice. The steam requirement (variant/1) for heating vacuum pans N as well as values of T, X and Y and S according to equation number (2) to (7) has been calculated and listed in Table 3, taking into account the differences in density of the juice flowing from the evaporator. No turbocompressor is needed when the juice density is kept at 55° Bx because vapours for different technological purposes will thicken juice and steam will not flow to the condenser. Steam reception for massecuite boiling is sufficient to evaporate, in the evaporation plant, 93.182 t/h of water from juice. The use of a turbocompressor allows to abtain a higher optimal thick juice density and a considerably lower steam consumption in the evaporation plant.

Table 3. First variant. Changes in the amount of steam for vacuum pans N, amount of steam compressed in turbocompressor T, amount of steam introduced the first evaporator without turbocompressor X, amount of steam introduced the first evaporator with turbocompressor Y, economy in comsumption of steam introduced into evaporator $X-Y$ due to the application of a turbocompressor at different dry matter contents of thick juice, expressed in t/h

Thick juice °Bx	W	N	$0.826\,N$	T	X	Y	$X-Y$
70°	100.000	14.528	12.000	18.944	40.764	36.028	4.736
65°	98.078	16.855	13.922	12.268	41.447	38.355	3.092
60°	95.833	19.572	16.167	4.688	42.244	41.072	1.172
55°	93.182	22.782	18.818	—	43.187	43.187	0.000

A second variant can be considered when heating raw juice with vapours from vacuum pans G instead of vapours from the 4th stage of evaporator D_5. Depending on the consumption of vapours from vacuum pans it is possible to reduce the requirement of exhaust steam from the turbine to the evaporation plant. In case when $G_5 = D_5 = 3$ t/h, in this particular variant it is necessary to increase the amount of steam compressed in the turbocompressor by 4 G that is by 12 t/h to obtain a suitable concentration of juice. The results are shown in Table 4 — second variant.

When a turbocompressor is used and raw juice is heated by vapours from vacuum pans in the amount of 3 t/h, the steam requirement of the

Table 4. Second variant. Changes in the consumption of steam for the first evaporator using a turbocompressor and heating raw juice with vapours $G = 3$ t/h from vacuum pans depending on dry matter content of thick juice, expressed in t/h

Thick juice °Bx	W	N	0.826 N	T	X	Y	$X-Y$	G
70°	100.000	14.528	12.000	30.944	40.764	33.028	7.736	3.0
65°	98.078	16.855	13.922	24.368	41.447	35.355	6.092	3.0
60°	95.833	19.572	16.167	16.688	42.244	38.072	4.172	3.0
55°	93.182	22.782	18.818	7.620	43.187	41.282	1.905	3.0

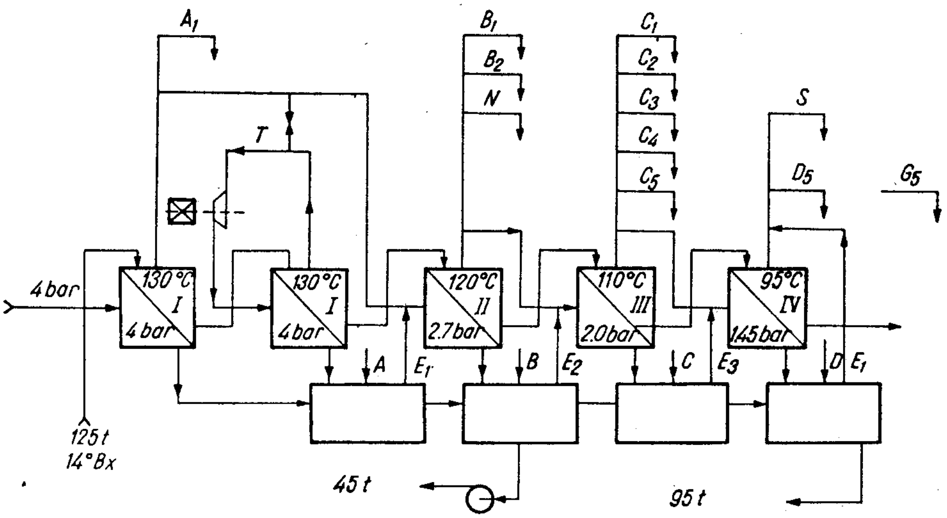

Fig. 1. Scheme of a four-effect evaporation plant system with turbocompressor and condensate utilisation

