Scientific Review – Engineering and Environmental Sciences (2021), 30 (3), 424–438
Sci. Rev. Eng. Env. Sci. (2021), 30 (3)
Przegląd Naukowy – Inżynieria i Kształtowanie Środowiska (2021), 30 (3), 424–438
Prz. Nauk. Inż. Kszt. Środ. (2021), 30 (3)
http://iks.pn.sggw.pl
DOI 10.22630/PNIKS.2021.30.3.36

Yevhen DMYTRENKO[1], Ihor YAKOVENKO[1], Oleg FESENKO[2]

[1] National University of Life and Environmental Sciences of Ukraine, Faculty of Construction and Design
[2] The State Research Institute of Building Constructions

Strength of eccentrically tensioned reinforced concrete structures with small eccentricities by normal sections

Key words: reinforced concrete structures, deformation model, two-line stress-strain diagram, non-eccentric tension with small eccentricities, normal cross sections

Introduction

Experimental-theoretical researches and improvement of the calculation theory of reinforced concrete structures by the action of diverse influences, such as strength, rigidity and crack resistance is a very important scientific problem in civil engineering.

New concepts for the calculation of reinforced concrete structures are being developed (Pavlikov, Kochkarev & Harkara, 2019), existing methods are being improved (Kolchunov & Yakovenko, 2016; Karpyuk, Kostyuk & Semina, 2018; Karpiuk, Somina & Antonova, 2019), including with the involvement of the calculation apparatus of fracture mechanics (Iakovenko, Kolchunov & Lymar, 2017). In accordance with the current building codes of Ukraine in the field of design of reinforced concrete structures, i.e. the DBN V.2.6-98:2009 standard (Ukrayinskyy naukovo-doslidnyy i navchalnyy tsentr problem standartyzatsiyi, sertyfikatsiyi ta yakosti [SE UkrNDNC], 2011a) and the DSTU B.V.2.6-156:2010 standard (SE UkrNDNC, 2011b), normal cross sections of reinforced concrete elements are calculated according to the boundary conditions of the first and second groups using the deformation method.

The main feature of this method is the solution of a system of non-linear equations. Therefore, the use of numerical calculation methods, modern software packages for their implementation is an integral part of this process. A characteristic feature of the above normative documents is that they practically do not consider cases of stress-strain state (SSS) of normal sections with off-center tension. In particular, this applies to the case of small eccentricities, there are no practical recommendations for the use of the deformation method. In practical manuals for building codes (Babayev, Bambura & Pustovoitova, 2015; Bambura, Pavlikov, Kolchunov, Kochkarev & Yakovenko, 2017; Wojciechowski & Zhuravsky, 2017) this issue is also insufficiently covered. The appearance of the first cracks (Bambura et al., 2017; Iakovenko & Kolchunov, 2017) in reinforced concrete structures significantly affects their rigidity. This further significantly reduces the load-bearing capacity of both normal and inclined sections. A special influence on the appearance and spread of cracks has a complex resistance – torsion with bending (Dem'Yanov, Yakovenko, Kolchunov, 2017; Dem'Yanov, Kolchunov, Iakovenko & Kozarez, 2019).

Topicality

Eccentrically stretched elements are quite common among reinforced concrete structures. These types include a fairly wide class of reinforced concrete structures – monolithic walls of tanks and hoppers rectangular in plan, the lower belts of non-skeletal trusses, pipeline structures, walls of cylindrical silos, granaries, arched structures, and so on.

When determining the SSS of normal cross sections of such elements due to the lower height of the compressed zone of concrete, its compressive strength is not fully used, as is the case with off-center compression or bending.

Execution of modern calculations of reinforced concrete structures in computer-aided design systems (Kolchunov, Yakovenko & Dmitrenko, 2016; Iakovenko et al., 2017) is based on the construction of finite-element models of rod (which simulate the work of beams, columns) and plate shell finite elements (which simulate the work of plates, diaphragms, walls).

Therefore, the search for a rational design on the basis of real research and the results of the SSS calculation of normal cross-sections of eccentrically stretched reinforced concrete structures by the deformation method is an actual task. The obtained results (Bambura et al., 2017; Kolchunov, Dem'yanov, Iakovenko & Garba, 2018) are of practical importance in the implementation of the proposed algorithms for calculations of reinforced concrete structures in modern software packages, based on the deformation model of reinforced concrete (Bambura et al., 2017) and conducting a more accurate, reliable analysis of such structures (Kolchunov et al., 2018).

Material and methods

Study area. Output data. To conduct a numerical study, the authors chose a rectangular reinforced concrete normal section of the slab fragment with double reinforcement with rod reinforcement (Fig. 1). The class of heavy concrete – C16/20, the class of longitudinal working reinforcement – A400C.

The following parameters were varied: the height of the section h and the coefficient of the accepted reinforcement ρ.

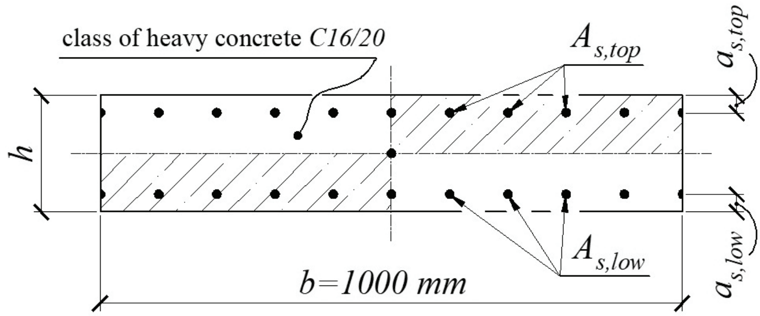


FIGURE 1. Geometric dimensions and normal section reinforcement scheme of reinforced concrete slab

This type of section is chosen due to the wide application in the modeling of both rod and plate (shell) types of finite elements. These types of finite elements make up the calculation models of buildings and structures when performing their calculation in modern software packages (for example, the PC family "Lira-CAD"). The initial data of the task are presented in Table 1.

Reinforcement of the section is taken symmetrical to prevent the impact on the external eccentricity (from external forces relative to the geometric center of gravity of the section) additional eccentricity (from the displacement of the center of gravity of the section due to asymmetric reinforcement). It should be noted that with small external eccentricities, this effect is quite significant and can in some cases lead to a change in the SSS cross section.

