Disk induction motor free rotor stability criterion

Sergey Yeroshin, Sergey Miroshnik

Volodymyr Dahl East-Ukrainian National University,
Molodizhny bl., 20a, Lugansk, 91034, Ukraine,
e-mail: Sergey.Yeroshin@gmail.com, SMiroshnik@mail.ru

Received September 17.2013: accepted October 11.2013

Summary. Mathematical model of disk induction motor free rotor motion has been enhanced. This model accounts for the effect of moving electromagnetic forces and the forces, counteracting the movement, such as viscous friction force and sliding friction force. Based on new equations of free rotor movement, the criterion of its steady motion, including design-engineering characteristics of the electrical machine, has been specified.
Key words. Free circular rotor, stability criterion.

INTRODUCTION

One of the upcoming trends in modern machine-building industry is generating machines with direct drive of working tool or operating device [11, 18]. Using a special structure disk induction motor (DIM), circular tool can be set in steady rotational motion and kept in space without mechanical support and electrical contact through the magnetic forces [24, 25]. Such an electrical machine will enable to enhance direct drive constructions efficiency through combining rotor functions of the motor with working tool or operating device of technological machine. The problem of operating device stability control arises when developing technological machines based on DIM with no mechanical support rotor [2, 5]. When a rolling rotor is under the action of external forces, which generate its mass-center displacement, it should resist them, and when external force is not applied the rotor should return in equilibrium position. Since perturbing factors are always present in reality, the research of stability gains the utmost theoretical and practical importance.

METHODS OF RESEARCH

Stability theory has been created by many mathematicians, mechanics and physicists. Mathematician A.M. Lyapunov [2, 12] made significant contribution in the stability theory.

We suggest to use displacement of the mass center from its initial position as a parameter to evaluate circular operating element (COE) motion stability. Let the initial position be the one at which symmetry axis of the operating element and the end stator of electric machine coincide.

We denominate real variable determining DIM rotor displacement as *e*. We assume that rotor motion (i.e. change of *e* with time *t*) is described with independent differential equation, i.e. the equation that doesn't contain independent variable [5, 14]:

$$\frac{de}{dt} = f(e), \qquad (1)$$

where: $f(e)$ – is a known function of the variable *e*.

Function $e(t;e^0)$ is the solution of this equation at $e\big|_{t=0} = e^0$. Then, according to Lyapunov's definitions, equilibrium position e_0 is called stable if there is such a $\delta_0 > 0$, that at $\left|e^0 - e_0\right| < \delta_0$ there is a solution $e(t;e^0)$ on the whole distance $0 \le t \le \infty$. Also, for any $\varepsilon > 0$ there must exist such a $\delta_0 = \delta(\varepsilon) > 0$ that if the condition $\left|e^0 - e_0\right| < \delta_0$ is fulfilled, then $\left|e(t;e^0) - e_0\right| \le \varepsilon$ [7, 13, 14].

It means that if COE mass center in the initial time point is located close enough to the equilibrium (i.e. value $\left|e^0 - e_0\right|$ is little), then describing a path in all subsequent time points it will remain close to the equilibrium position.

Equilibrium e_0 is called asymptotically stable if it is stable according to Lyapunov's definition, and if at sufficiently small $\left|e^0 - e_0\right|$ the following condition is fulfilled [7, 13]:

$$\lim_{t\to\infty} e(t;e^0) = e_0\,. \tag{2}$$

That is, if COE mass center is displaced in relation to equilibrium, then it will tend to return in the equilibrium position with the course of time.

According to Lyapunov's motion stability theorem it is essential to know when the real components of roots of characteristic equation will be negative. The solution of this problem not involving the direct calculation of characteristic equation roots, is of the greatest interest [14].

This problem was first put by D. Maxwell, and it was he, who gave solution to third-order equation, but in general this problem was solved by E. Raus. His solution is algorithmic. The analytical solution was obtained by A. Hurwitz [14].

Stability theory includes other methods and criteria allowing to evaluate mechanical systems motion stability based on qualitative analysis of motion differential equations. E.g., the methods of Vyshnegradskiy and Michailov are based on graphical representation of stability conditions [9, 16]. Unlike the mentioned above methods, Hurwitz criterion is algebraic, thus is more convenient to use and has become widely spread.