A_1 — vapours from 1st stage for heating juice before evaporation
B_1 — vaporu from 2nd stage for heating juice before evaporation
B_2 — vapours from 2nd stage for heating extractor
N — vapours from 2nd stage for heating vacuum pans
C_1 — vapours from 3rd stage for heating juice before evaporation
C_2 — vapours from 3rd stage for heating extractor
C_3 — vapours from 3rd stage for heating juice before filtration
C_4 — vapours from 3rd stage for heating thick juice and syrups
C_5 — vapours from 3rd stage for heating raw juice
D_5 — vapours from 4th stage for heating raw juice
S — vapours from 4th stage going to the condenser
G_5 — vapours from vacuum pans for heating raw juice
T — vapours from 1st stage compressed from 2.7 bar to 4 bar
E — vapours obtained from expension of condensates

That hot condensates are used to feed boilers and the remaining condensates are used to heat raw juice, to supply the extractor, for sugar affination and to sweeten-off cake after first carbonatation evaporation plan can be reduced by 19%, that is 7.7 t/h for every 100 t processed beets. When using a turbocompressor without heating raw juice with vapours from the vacuum pans — the steam economy will be 11%, that is 4.7 t for 100 t beets. The above requires a very good concentration of juice in the evaporation plant to about 70° Bx. A reduction of juice concentration to 60° Bx reduces steam economy in the first variant to 1 t/h, and in the second variant 4 t/h only instead of 7.7 t/h steam economy are obtained — using a turbocompressor and heating raw juice with vacuum pans vapours in the amount $G = 3$ t/h.

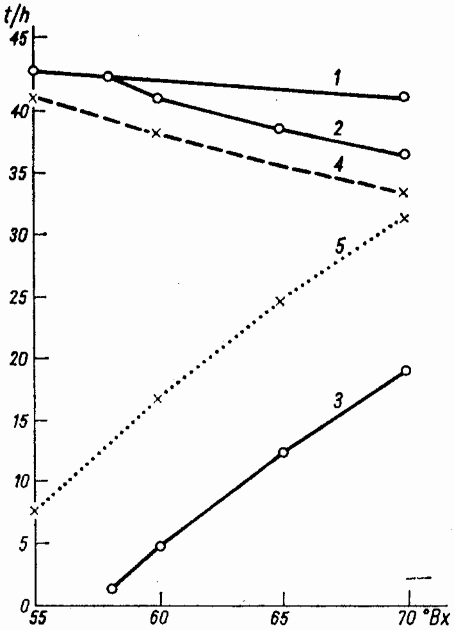

Fig. 2. Changes in steam consumption depending on thick juice concentration with application of turbocompressor.

1 — Steam consumption for thickening juice in evaporation plant without turbocompressor
2 — Steam consumption for thichening juice in evaporation plant with turbocompressor
3 — Amount of steam compressed by the turbocompressor to concentrate juice
4 — Steam comsumption for juice thickening with turbocompressor and heating raw juice with vapours from vacuum pans
5 — Amount of steam compressed by turbocompressor during heating raw juice with vapours from vacuum pans

The diagram od Fig. 2 presents the results taken from Table 3 and 4 (variant first and second) showing differences in the consumption of exhaust steam from the turbine used for the evaporator depending on the concentration of thick juice. The amount of steam in t/h is plotted on the y-axis and the concentration of thick juice is given on the x-axis. The first curve corresponds to the consumption of steam by the evaporation plant without turbocompressor, curve 2 is steam consumption when a turbocompressor is used and curve 4 gives the above when vapours from vacuum pans are used to heat raw juice (variant second). Curve 3 shows the amount of steam compressed by a turbocompressor and curve 5 the amount of compressed steam T t/h (second variant). If thick juice is concentrated to 70° Bx, steam should be introduced to both apparatuses first effect (Fig. 1) heated by exhaust steam from the turbines approximately 30 t/h and by compressed steam approximately 30 t/h (curve 4, 5, Fig. 2).

In a correct evaporation plant system, in order to obtain maximum steam economy, graded expansion of condensates is applied and the obtained vapour is used in the next effect as shown on Fig. 1. In variant first and second such a solution is not considered.