The calculated diagrams for concrete (Fig. 2a) and reinforcement (Fig. 2b) are accepted as bilinear with the corresponding parameters specified in the current building codes, i.e. the DBN V.2.6-98:2009 standard.

TABLE 1. Physico-mechanical characteristics and initial data for calculation of reinforced concrete cross section

Characteristics of reinforced concrete section	Value		
– sectional height – h [cm]	20	16	12
– sectional width – b [cm]	100		
– area of the longitudinal reinforcement at the top of the section – $A_{s,top}$ [cm^2]	11.31	6.5	2.25
– the area of the longitudinal reinforcement at the bottom of the section – $A_{s,low}$ [cm^2]	11.31	6.5	2.25
– the distance from the upper face of the slab to the reinforcement axis at the top of the section – $a_{s,top}$ [cm]	3		
– the distance from the lower face of the slab to the reinforcement axis at the bottom of the section – $a_{s,low}$ [cm]	3		
– section reinforcement coefficient – ρ [%]	1.33	1.0	0.5
– the coefficient of reinforcement to concrete reduction – α_s	7.407		

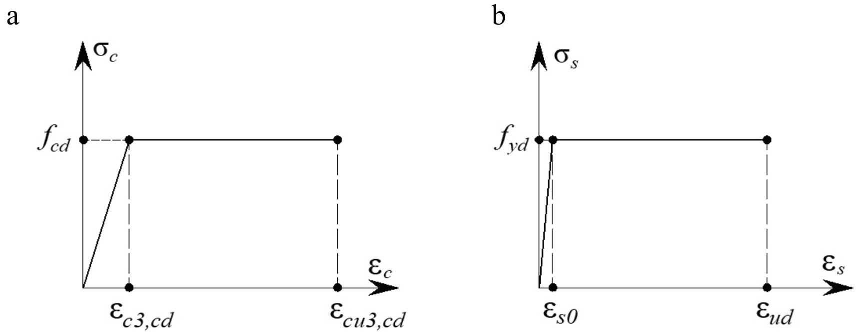


FIGURE 2. Bilinear diagrams of the stress-strain state of materials “σ – ε” are accepted at modeling: a – for concrete; b – for reinforcement

Research methodology

Stress-strain state of reinforced concrete rectangular section is flat non--eccentric tension.

This type of SSS is quite common in determining the required area of reinforcement in shell reinforced concrete elements. In the implementation of such calculations by the method of Wood–Armer (Shin, Bommer, Deaton & Alemdar, 2009) there is a problem of significant duration of their implementation due to the large number of calculated combinations of efforts in comparison with the method of Prof. N.I. Karpenko (Karpenko, 1996). That is why the developers of computerized algorithms for calculating reinforced concrete structures by the Wood–Armer method is also an important task to reduce the time of calculations and their high accuracy.

The system of non-linear equilibrium equations (in general case), which describes the operation of a normal reinforced concrete section under load (the second form of equilibrium), with a triangular diagram of the compressed zone of concrete (Fig. 3a), has the following form:

$$\begin{cases} \dfrac{b \cdot E_{cd} \cdot \varepsilon_{c(1)}^{2}}{2 \cdot \chi} + \sum_{i=1}^{n} A_{si} \cdot \sigma_{si} - N = 0 \\ \dfrac{b \cdot E_{cd} \cdot \varepsilon_{c(1)}^{3}}{3 \cdot \chi^{2}} + \sum_{i=1}^{n} A_{si} \cdot \sigma_{si} \cdot \\ \cdot \dfrac{\varepsilon_{c(1)} - \chi \cdot z_{si}}{\chi} - M = 0 \end{cases} \tag{1}$$

where:
E_{cd} – calculated modulus of elasticity of concrete in compression [MPa],
N – longitudinal force acting on the section under consideration,
M – external moment and longitudinal force acting on the section under consideration.

The external moment is dependent by next formula:

$$M = N \cdot (y + e_0 - x_1) \tag{2}$$

where:
y – distance from the extreme stretched edge of concrete to the center of section gravity [cm],

e_0 – eccentricity of external force application of relative to the center of section gravity [cm],

b – section width [cm],

χ – curvature of the bend axis in cross section is determined by the formula:

$$\chi = \frac{1}{r} = \frac{(\varepsilon_{c(1)} - \varepsilon_{c(2)})}{h} \tag{3}$$

In Eq. (3) the following notation is accepted:

$\varepsilon_{c(1)}$ – deformation of compressed fiber concrete,

$\varepsilon_{c(2)}$ – average deformations of stretched concrete fiber,

z_{si} – the distance of the *i*-th rod or layer of reinforcement from the most compressed edge of the section [cm],

z_{si} – the area of the *i*-th rod or layer of reinforcement [cm^2],

A_{si} – stresses in the *i*-th layer of rods or reinforcement [MPa], are determined by the deformation diagram of the reinforcement (Fig. 2b) depending on the corresponding strains ε_{si}, which are determined by the following dependence:

$$\varepsilon_{si} = \chi \cdot (x_1 - z_{si}) \tag{4}$$

where the height of the compressed zone of concrete – x_1 [cm] is defined as follows:

$$x_1 = \frac{\varepsilon_{c(1)}}{\chi} \tag{5}$$

In the case, when compressed zone of concrete has a trapezoidal diagram, the system of non-linear equations of the section equilibrium (Fig. 3b) has the following form:

$$\begin{cases} \dfrac{b \cdot f_{cd}}{2 \cdot \chi} \cdot (2 \cdot \varepsilon_{c(1)} - \varepsilon_{c3,cd}) + \\ + \sum\limits_{i=1}^{n} A_{si} \cdot \sigma_{si} - N = 0 \\ \dfrac{b \cdot f_{cd}}{3 \cdot \chi^2} \cdot (3 \cdot \varepsilon_{c(1)} \cdot \varepsilon_{c3,cd} - 2 \cdot \varepsilon_{c3,cd}^2) + \\ + \sum\limits_{i=1}^{n} A_{si} \cdot \sigma_{si} \cdot \dfrac{\varepsilon_{c(1)} - \chi \cdot z_{si}}{\chi} - M = 0 \end{cases} \tag{6}$$

where:

f_{cd} – the calculated compressive strength of concrete [MPa],

$\varepsilon_{c3,cd}$ – ultimate elastic deformations of concrete compression (Fig. 2a).