Works [23, 27] studied free circular rotor mass center motion through differential equations taking into account environment resistance forces. Their values are in direct proportion to rotor motion speed. The criterion, determining the range of variation of certain system parameters that affect stability, has been obtained. This criterion is of little informativity since it doesn't account for engineering-design characteristics of electromagnetic system.

The second disadvantage of this criterion is that it leaves out of account sliding friction forces that often take place in technical systems. The work [30] shows that when rotor moves over the work space in air, coefficient of sliding friction significantly exceeds coefficient of air resistance. Moreover, the impact of rotor spin motion on viscous and sliding friction forces is not taken into account.

The purpose of current research is to obtain functional dependance of DIM free rotor stability criterion on design-engineering factors, which electric machines are characteristic of, and also on viscous friction and sliding friction forces.

It is necessary to allow for the forces acting upon rotor, their values, direction and law of variation to research motion stability of electromechanical system.

RESULTS OF RESEARCH

The research of forces and torques acting on circular rotor in rotating field has demonstrated that rotor center displacement vector and its feedback do not coincide in direction [22].

When rotor center is displaced in relation to stator axis by the value *e*, the DIM rotor is under the action of tangential $\overline{F}_\tau$ and radial $\overline{F}_r$ components of electromagnetic forces $\overline{F}$ main vector [22, 28]. The line of action of the force $\overline{F}_r$ passes through rotor rotation center and, thus, doesn't generate torque. If the direction of $\overline{F}_r$ is opposite to the offset, the force stabilizes rotor motion, i.e. it will tend to return the rotor in equilibrium. The force $\overline{F}_\tau$ is directed perpendicularly to the displacement and is always destabilizing.

We consider the movement of free circular rotor with depth *h*, outer radius R_{RO} and inner radius R_{RI}, under the action of spinning axisymmetrical magnetic field of stator with outer and inner radiuses R_{SO} and R_{SI} correspondingly. We also assume that $R_{RI} - R_{SI} > e$ and $R_{SO} - R_{RO} > e$. Rotor depth *h* is much less than its outer radius R_{RO}.

Fig. 1 shows the position of circular rotor over the stator surface at any given time *t*. Coordinates *x* and *y* determine the current position of rotor center O_1 in fixed coordinate system connected with stator center *O*.

The moment of electromagnetic forces $M_{o\tau}$ set rotor in spinning motion with constant angle velocity ω_r in regard to its mass center O_1. Under the action of electromagnetic force $\overline{F}$ the rotor moves with velocity $\overline{v}$ in relation to stator.

According to the definition, elastic system stiffness is force-motion ratio [19], where motion is caused by that force. As in the investigated system an argument is rotor center motion causing its feedback, then the notion of stiffness is remains the same.

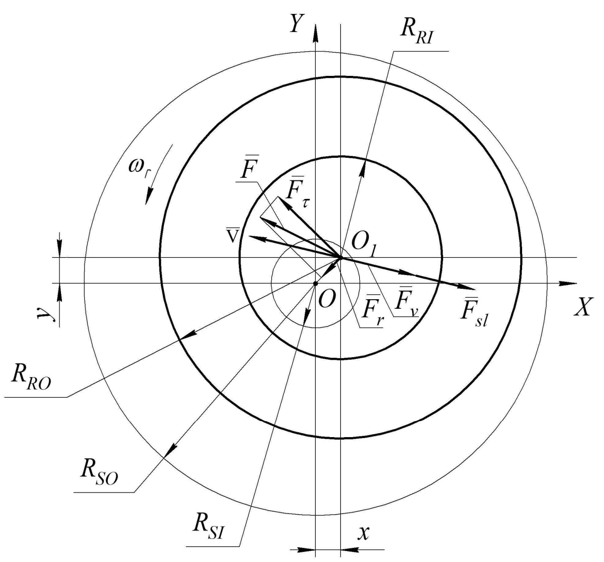

Fig. 1. Flow pattern of force on circular rotor

Radial $\overline{F}_r$ and tangential $\overline{F}_\tau$ components of the main vector of electromagnetic forces $\overline{F}$ are determined by radial D_r and tangential D_τ stiffness:

$$D_r = \frac{dF_r}{de}, \tag{3}$$

$$D_\tau = \frac{dF_\tau}{de}. \tag{4}$$

Taking into account the fact that force $\overline{F}_r$ counteracts the rotor displacement, whereas $\overline{F}_\tau$ is always perpendicular to the displacement direction, we determine the projection of vector $\overline{F}$ on coordinate axes OX and OY [22, 28] as follows:

$$F_x = -D_r x - D_\tau y, \tag{5}$$

$$F_y = D_\tau x - D_r y. \tag{6}$$

Aside from the considered moving electromagnetic forces the rotor is under the action of rotor environment viscous friction force $\overline{F}_v$. Also, when circular rotor is moving on horizontal surface it is under the action of gravitation force that causes normal feedback and, as a result, generates sliding friction force $\overline{F}_{sl}$ counteracting motion [28].

Let us set up differential equations of circular rotor movement when it is under the action of electromagnetic and motion resistance forces [3, 20]:

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} &= F_{vx} + F_{sl\,x} + F_x, \\ m\frac{d^2y}{dt^2} &= F_{vy} + F_{sl\,y} + F_y, \\ I\frac{d^2\varphi}{dt^2} &= M_v + M_{sl} + M_{o\tau}, \end{aligned}\right\} \tag{7}$$

where: *m* – is rotor mass kg, *I* – second moment of circular rotor kg m^{2}, F_{vx} and F_{vy} – projections of viscous friction forces on coordinate axes OX and OY *N*, $F_{sl\,x}$ and $F_{sl\,y}$ – projections of sliding forces on coordinate axes OX and OY *N*, M_v – moment of forces of viscous friction *Nm*, M_{sl} – moment of forces of sliding friction *Nm*, $M_{o\tau}$ – spinning moment of electromagnetic forces *Nm*.

Since the relationship $h/R_{RO} << 1$ is correct for the considered circular rotor, the motion drag forces are insignificant and not taken into account.

We assume that elementary force $d\overline{F}_v$ of viscous friction of rotor surface (fig. 2) elementary deck *A* on liquid or gaseous environment is proportional to the first degree of velocity and is always directed oppositely to movement. This force is derived from the expression [17]:

$$d\overline{F}_v = -\nu \overline{\mathrm{v}}_{\mathrm{A}} dS_r, \tag{8}$$

where: ν – is a drag factor $kg/_{s \cdot m^2}$, $\overline{\mathrm{v}}_{\mathrm{A}}$ – rotor surface elementary deck *A* motion velocity m/s, $dS_r = \rho d\rho d\varphi$ – elementary deck *A* area m^2.

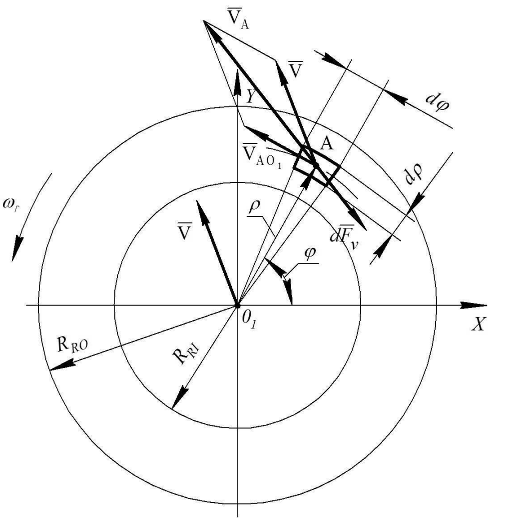

Fig. 2. Design pattern for determining the forces and torques of viscous friction

Since the rotor is in plain-parallel motion, the elementary deck *A* speed in relation to stator will be determined by the expression [3, 19, 20]:

$$\overline{\mathrm{v}}_{\mathrm{A}} = \overline{\mathrm{v}} + \overline{\mathrm{v}}_{\mathrm{AO_1}}, \tag{9}$$

where: $\overline{\mathrm{v}}_{\mathrm{AO_1}}$ – is relative speed of deck *A* around rotor rotation center $m/_s$.