Table 5. Third variant. Changes in steam consumption depending on thick juice concentration using a turbocompressor and graded condensates expansion, expressed in t/h

Thick juice °Bx	W	N	0.826 N	T	X	Y	$X-Y$
70°	100.000	14.528	12.000	28.944	39.264	32.028	7.236
65°	98.078	16.855	13.922	22.368	39.947	34.355	5.592
60°	95.833	19.572	16.167	14.688	40.744	37.072	3.672
55°	93.182	22.782	18.818	5.620	41.687	40.282	1.405
50°	90.000	26.634	22.000	—	44.134	44.134	—

In the case of a graded expansion of condensates we can assume for simplification purposes that for example: $E_1 = E_2 = E_3 = E_4 = 1$ t/h. In this case the respective equations (3 to 6) will be changed (8 to 11): instead of equation (3)

$W = (A-E_1)+2(B-E_2)+2N+3(C-E_3)+4(D-E_4)+T$ (8) instead of equation (4)

$$T = 112 - (A-E) - 2(B-E) - 3(C-E) - 4(D-E) - 2.826\ N$$

or $T = 112-A-2B-3C-4D-2.826\ N+10\ E$ (9)

instead of equation (5) $X = Z+A+B+N+C+D+S - 4E$ (10)

instead of equation (6) $Y = Z+A+B+N+C+D - 4E$ (11)

equation (7) $X-Y = S = \frac{T}{4}$ is the only one to remain unchanged. Now, it is necessary to replace Table 3 with a list of appropriately amended results. This is third variant of steam consumption using a turbocompressor and graded expansion of condensates — Table 5. Comparing the third variant (Table 5) with data given in Table 3 (first variant) we obtain an increase of steam economy of 2.5 t/h at a high juice concentration to 70° Bx and the use of a turbocompressor and graded condensates, the steam requirement for the evaporation plant is $Y = 32$ t/h, whereas without a turbocompressor and proper condensates expansion the steam requirement is $X = 41$ t/h (see Table 3). It has to be added that without a turbocompressor such a high juice concentration is usually impossible. At 60° Bx without turbocompressor and condensates expansion the steam requirement is $X = 42$ f/h that is 10 t steam more per hour. This means that the economy of steam reaches 25%. When, in addition to proper graded condensates expansion raw juice is heated with vapours from vacuum pans the results are still better (variant fourth). Tof example, by using vapours from vacuum pans $G = 3$ t/h, steam consumption per evaporation plant can be reduced to $Y = 29$ t/h provided the density of thick juice used for boiling massecuite is of 70° Bx. The fourth variant is presented in Table 6.

Table 6. Fourth variant. Changes in steam consumption depending on thick juice concentration using a turbocompressor, graded condensates expansion and heating raw juice with vapours from vacuum pans to an amount of $G = 3$ t/h, expressed in t/h

Thick juice °Bx	W	N	$0.826\,N$	T	X	Y	$X-Y$	G
70°	100.000	14.528	12.000	39.944	39.014	29.028	9.986	3.000
65°	98.078	16.855	13.922	33.368	39.697	31.355	8.342	3.000
60°	95.833	19.572	16.167	25.684	40.499	34.078	6.421	3.000
55°	93.182	22.782	18.818	16.620	41.437	37.282	4.155	3.000
50°	90.000	26.634	22.000	5.732	42.567	41.134	1.433	3.000

The heat economies of the above evaporation system are the highest but the compressing of 40 ton steam per hour calls for a proper propulsion power for the turbocompressor. As the steam requirement of the evaporator is relatively very low, about 30 tons per hour, in modern sugar factories it is advisable to use steam boilers and turbines for a pressure of 70 to 80 bars and superheated steam at 450°C. These steam boilers and turbogenerators can produce enough power for a sugar factory with turbocompressor. A comparison of steam consumption in the third and fourth variant (Tables 5 and 6) is given in Fig. 3. The presented mathematical model of a four-effect evaporator with turbocompressor makes it easy to compare the results of four variants of working conditions and the advantages resulting from an adequate juice concentration.

Taking the above into consideration we come the following conclusions:

1. The lowest steam consumption in a sugar factory is achieved by taking full advantage of vapours from the expansion of condensates, using condensates to heat raw juice and next to supply the extractor with simultaneously steam turbocompressing.

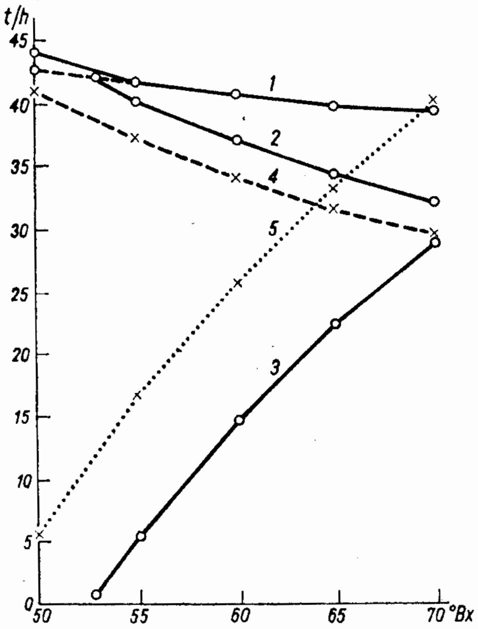

Fig. 3. Changes in steam consumption depending on thick juice concentration with the use of a turbocompressor and graded expansion of condensates.