The tensile performance of concrete was not taken into account in accordance with Chapter 4.1.1 the DSTU B V.2.6--156:2010 standard.

The solution of the equations system (6) was performed according to the method presented in Appendix A of the DSTU B V.2.6-156:2010 standard for one section repeatedly by finding a balance between external forces M and N and forces arising in concrete and reinforcement. The implementation of the above algorithm was performed by selecting to the fixed value of the strains of the more compressed face of concrete $\varepsilon_{c(1)}$ the corresponding strains of the

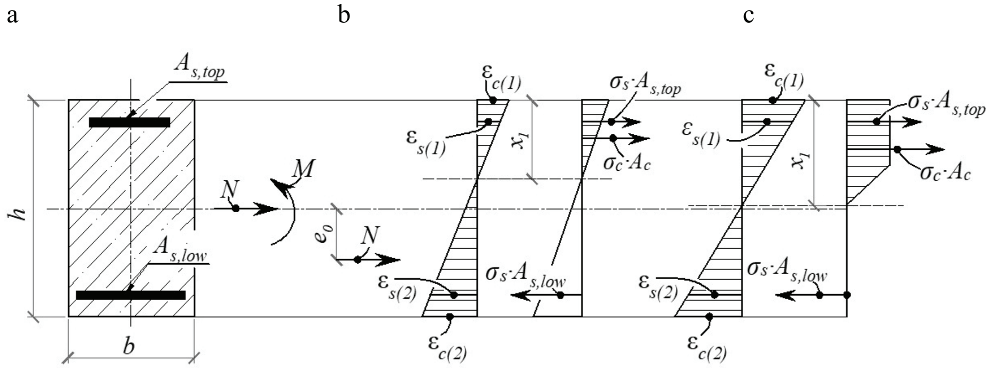


FIGURE 3. The scheme of efforts and the calculated schemes of stresses and strains at eccentric stretching of a reinforced concrete rod: a – cross section of the rod; b – with a triangular diagram of the compressed zone; c – with a trapezoidal diagram of the compressed zone

stretched face $\varepsilon_{c(2)}$, at which equilibrium occurs.

Selection is performed using a combination of numerical methods – methods of successive approximations, half division, secant. The selection step is accepted $0.01 \cdot \varepsilon_{c(1)}$ (in contrast to the step recommended $0.1 \cdot \varepsilon_{c(1)}$ in Appendix A of the DSTU B V.2.6-156:2010 standard). To determine the strength of the cross section, a state diagram "$N - \varepsilon_{c(1)}$" was constructed at characteristic points $\varepsilon_{c(1)}$, the maximum of which was equal to the maximum value of the longitudinal force N, perceived by the cross section.

The maximum value of the bending moment, which perceives the cross section at a given combination of external forces is determined by the formula:

$$M = N \cdot e_0 \tag{7}$$

At the above value of the strain step $\varepsilon_{c(1)}$, the diagram "$N - \varepsilon_{c(1)}$" must have 100 points. At each step $\varepsilon_{c(1)}$ in the selection of strains $\varepsilon_{c(2)}$, all other parameters of the SSS cross section (χ, x_1, σ_{si}) were calculated and then substituted into the first equation of the system – Eq. (1) or (6), while N assumed to be zero.

Next, the obtained value of the internal force N, which perceives the cross section, was multiplied by the eccentricity of external forces e_0. The resulting bending moment M was substituted into the second equation of the system – Eq. (1) or (6) and when the above SSS parameters of the cross section were calculated, the equilibrium was checked. Thus, in the chosen approach, the criterion for finding the equilibrium of the equations system (1) and (6) is the proximity to zero of the result of the second equation (equation of moments), namely the condition:

$$\sum M \le \Delta M \tag{8}$$

where ΔM is an error in solving a system of equations [kN·m].

Results and discussion

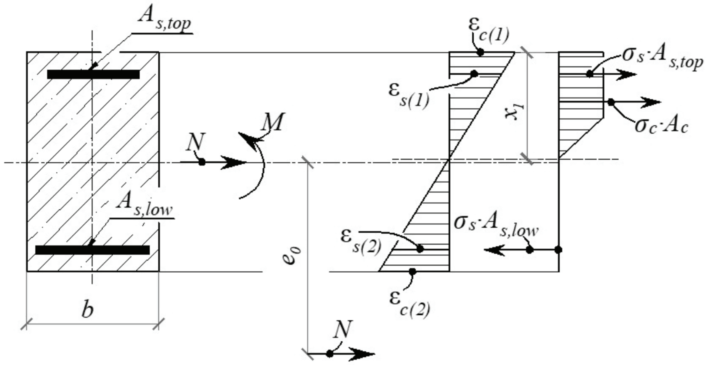
The authors consider two possible forms of equilibrium of reinforced concrete section under the action flat non-eccentric tension:

– non-eccentric tension with large eccentricities, the strains diagram is ambiguous, the work of the section is similar to that which bends (Fig. 4a);
– non-eccentric tension with small eccentricities – the line of action of the external tensile force is between the rods of the longitudinal reinforcement of the section, the section is almost completely stretched with a relatively small height of the compressed zone of concrete (Fig. 4b).

According to the first form of equilibrium of the experimental cross section (Fig. 4a) – under the action of external forces N = –2,000 kN, M = = 600 kN·m, e_0 = 30 cm, e_0/h = 1.5 (the line of action of the external force is outside the cross section) the equilibrium is at the characteristic points $\varepsilon_{c(1)}$ of the state diagram "$N - \varepsilon_{c(1)}$" (Fig. 5). The bearing capacity of the reinforced concrete section is – N_{max} = 169.5 kN, M_{max} = 50.03 kN·m.