Having denominated velocity vector $\overline{\mathrm{v}}$ projections on coordinate axes OX and OY as $\mathrm{v_x}$ and $\mathrm{v_y}$ correspondingly, we find projections $\mathrm{v_{Ax}}$ and $\mathrm{v_{Ay}}$ of vector $\overline{\mathrm{v}}_{\mathrm{A}}$ on the same axes, allowing for the fact that $\mathrm{v_{AO_1}} = \rho\omega_r$:

$$\mathrm{v_{Ax}} = \mathrm{v_x} - \rho\omega_r \sin\varphi, \tag{10}$$

$$\mathrm{v_{Ay}} = \mathrm{v_y} + \rho\omega_r \cos\varphi. \tag{11}$$

Taking into account (Eq. 10) and (Eq. 11), we also obtain projections of vector $d\overline{F}_v$ on axes OX and OY from the expression (Eq. 8):

$$dF_{vx} = -\nu(\mathrm{v_x} - \rho\omega_r \sin\varphi)\rho d\rho d\varphi, \tag{12}$$

$$dF_{vy} = -\nu(\mathrm{v_y} + \rho\omega_r \cos\varphi)\rho d\rho d\varphi. \tag{13}$$

Having integrated the expression (Eq. 12) and (Eq. 13), we determine the projections of viscous friction force acting on the rotor [15]:

$$F_{vx} = -\nu S_r \mathrm{v_x}, \quad F_{vy} = -\nu S_r \mathrm{v_y}, \tag{14}$$

where: $S_r = \pi(R_{RO}^2 - R_{RI}^2)$ – is circular rotor surface area m^2.

From the expression (Eq. 14) it is seen that viscous friction force depends on environment properties, rotor area and its motion speed in relation to stator, but doesn't depend on rotation frequency.

Let us find torque dM_v of viscous friction elementary force $d\overline{F}_v$ in regard to rotor rotation center:

$$dM_v = -dF_{vx}\rho\sin\varphi + dF_{vy}\rho\cos\varphi. \tag{15}$$

Having put expression (Eq. 12) and (Eq. 13) into (Eq. 15), we obtain:

$$\begin{aligned} dM_v = \nu((\mathrm{v_x} - \rho\omega_r \sin\varphi)\rho^2 \sin\varphi - \\ -(\mathrm{v_y} + \rho\omega_r \cos\varphi)\rho^2 \cos\varphi) d\rho d\varphi \end{aligned}. \tag{16}$$

Having integrated (Eq. 16), we get:

$$M_v = -\frac{1}{2}\nu\omega_r S_r (R_{RO}^2 + R_{RI}^2). \quad (17)$$

At $h/R_{RO} << 1$, distribution of standard pressure, which the moving rotor exerts on insulating substrate, is close to equilibrium. In this case sliding friction force, acting on rotor, is derived from the expression [6, 21]:

$$\overline{F}_{sl} = -\frac{\mu mg}{S_r}\int_0^{2\pi}\int_{R_{RI}}^{R_{RO}} \frac{\overline{\mathrm{v}} + \overline{\omega}_r \times \overline{\rho}}{\left|\overline{\mathrm{v}} + \overline{\omega}_r \times \overline{\rho}\right|} d\rho d\varphi, \quad (17)$$

where: μ – is sliding friction ratio, which we will consider as constant, m – circular rotor mass *kg*, g – gravitational acceleration m/s^2.

In works [6, 21] the expression (Eq. 17) has been integrated, based on this we can write the following:

$$F_{sl\,x} = -F_{sl}\frac{\mathrm{v_x}}{\mathrm{v}}, \; F_{sl\,y} = -F_{sl}\frac{\mathrm{v_y}}{\mathrm{v}}, \quad (18)$$

where: $\mathrm{v} = \sqrt{\mathrm{v_x^2} + \mathrm{v_y^2}}$ – rotor mass center speed vector modulus m/s, F_{sl} – sliding friction force vector modulus *N*.

Having denoted the ratio between circular rotor inner radius and its outer radius as $\alpha = R_{RI}/R_{RO}$ and having done the transformations [10, 15], we obtain projections of sliding friction force on coordinate axes:

$$F_{sl\,x} = -\frac{\mu mg}{\omega_{\mathrm{r}} R_{RO}(1+\alpha)}\mathrm{v_x}, \quad (19)$$

$$F_{sl\,y} = -\frac{\mu mg}{\omega_{\mathrm{r}} R_{RO}(1+\alpha)}\mathrm{v_y}. \quad (20)$$

From (Eq. 19) and (Eq. 20) it is obvious that as rotor spin angle frequency rises, the sliding friction force falls, at simultaneous rotation and motion of the circular rotor.