1 — Steam consumption for thickening juice in an evaporation plant without turbocompressor

2 — Steam consumption for thickening juice in an evaporation plant with turbocompressor

3 — Amount of steam compressed by turbocompressor for the concentration of juice

4 — Steam consumption for juice thickening with turbocompressor and heating raw juice with vapours from vacuum pans

5 — Amount of steam compressed by turbocompressor during heating raw juice with vapours from vacuum pans

2. In some sugar factories utilization of heat of vapours from vacuum pans for heating raw juices results in a further economy of 3 ton steam per 100 ton processed beets.

3. The above solutions enable the use, for crystallisation purposes, syrups and juices of slightly lower concentration, e.g. 65° Bx.

4. The use of a turbocompressor enables operation of a three-effect evaporation plant without fourth effect under reduced pressure, which makes the technological process easier and compensates the cost of the turbocompressor.

5. The above examples are an illustration for a mathematical model as in each case, the scheme of the evaporation system and the process used determine the consumption of steam used in a sugar factory for heating purposes in the respective processes.

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Author address: P. P. Cukrownie Mazowieckie, Płock.

S. M. Zagrodzki jr

MODEL MATEMATYCZNY WYPARKI CZTERODZIAŁOWEJ Z TURBOSPRĘŻARKĄ

P. P. Cukrownie Mazowieckie, Płock

Streszczenie

W przemyśle cukrowniczym, przy wykorzystaniu ciepła skroplin i oszczędnym zużyciu ciepła do celów technologicznych w układach stacji wyparnej konieczne jest stosowanie turbosprężarek, aby zagęścić sok do optymalnego stężenia bez kierowania oparów do skraplacza.

Opracowano model matematyczny wyparki czterodziałowej z turbosprężarką, na podstawie którego można obliczyć korzyści wynikające z zastosowania turbosprężarki w różnych wariantach układów stacji wyparnej. Rozpatrzono zależność zapotrzebowania pary w cukrowni od stężenia soku gęstego, od stopnia wykorzystania oparów z warników do zagrzewania soku surowego oraz od stosowania sukcesywnego rozprężania skroplin i stosowania ich do zagrzewania soku, a następnie do zasilania ekstraktora krajanki.

Na podstawie opracowanego modelu matematycznego można wyliczyć wielkość zużycia pary w zależności od stopnia zagęszczenia soku w wyparce. Zagęszczając sok gęsty do 70° Bx zamiast do 55° Bx można zaoszczędzić ponad 4 t pary na 100 t przerabianych buraków. Stosując dodatkowo ogrzewanie soku surowego oparami z warników można powiększyć oszczędności do ponad 7 t pary na 100 t buraków. Najmniejsze zużycie pary w cukrowni uzyskuje się przy jednoczesnym wykorzystaniu oparów z rozprężania skroplin, stosowaniu skroplin do ogrzewania soku surowego i zasilania ekstraktora oraz ogrzewaniu soku surowego oparami z warników.

Wyliczono, że dzięki zastosowaniu turbosprężarki możliwe jest w takich warunkach zmniejszenie zużycia pary od 4 do 10 t na 100 t przerobionych buraków. W najlepszym układzie wyparki z turbosprężarką zapotrzebowanie pary do stacji wyparnej może być zmniejszone do 29 t na 100 t buraków zamiast 39 t bez turbosprężarki. Zastosowanie turbosprężarki może pozwolić na dobrą pracę trójdziałowej wyparki ciśnieniowej bez czwartego działu pod zmniejszonym ciśnieniem. Taki układ stacji wyparnej wymaga mniejszych nakładów inwestycyjnych, co wyrównuje koszty inwestycyjne związane z zastosowaniem turbosprężarki.

Przedstawione przykłady liczbowe i wykresy stanowią ilustrację modelu matematycznego. Rzeczywisty układ wyparki i sposób prowadzenia procesu technologicznego determinują wielkość zużycia pary do celów grzejnych w poszczególnych stadiach. Zastosowanie turboskrężarki ułatwia automatyzację gospodarki cieplnej oraz umożliwia utrzymanie optymalnych warunków wszystkich procesów.