If we consider the action of another combination of external forces N = = –2,000 kN, M = 120 kN·m, e_0 = 6.0 cm, e_0/h = 0.3 (under the action of reduced by 5 times M), there is a second form of equilibrium (Fig. 4b) at cross section.

a



b

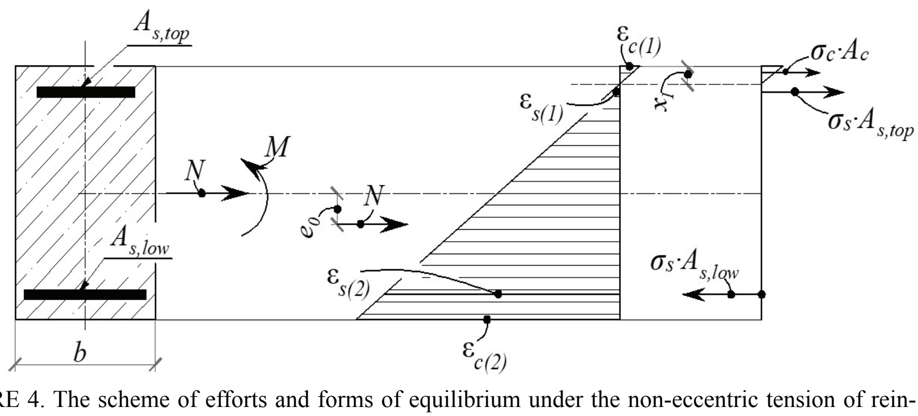


FIGURE 4. The scheme of efforts and forms of equilibrium under the non-eccentric tension of reinforced concrete section: a – with large eccentricities; b – with small eccentricities

To more clearly display the results of calculations using the algorithm under consideration, the graphs of dependence on the values $\sum M$ of strains of the more tension section face $\varepsilon_{c(2)}/\varepsilon_{c,\max}$, which correspond to the values of the more compressed section face $\varepsilon_{c(1)}$, are constructed. The values $\varepsilon_{c,\max}$ correspond to the largest values of tensile strains $\varepsilon_{c(2)}$, at which there is a rupture of the reinforcement in the tensioned zone of concrete.

According to the data obtained with this combination of efforts, the equilibrium is also established (graph $\sum M$ of $\varepsilon_{c(2)}/\varepsilon_{c,\max}$ the intersection of the abscissa) at all 100 characteristic points of the state diagram "$N - \varepsilon_{c(1)}$" (Fig. 7), even at the highest values $\varepsilon_{c(1)} = \varepsilon_{cu,3d}$ (at values $\varepsilon_{c(2)}$ close to the maximum $\varepsilon_{c,\max}$ – Fig. 6, Curve 1).

The bearing capacity of the section is $N_{\max} = 484.3$ kN, $M_{\max} = 29.06$ kN·m.

With a further decrease in the value of the external moment, with a combination of external forces $N = -2{,}000$ kN, $M = 30$ kN·m, $e_0 = 1.5$ cm, $e_0/h = 0.075$ cm, the equilibrium is no longer at all points of the diagram "$N - \varepsilon_{c(1)}$".

In this case, when $\varepsilon_{c(1)} = \varepsilon_{cu,3d}$ and $\varepsilon_{c(1)} = 0.8 \cdot \varepsilon_{cu,3d}$ the graph $\sum M$ of the dependence $\varepsilon_{c(2)}/\varepsilon_{c,\max}$ does not intersect the abscissa axis (Fig. 8, Curves 1, 2). The equilibrium is starting from $\varepsilon_{c(1)} = 0.56 \cdot \varepsilon_{cu}$.

In Figure 9 section of the descending branch of the diagram at values $\varepsilon_{c(1)} > 1.7 \cdot 10^{-3}$ corresponds to the values of strains $\varepsilon_{c(1)}$, in which the search for equilibrium between external and internal forces occurs rupture of the reinforcement in the tensioned zone, i.e. the equations system (1) or (6) has no solution.

Comparing the state diagrams of the cross section "$N - \varepsilon_{c(1)}$" at different eccentricities e_0 of external load application (Figs. 5, 7, 9), it should be noted that when the eccentricity for the experimental reinforced concrete section decreases, the length of the inclined section

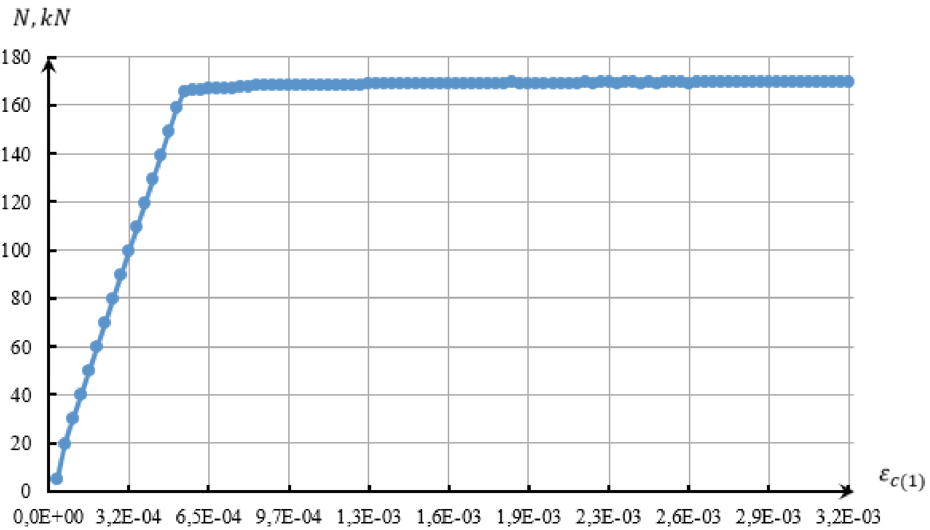


FIGURE 5. State diagram "$N - \varepsilon_{c(1)}$" of the experimental section at $e_0 = 30$ cm