Moment of friction is determined by the expression [6, 21]:

$$\overline{M}_{sl} = -\frac{\mu mg}{S_r}\int_0^{2\pi}\int_{R_{RI}}^{R_{RO}} \rho \times \frac{\overline{\mathrm{v}} + \overline{\omega}_r \times \overline{\rho}}{\left|\overline{\mathrm{v}} + \overline{\omega}_r \times \overline{\rho}\right|} d\rho d\varphi. \quad (21)$$

According to works [6, 21], after being integrated the expression (Eq. 21) looks like:

$$\overline{M}_{sl} = -M_{sl}\frac{\overline{\omega}_r}{\omega_r}. \quad (22)$$

After the calculations have been done friction torque is determined according to the following expression [1, 15]:

$$M_{sl} = -\frac{2}{3}\mu mgR_{PH}\frac{1-\alpha^3}{1-\alpha^2}. \quad (23)$$

Having introduced the following denominations: $\vartheta = \nu S_r$, $\upsilon = \frac{\mu mg}{\omega_{\mathrm{r}} R_{RO}(1+\alpha)}$ and $\beta = \frac{1}{2}\nu S_r (R_{RO}^2 + R_{RI}^2)$ we put down the system of equations (Eq. 7), with allowance for the obtained above forces and moments of viscous and sliding friction, as well as electromagnetic forces and their torque:

$$\left.\begin{aligned} m\frac{d^2x}{dt^2} + (\vartheta+\upsilon)\frac{dx}{dt} &= -D_r x - D_\tau y, \\ m\frac{d^2y}{dt^2} + (\vartheta+\upsilon)\frac{dy}{dt} &= D_\tau x - D_r y, \\ I\frac{d^2\varphi}{dt^2} + \beta\frac{d\varphi}{dt} + M_{sl} &= M_{o\tau}. \end{aligned}\right\} \quad (24)$$

Rotational moment $M_{o\tau}$ of electromagnetic forces is determined from the expression [22]:

$$M_{o\tau} = \pi C_\tau B^2 (R_{RO}^4 - R_{RI}^4)\left(\frac{1}{2} + \frac{e^2}{R_{RO}^2 + R_{RI}^2}\right), \quad (25)$$

where: $C_\tau = C\sin\psi$ and $C = \frac{(\omega_s - \omega_r)h}{2\rho_r}$, B – is averaged value of induction density in the working zone of DIM stator *tesla*, ψ – angle between the normal line to stator slot and the radius of the DIM working zone *radian*, ω_s – angle velocity the electromagnetic field of the stator s^{-1}, ρ_r – DIM rotor resistivity constant $Ohm m$.

Since the displacement $e << R_{RO}$ and $e << R_{RI}$, then it is possible to assign the addend $\frac{e^2}{R_{RO}^2 + R_{RI}^2} = 0$ with quite high precision. Then the third equation of the system does not depend on the x and y coordinates and thus does not affect the rotor motion trajectory.

The first two equations of the set (Eq. 24) include only coordinates x and y of rotor mass center current position. Solution of these two equations determines rotor trajectory and, consequently, its displacement in relation to stator. The third equation of the set does not depend on x and y coordinates and thus doesn't affect rotor trajectory. Besides, as rotor rotates with constant angle frequency ($\omega_p = const$), the product $I\frac{d^2\varphi}{dt^2} = 0$ and resistance forces torque are balanced by electromagnetic forces torque. Since the resistance torque values does not depend on displacement *e,* electromagnetic torque $M_{o\tau}$ remains unchanged.

In this case, it is enough to consider the first two equations of the set (Eq. 24) to analyze rotor motion stability. As this set of equations is linear and independent, we use Hurwitz criterion [14] to analyze its stability.