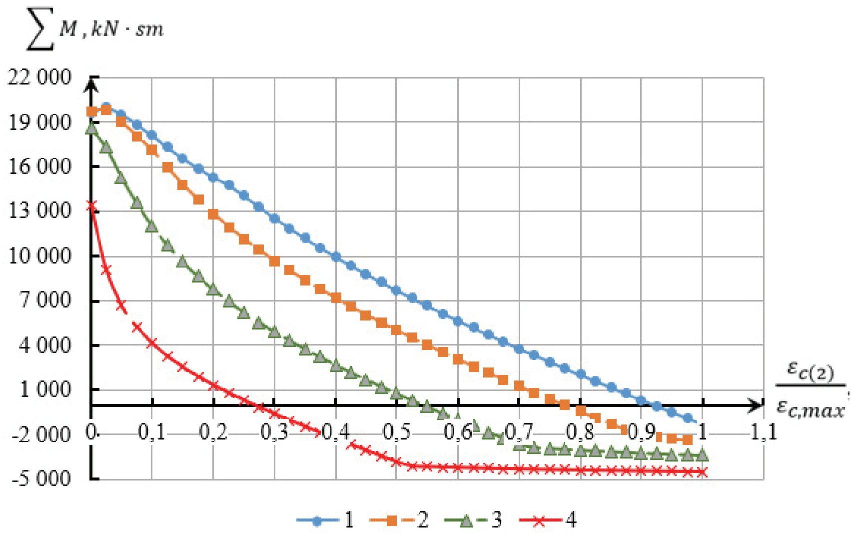


FIGURE 6. Graphs $\sum M$ of dependence on $\varepsilon_{c(2)}/\varepsilon_{c,\max}$ values, when e_0 = 6.0 cm: 1 – at $\varepsilon_{c(1)} = \varepsilon_{cu,3d}$; 2 – at $\varepsilon_{c(1)} = 0.8 \cdot \varepsilon_{cu,3d}$; 3 – at $\varepsilon_{c(1)} = 0.5 \cdot \varepsilon_{cu,3d}$; 4 – at $\varepsilon_{c(1)} = 0.2 \cdot \varepsilon_{cu,3d}$

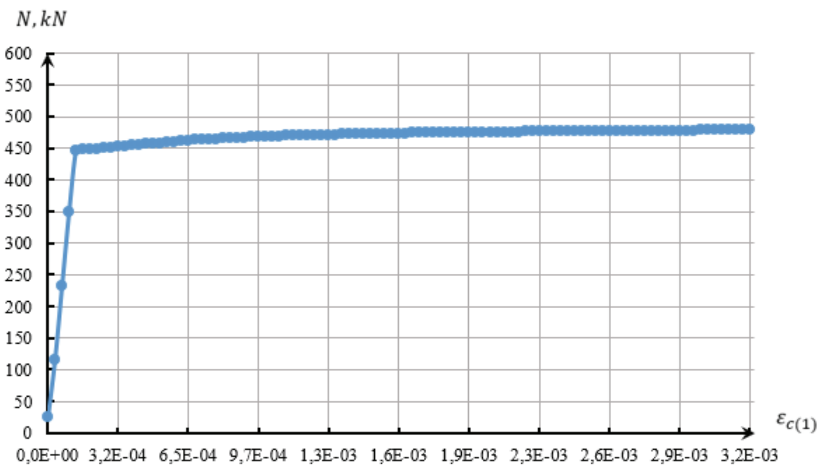


FIGURE 7. State diagram "$N - \varepsilon_{c(1)}$" of the experimental reinforced concrete section at e_0 = 6.0 cm

decreases until its complete disappearance. So at eccentricity e_0 = 30 cm its length makes from $\varepsilon_{c(1)}$ = 0 to $\varepsilon_{c(1)}$ = $=5.9 \cdot 10^{-4}$; when e_0=6.0 cm – from $\varepsilon_{c(1)}$=0 to $\varepsilon_{c(1)} = 1.029 \cdot 10^{-4}$. At eccentricity e_0 = = 1.5 cm, it is absent in general.

Also, when the external eccentricity decreases e_0 (reduction of the bending moment M at a constant longitudinal force N), the value $\varepsilon_{c(1)}$ at which the chosen system of equations has a solution decreases, i.e. the maximum relative deformations of compressed concrete $\varepsilon_{c(1)}$ decrease by force. This is fully consistent with the physical picture of the experimental process – with increasing

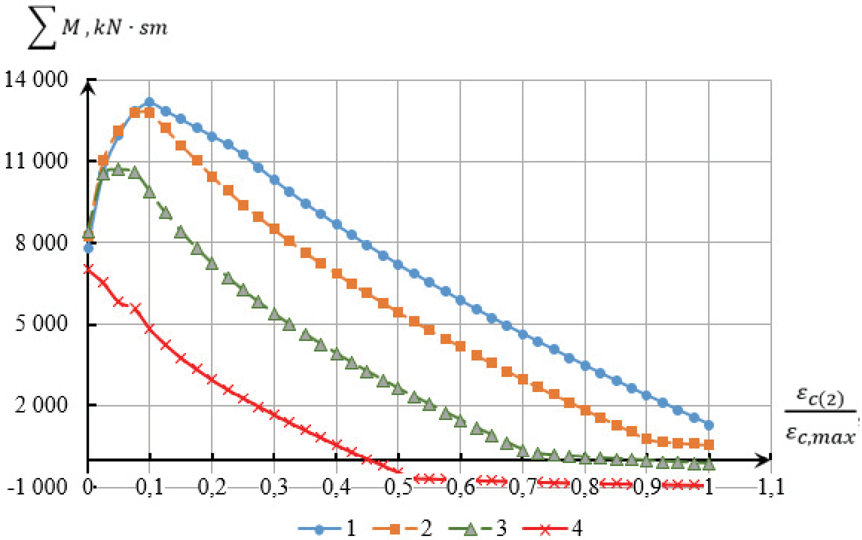


FIGURE 8. Graphs $\sum M$ of dependence on $\varepsilon_{c(2)}/\varepsilon_{c,\max}$ values, when e_0 = 1.5 cm: 1 – at $\varepsilon_{c(1)} = \varepsilon_{cu,3d}$; 2 – at $\varepsilon_{c(1)} = 0.8 \cdot \varepsilon_{cu,3d}$; 3 – at $\varepsilon_{c(1)} = 0.5 \cdot \varepsilon_{cu,3d}$; 4 – at $\varepsilon_{c(1)} = 0.2 \cdot \varepsilon_{cu,3d}$

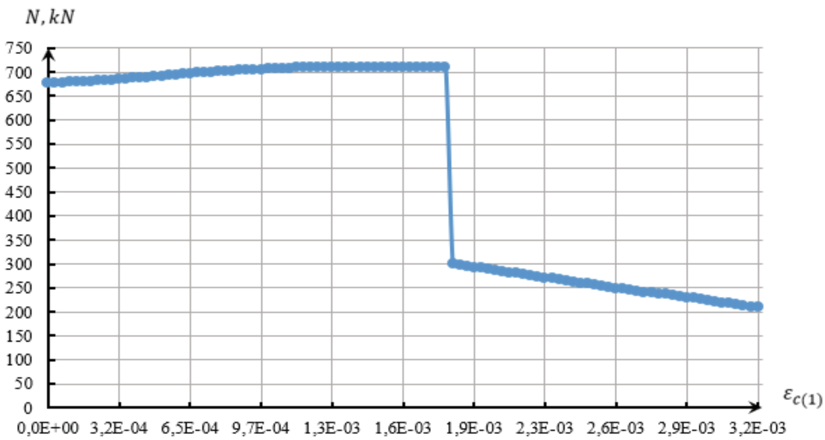


FIGURE 9. State diagram "$N - \varepsilon_{c(1)}$" of the experimental reinforced concrete section at e_0 = 1.5 cm

external tensile force N, the tensioned cross-sectional area increases, the impact of compressed concrete on the overall cross-sectional strength decreases.

If we continue to gradually reduce the eccentricity e_0 of the external forces application, the number of characteristic points $\varepsilon_{c(1)}$ at which the equilibrium will be in the cross section will also decrease. The results of experimental cross sections calculations are presented in Table 2. But even at sufficiently small values e_0, close to the transition limit of SSS cross section from non-eccentric tension with small eccentricities to central tensile using the method presented in Appendix A

TABLE 2. The results of numerical calculations of experimental reinforced concrete sections

Geometric cross-sectional dimensions $b \times h$ [cm]	External efforts			Curvature χ [cm^{-1}]	Strains $\varepsilon_{c(1)}$ (equilibrium)	Compressed cross-sectional area height x [cm]	Upper reinforcement $\sigma_{s,top}$ [MPa]	Lower reinforcement $\sigma_{s,low}$ [MPa]	Bearing capacity of section		ρ [%]
	M [kN·m]	N [kN]	e_0 [cm]						N_{int} [kN]	M_{int} [kN·m]	
20 × 100	600	–2 000	30	1.15E-03	$1.0 \cdot \varepsilon_{cu}$	2.81	–45.8	–364.0	169.5	50.85	1.3
	300	–2 000	15	1.27E-03	$1.0 \cdot \varepsilon_{cu}$	2.54	–122.5	–364.0	284.2	42.63	1.3
	150	–2 000	7.5	1.43E-03	$1.0 \cdot \varepsilon_{cu}$	2.25	–224.4	–364.0	429.5	32.21	1,3
	75	–2 000	3.75	1.60E-03	$1.0 \cdot \varepsilon_{cu}$	2.01	–332.5	–364.0	577.0	21.64	1.3
	50	–2 000	2.5	1.40E-03	$0.86 \cdot \varepsilon_{cu}$	1.73	–364.0	–364.0	647.9	16.20	1.3
	40	–2 000	2	1.59E-03	$0.7 \cdot \varepsilon_{cu}$	1.44	–364.0	–364.0	678.5	13.57	1.3
	30	–2 000	1.5	1.56E-03	$0.56 \cdot \varepsilon_{cu}$	1.16	–364.0	–364.0	711.2	10.67	1.3
	20	–2 000	1	1.54E-03	$0.41 \cdot \varepsilon_{cu}$	0.86	–364.0	–364.0	746.1	7.46	1.3
	10	–2 000	0.5	1.49E-03	$0.25 \cdot \varepsilon_{cu}$	0.54	–364.0	–364.0	783.4	3.92	1.3
	5	–2 000	0.25	1.47E-03	$0.17 \cdot \varepsilon_{cu}$	0.37	–364.0	–364.0	803.0	2.01	1.3
	1	–2 000	0.05	1.23E-03	$0.07 \cdot \varepsilon_{cu}$	0.18	–364.0	–364.0	819.2	0.41	1.3
	0.5	–2 000	0.025	1.25E-03	$0.05 \cdot \varepsilon_{cu}$	0.13	–364.0	–364.0	821.3	0.21	1.3

TABLE 2, cont.