For this purpose we convert the first two equations of the set (Eq. 24) into the fourth-order differential equation:

$$\begin{aligned} & m^2 y^{(4)} + 2m(\vartheta + \upsilon)y^{(3)} + \\ & + (2mD_r + (\vartheta + \upsilon)^2)y^{(2)} + \\ & + 2D_r(\vartheta + \upsilon)y^{(1)} + (D_r^2 + D_\tau^2)y = 0. \end{aligned} \tag{26}$$

The characteristic equation corresponding to (Eq. 26) is as follows:

$$a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4 = 0 . \tag{27}$$

where: $a_0 = m^2$, $a_1 = 2m(\vartheta + \upsilon)$,
$a_2 = 2mD_r + (\vartheta + \upsilon)^2$,
$a_3 = 2D_r(\vartheta + \upsilon)$, $a_4 = D_r^2 + D_\tau^2$.

Let us set up Hurwitz determinant from these coefficients:

$$\Delta = \begin{vmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix} . \tag{28}$$

According to Hurwitz criterion the system is stable when all main diagonal minors of determinant (Eq. 28) are greater than zero. If at least one minor is equal to zero, the system is in the state of indifferent equilibrium, and if it is less than zero - the system is unstable.

Let us find the minors of the matrix (Eq. 28):

$$\begin{aligned} & \Delta_1 = 2m(\vartheta + \upsilon) > 0 , \\ & \Delta_2 = 2m(\vartheta + \upsilon)(mD_r + (\vartheta + \upsilon)^2) > 0 , \\ & \Delta_3 = 4m(\vartheta + \upsilon)^2(D_r(\vartheta + \upsilon)^2 - mD_\tau^2) > 0 , \\ & \Delta_4 = \Delta_3 a_4 > 0 . \end{aligned} \tag{29}$$

The last two inequations (Eq. 29) are fair when the following condition is fulfilled:

$$\frac{(\vartheta + \upsilon)^2 D_r}{mD_\tau^2} > 1 . \tag{30}$$

Having put the expressions for ϑ and υ into (Eq. 30) we obtain:

$$\left(vS_r + \frac{\mu mg}{\omega_r R_{RO}(1 + \alpha)} \right)^2 \frac{D_r}{mD_\tau^2} > 1 . \tag{31}$$

Equation 31 determines the range of variation of electromechanical system parameters that provide its stability, and it is circular rotor steady motion criterion (stability criterion).

Physical meaning of the expression (Eq. 31) consists in the fact that if the inequity is realized then rotor rotation center tends to take the stator center position under the action of tangential $\overline{F}_\tau$ and radial $\overline{F}_r$ forces. At that, as the displacement value *e* decreases, F_τ and F_r forces and rotor mass center motion velocity falls, too, but at $e = 0$ these values are equal to zero, which corresponds to steady rotation motion. If the inequity (Eq. 31) is not realized, the opposite process takes place, i.e. the rotor mass center moves away from stator center until the rotor falls outside the action of electromagnetic forces.

Radial and tangential forces are derived from the expressions [29]:

$$F_r = \frac{\omega_s sh}{\rho_r} e\pi\left(B_O^2 R_{RO}^2 \cos\psi_O - B_I^2 R_{RI}^2 \cos\psi_I\right), \quad (32)$$

$$F_\tau = \frac{\omega_s sh}{\rho_r} e\pi\left(B_O^2 R_{RO}^2 \sin\psi_O - B_I^2 R_{RI}^2 \sin\psi_I\right), \quad (33)$$

where: s – is rotor slip, B_O and B_I – induction density nearby the outer and inner stator contours correspondingly *tesla,* ψ_O and ψ_I – angle between the normal line to stator slot and the radius on outer and inner contour correspondingly *radian*.

To evaluate the circular rotor motion stability degree we introduce the safety factor K_{sf} that shows how many times left part of the inequity (Eq. 31) is bigger than 1. The greater K_{sf} is, the more steadily rotor moves. If K_{sf} is equal to one, it corresponds to the state of motion critical stability. If that coefficient is less than one, rotor motion is unsteady, it tends to fall outside the boundaries of stator magnetic field.

Using expressions (Eq. 32) and (Eq. 33), we rewrite criterion (Eq. 31) as follows:

$$K_{sf} = \frac{\rho_r}{\pi m \omega_s sh}\left(\nu S_r + \frac{\mu mg}{\omega_s R_{RO}(1-s)(1+\alpha)}\right)^2 \times \\ \times \frac{B_O^2 R_{RO}^2 \cos\psi_O - B_I^2 R_{RI}^2 \cos\psi_I}{(B_O^2 R_{RO}^2 \sin\psi_O - B_I^2 R_{RI}^2 \sin\psi_I)^2} > 1. \quad (34)$$

Stability criterion (Eq. 34) allows for the impact of rotor mass, its radial sizes and depth, rotor slip, electrical resistance, environmental resistance, distribution of induction density values in the running clearance, DIM stator slot direction.