Geometric cross-sectional dimensions $b \times h$ [cm]	External efforts			Curvature χ [cm^{-1}]	Strains $\varepsilon_{c(1)}$ (equilibrium)	Compressed cross-sectional area height x [cm]	Upper reinforcement $\sigma_{s,top}$ [MPa]	Lower reinforcement $\sigma_{s,low}$ [MPa]	Bearing capacity of section		ρ [%]
	M [kN·m]	N [kN]	e_0 [cm]						N_{int} [kN]	M_{int} [kN·m]	
16 × 100	600	–2 000	30	1.32E-03	$1.0 \cdot \varepsilon_{cu}$	2.44	–154.8	–364.0	81.5	24.46	1.0
	300	–2 000	15	1.42E-03	$1.0 \cdot \varepsilon_{cu}$	2.27	–219.0	–364.0	141.6	21.24	1.0
	150	–2 000	7.5	1.57E-03	$1.0 \cdot \varepsilon_{cu}$	2.05	–312.5	–364.0	224.7	16.85	1.0
	75	–2 000	3.75	2.11E-03	$1.0 \cdot \varepsilon_{cu}$	1.53	–364.0	–364.0	312.6	11.72	1.0
	50	–2 000	2.5	2.10E-03	$0.76 \cdot \varepsilon_{cu}$	1.17	–364.0	–364.0	354.7	8.87	1.0
	30	–2 000	1.5	2.01E-03	$0.51 \cdot \varepsilon_{cu}$	0.82	–364.0	–364.0	395.7	5.94	1.0
	20	–2 000	1	2.00E-03	$0.38 \cdot \varepsilon_{cu}$	0.62	–364.0	–364.0	419.2	4.19	1.0
	10	–2 000	0.5	1.97E-03	$0.24 \cdot \varepsilon_{cu}$	0.393	–364.0	–364.0	444.9	2.22	1.0
	5	–2 000	0.25	1.83E-03	$0.16 \cdot \varepsilon_{cu}$	0.283	–364.0	–364.0	458.7	1.15	1.0
	1	–2 000	0.05	1.72E-03	$0.07 \cdot \varepsilon_{cu}$	0.13	–364.0	–364.0	470.2	0.24	1.0
	0.5	–2 000	0.025	1.75E-03	$0.05 \cdot \varepsilon_{cu}$	0.092	–364.0	–364.0	471.72	0.12	1.0
12 × 100	600	–2 000	30	2.44E-03	$1.0 \cdot \varepsilon_{cu}$	1.33	–364.0	–364.0	25.0	7.49	0.5
	300	–2 000	15	2.82E-03	$1.0 \cdot \varepsilon_{cu}$	1.15	–364.0	–364.0	43.8	6.57	0.5
	150	–2 000	7.5	3.05E-03	$0.86 \cdot \varepsilon_{cu}$	0.91	–364.0	–364.0	70.0	5.25	0.5
	75	–2 000	3.75	2.98E-03	$0.61 \cdot \varepsilon_{cu}$	0.66	–364.0	–364.0	98.9	3.71	0.5
	50	–2 000	2.5	2.93E-03	$0.48 \cdot \varepsilon_{cu}$	0.53	–364.0	–364.0	114.3	2.86	0.5
	30	–2 000	1.5	2.82E-03	$0.35 \cdot \varepsilon_{cu}$	0.40	–364.0	–364.0	129.5	1.94	0.5
	20	–2 000	1	2.82E-03	$0.27 \cdot \varepsilon_{cu}$	0.31	–364.0	–364.0	140.0	1.40	0.5
	10	–2 000	0.5	2.63E-03	$0.18 \cdot \varepsilon_{cu}$	0.221	–364.0	–364.0	151.1	0.76	0.5
	5	–2 000	0.25	2.65E-03	$0.13 \cdot \varepsilon_{cu}$	0.159	–364.0	–364.0	157.2	0.39	0.5
	1	–2 000	0.05	2.74E-03	$0.06 \cdot \varepsilon_{cu}$	0.07	–364.0	–364.0	162.4	0.08	0.5
	0.5	–2 000	0.025	2.43E-03	$0.04 \cdot \varepsilon_{cu}$	0.053	–364.0	–364.0	163.118	0.04	0.5

of the DSTU B V.2.6-156:2010 standard, you can find a balance between external and internal forces and estimate the load-bearing capacity of the section, which is completely determined by the area and tensile strength of the reinforcement. But to successfully find a solution to the system of Eq. (1) or (6) you need to reduce the selection step from the recommended $\varepsilon_{c(1)} = 0.1 \cdot \varepsilon_{cu}$ to the smallest, for example, to $\varepsilon_{c(1)} = 0.01 \cdot \varepsilon_{cu}$.

An alternative option in this case may be to calculate the load-bearing capacity of the reinforced concrete section according to the formulas of the algorithm presented in the previous building codes, i.e. the SNiP 2.03.01-84* standard (Gosudarstvennyy stroitelnyy komitet SSSR [Gosstroy SSSR], 1989), which is based on the method of limiting forces. An additional argument in favor of the calculation of this option is its higher speed compared to the iterative algorithm of the deformation method, which may be useful when performing calculations of reinforcement of rectangular sections by the Wood–Armer method (Shin et al., 2009).

Conclusions

During performing calculations on the strength of normal sections by the deformation method of non-eccentrically tensioned elements of reinforced concrete structures with small eccentricities, an equilibrium was found between internal and external forces only with a two-digit diagram of the distribution of relative longitudinal strains (in the presence of a compressed zone availability).

It is established that with a decrease in the eccentricity of the application of force, the compressed zone of concrete in cross section decreases until its complete disappearance. The cross section of the reinforced concrete structure becomes completely tensioned. An equilibrium between external and internal forces cannot be found with the help of the deformation method realization, proposed in Appendix A of the DSTU B V.2.6-156:2010 standard.

Options for solving this task without significant loss of calculation accuracy are proposed, the most appropriate of which is the transition to the method of limiting forces (which was adopted as the main in previous building codes, i.e. the SNiP 2.03.01-84* standard) and further calculation by this method.

References

Babayev, V.M., Bambura, A.M., Pustovoitova, O.M. (2015). *Praktichnij rozrahunok elementiv zalizobetonnih konstrukcij za DBN V.2.6-98:2009 u porivnjanni z rozrahunkami za SNiP 2.03.01-84* i EN 1992-1-1 (Eurocode 2)* [*Practical calculation of elements of reinforced concrete structures according to DBN B.2.6-98: 2009 in comparison with calculations according to SNiP 2.03.01-84* and EN 1992-1-1 (Eurocode 2)*]. Kharkiv: Publishing Golden Pages.

Bambura, A.M., Pavlikov, A.M., Kolchunov, V.I., Kochkarev, D.V. & Yakovenko, I.A. (2017). *Praktichnij posibnik iz rozrahunku zalizobetonnih konstrukcij za dijuchimi normami Ukraïni (DBN V.2.6-98:2009) ta novimi modeljami deformuvannja, shho rozrobleni na ïhnju zaminu* [*Practical guide to the calculation of reinforced concrete structures – according to the current standards of Ukraine (DBN B.2.6-98:2009) and new models of deformation, designed to replace them*]. Kyiv: Publishing Toloka.

Dem'Yanov, A., Kolchunov, V., Iakovenko, I. & Kozarez, A. (2019). Load bearing capacity calculation of the system "reinforced concrete beam – deformable base" under torsion with bending. *E3S Web of Conferences*, *97*, 1-8. https://doi.org/10.1051/e3sconf/20199704059

Dem'Yanov, A.I., Yakovenko, I.A. & Kolchunov, V.I. (2017). The development of universal short dual-console element for resistance of reinforced concrete structures under the action torsion with bending. *Izvestiya Vysshikh Uchebnykh Zavedenii, Seriya Teknologiya Tekstil'noi Promyshlennosti*, *370*(4), 246-251.

Gosudarstvennyy stroitelnyy komitet SSSR [Gosstroy SSSR] (1989). *Betonnye i zhelezobetonnye konstrukcii* (SNiP 2.03.01-84*) [*Concrete and reinforced concrete structures. Building codes and regulations* (SNiP 2.03.01-84*)]. Moskva: CITP Gosstroya SSSR.

Iakovenko, I.A. & Kolchunov, V.I. (2017). The development of fracture mechanics hypotheses applicable to the calculation of reinforced concrete structures for the second group of limit states. *Journal of Applied Engineering Science*, *15*(3), 371-380. https://doi.org/10.5937/jaes15-14662

Iakovenko, I., Kolchunov, V. & Lymar, I. (2017). Rigidity of reinforced concrete structures in the presence of different cracks. *MATEC Web of Conferences*, *116*, 02016. EDP Sciences. https://doi.org/10.1051/matecconf/201711602016

Karpenko, N.I. (1996). *Obshhie modeli mehaniki zhelezobetona* [*General models of reinforced concrete mechanics*]. Moskva: Publishing Stroiizdat.

Karpiuk, V., Somina, Yu. & Antonova, D. (2019). Calculation models of the bearing capacity of span reinforced concrete structures support zones materials. *Science Forum: Actual Problems of Engineering Mechanics*, *968*, 209-226. https://doi.org/10.4028/www.scientific.net/MSF.968.209

Karpyuk, V.M., Kostyuk, A.I. & Semina, Yu.A. (2018). General case of nonlinear deformation-strength model of reinforced concrete structures. *Strength of Materials*, *50*(*3*), 453-464. https://doi.org/10.1007/s11223-018-9990-9

Kolchunov, V.I., Dem'yanov, A., Iakovenko, I. & Garba, M. (2018). Bringing the experimental data of reinforced concrete structures crack resistance in correspondence with their theoretical values. *Science & Construction*, *1*(15), 42-49.

Kolchunov, V.I. & Yakovenko, I.A. (2016). About the violation solid effect of reinforced concrete in reconstruction design of textile industry enterprises. *Izvestiya Vysshikh Uchebnykh Zavedenii. Teknologiya Tekstil'noi Promyshlennosti*, *2016*(3), 258-263.

Kolchunov, V.I., Yakovenko, I.A. & Dmitrenko, E.A. (2016). The analytical core model formation of the nonlinear problem bond armature with concrete. *Academic Journal. Industrial Machine Building, Civil Engineering*, *2*(47), 125-132.

Pavlikov, A., Kochkarev, D. & Harkava, O. (2019). Calculation of reinforced concrete members strength by new concept. In *Proceedings of the fib Symposium 2019: Concrete – Innovations in Materials, Design and Structures* (pp. 820-827). Lausanne: International Federation for Structural Concrete.

Shin, M., Bommer, A., Deaton, J. & Alemdar, B. (2009). Twisting moments in two-way slab. *Concrete International*, *78*, 35-40.

Ukrayinskyy naukovo-doslidnyy i navchalnyy tsentr problem standartyzatsiyi, sertyfikatsiyi ta yakosti [SE UkrNDNC] (2011a). *Betonni ta zalizobetonni konstrukcii̇. Osnovni polozhennja* (DBN V.2.6-98:2009) [*Concrete and reinforced concrete structures. Basic provisions* (DBN V.2.6-98:2009)]. Kyiv: Minrehionbud Ukrayiny.

Ukrayinskyy naukovo-doslidnyy i navchalnyy tsentr problem standartyzatsiyi, sertyfikatsiyi ta yakosti [SE UkrNDNC] (2011b). *Betonni ta zalizobetonni konstrukcii̇ z vazhkogo betonu. Pravila proektuvannja* (DSTU B V.2.6-156:2010) [*Concrete and reinforced concrete structures with heavy weight structural concrete. Design rules* (DSTU B V.2.6--156:2010)]. Kyiv: Minrehionbud Ukrayiny.

Wojciechowski, O.V., Zhuravsky, O.D. & Baida, D.M. (2017). *Rozrahunok zalizobetonnih konstrukcij z vikoristannjam sproshhenih diagram deformuvannja materialiv (za DSTU B V.2.6-156:2010). Chastina 1. Rozrahunok za I grupoju granichnih staniv* [*Calcu-

lation of reinforced concrete structures using simplified diagrams of deformation of materials (according to DSTU B V.2.6-156:2010). Part 1. Calculation of the 1st group of limit states]. Kyiv: Publishing KNUBA.

Summary

Strength of eccentrically tensioned reinforced concrete structures with small eccentricities by normal sections. It is implemented the method of normal rectangular sections slab (shell) reinforced concrete elements strength calculating with flat eccentric tensile strength using the deformation method. The results of the calculation are analyzed for the case of eccentric tension with small eccentricities with varying next parameters: the height of the cross section and the reinforcement coefficient. It is investigated the character of diagrams condition change of section "$N-\varepsilon_{c(1)}$" at gradual change of the stress-strain state from eccentric to the central tension. It is revealed that when the eccentricity of external forces decreases, the compressed zone of concrete decreases until its complete disappearance, and at rather small values of eccentricities of force application the balance between external and internal forces cannot be found by the method of current norms. An equilibrium is found between internal and external forces only at a two-digit diagram of the distribution of relative longitudinal deformations (in the case of a compressed zone). Variants of the given problem decision without considerable loss of calculations accuracy are offered, the most expedient of which is transition to algorithm of calculation by a method of limiting efforts. It was accepted as the basic in the previous building norms. The results of numerical calculations performed in the software complex "Lira-CAD" and the corresponding mathematical modeling confirmed the rationality and allowable accuracy of further calculations by this method.

Authors' address:
Yevhen Dmytrenko
(https://orcid.org/0000-0001-9737-943X)
National University of Life and Environmental Sciences of Ukraine
Faculty of Construction and Design
Construction Department
12B Heroiv Oborony Str., building 7, office 29, 03041 Kyiv
Ukraine
e-mail: zdmitrenko26@gmail.com

Ihor Yakovenko
(https://orcid.org/0000-0003-4256-9855)
National University of Life and Environmental Sciences of Ukraine
Faculty of Construction and Design
Construction Department
12B Heroiv Oborony Str., building 7, office 29, 03041 Kyiv
Ukraine
e-mail: i2103@ukr.net

Oleg Fesenko
(https://orcid.org/0000-0001-8154-2239)
The State Research Institute of Building Constructions
e-mail: oleg_for@ukr.net