The stability criterion allows for the impact of operating environment through environmental resistance ν and sliding friction μ coefficients. It is obvious from the criterion (Eq. 34) that as these coefficients rise rotor stability increases, and according to expressions (Eq. 14), (Eq. 19) and (Eq. 20), environmental resistance forces F_v and F_{sl} are proportional to ν and μ. Consequently, as the forces counteracting rotor motion increase, rotor stability rises as well.

From the rotation stability criterion (Eq. 34) and expression (Eq. 25) it is obvious that the ratio between rotor resistivity constant and its depth ρ_r/h affects the safety factor K_{sf} and driving torque $M_{o\tau}$ oppositely. As ρ_r/h rises, safety factor K_{sf} increases while torque $M_{o\tau}$ declines. To increase the driving torque $M_{o\tau}$ DIM rotor is produced from low ρ_r materials, like copper and aluminium. Rotor depth h is determined by its process value [4, 8].

From all has been said it follows that design-engineering characteristics improving DIM efficiency lead to decline of stability of rotor without mechanical support.

Stability criterion analysis showed that asymptotical stability condition is fulfilled when the following equation is adhered:

$$B_H^2 R_{PH}^2 \sin\psi_H - B_B^2 R_{PB}^2 \sin\psi_B = 0\,. \quad (35)$$

At that tangential force F_τ is equal to zero.

As it is obvious from (Eq. 35) the condition $F_\tau = 0$ is affected only by rotor radial sizes (R_{RO}, R_{RI}), elementary forces direction (angles ψ_O and ψ_I) and distribution of induction density in running clearance (B_O, B_I).

Rotor asymptotical stability always takes place when destabilizing force is absent, regardless of displacement *e* and is expressed with condition $F_\tau = 0$. At that stability criterion, regardless of other parameters, tends to infinity.

Let us consider three alternatives of DIM.

The first variant is characterized with conditions $R_{SO} - R_{RO} > e$ and $R_{RI} - R_{SI} > e$, i.e. rotor is always within stator magnetic field. Destabilizing force turns into zero when the equation (Eq. 35) is rendered.

This variant is the most common, since $F_\tau = 0$ is fulfilled in a broad range of variation for parameters included in (Eq. 35).

For the second variant we assume that $R_{SO} = R_{RO}$, and $R_{RI} - R_{SI} > e$. Then $F_\tau = 0$ will be fulfilled at:

$$\frac{R_{RO}}{R_{RI}} = \frac{B_I}{B_O}\sqrt{2\frac{\sin\psi_I}{\sin\psi_O}}. \tag{36}$$

Equation $R_{SO} = R_{RO}$ reduces the number of active parameters, that to a certain extend simplifies rotor stability maintenance. From (Eq. 36) it is obvious that at constant geometric parameters of stator and rotor, zero value of destabilizing force F_τ can be achieved by changing B_I and B_O induction density values on the outer and inner contours of stator correspondingly.

The third variant of DIM construction is characterized with equation $R_{SO} = R_{RO}$ and magnetic field uniformity in the working area, i.e. $B_O = B_I = const$. Rotor rotation stability condition takes the form of:

$$\frac{R_{RO}}{R_{RI}} = \sqrt{2\frac{\sin\psi_I}{\sin\psi_O}}. \tag{37}$$

The latter variant makes condition $F_\tau = 0$ absolutely stringent, in other words, the condition is set at accurate conformance of parameters that cannot be regulated in real machine.

To ensure steady motion of rotor it is necessary that center directed force F_r appears and the equation (Eq. 35) is adhered as the rotor shifts. This can be achieved if the force F_τ pattern of change along the radius will not coincide with the force F_r pattern of change [26]. At that the stator slot tilt angle ψ must functionally depend on the radius.

CONCLUSION

1. The free rotor motion mathematical model refinement enabled to specify the stable motion criterion that allows for the impact of characteristics, like: rotor mass, sizes, electrical resistance, environmental resistance and sliding friction ratios, distribution of induction density in DIM running clearance and stator slots direction. The new criterion enabled to determine the impact of DIM design and engineering characteristics on free rotor stable motion.

2. Based on the obtained stability criterion it has been detected that increase of viscous friction force and sliding friction force improves rotor motion stability. This property makes it possible to use DIM as a drive for circular operating devices of technological machines.

3. If the electric machine efficiency is preserved, free rotor stability rise is possible in such system of electromagnetic forces where the rotor shift from center doesn't lead to occurrence of tangential destabilizing force but causes only radial stabilizing force returning rotor to the center.

REFERENCES

1. **Ahiezer N.I., 1970.:** The elements of elliptical functions theory. "Nauka", Moscow, 304. (in Russian).
2. **Alfutov N.A., 2003.:** Motion and equilibrium state stability. Moscow, M.E. Bauman MSTU, 256 p. (in Russian).
3. **Bolotin S.V., Karapetyan A.V., Kutushev I.E., Tereshchev D.V., 2010.:** Theoretical mechanics. Dynamics. Publishing house "Academia", Moscow, 432 p. (in Russian).
4. **But D.A., 1990.:** Noncontact electric machines. "Vysshaya shkola", Moscow, 416 (in Russian).
5. **Chetaev N.G., 1990.:** Motion stability. Moscow, Nauka, 175 p. (in Russian).
6. **Farkash Z., Bartels G., Winger T., Wolf D.E., 2011.:** About friction force at progressive and rotational motion of flat body. Non-linear dynamic, Computer research institute, Izhevsk, V. 7, #1, p. 139-146. (in Russian).
7. **Fedoryuk M.V., 1985.:** Ordinary differential equations. "Nauka", Moscow, 448 (in Russian).
8. **Krumin Yu.K., 1969.:** Travelling magnetic field-conducting medium interaction. "Zinatne", Riga, 258 (in Russian).
9. **Kuropatkin P.V., 1973.:** Automatic control theory. :Vysshaya Shkola", Moscow, 528 (in Russian).
10. **Lurje A.I., 1961.:** Analytical mechanics, SIPML Press, Moscow, 824. (in Russian).

11. **Lushchyk V.D. 1993.:** Coincident electric machines and equipment. Kiev, Technika, 203. (in Russian).
12. **Lyapunov A.M., 1950.:** General task about motion stability. State publisher of techno-theoretical literature, Moscow – Leningrad, 472. (in Russian).
13. **Malkin I.G., 1966.:** Motion stability theory. “Nauka”, Moscow, 532. (in Russian).
14. **Merkin D.R., 2003.:** Introduction to motion stability theory. “Lan” Press, St.Petersburg, 304.
15. **Piskunov N.S., 1985.:** Differential and integral calculus. V. 2, Science Press, Moscow, 560. (in Russian).
16. **Postnikov M.M., 1981.:** Stable polynomials. “Nauka”, Moscow, 176. (in Russian).
17. **Strelkov S.P., 1975.:** General physics course. Mechanics. Science Press, Moscow, 560. (in Russian).
18. **Svecharnik D.V., 1988.:** Direct drive electrical machines: gearless electric drive. – Moscow, Energoatomizdat, 208. (in Russian).
19. **Targ S.M., 1986.:** Short course of theoretical mechanics. “Vysshaya shkola”, Moscow, 416. (in Russian).
20. **Vilke V.G., 2003.:** Theoretical mechanics. “lan” Press, St. Petersburg, 304. (in Russian).
21. **Waidman P.D., Malyotra Ch., 2011.:** About the final motion of sliding and spinning disks with simple Coulomb friction. Non-linear dynamics, , Computer research institute, Izhevsk, V. 7, #2, p. 339-365. (in Russian).
22. **Yeroshin S.S., 1998.:** Determining the forces acting on circular plate in rotating magnetic field. Collection of scientific works of East-Ukrainian State University. Machine building. EUSU Press, Luhansk, 13-21. (in Russian).
23. **Yeroshin S.S., 2007.:** The concept of generating machines and gears with direct drive of operating devices with no mechanical support. Theoretical engineering, Lviv, #2 (116), 27-32. (in Ukrainian).
24. **Yeroshin S.S., Breshev V.E., 2005.:** Machine efficiency improvement by using operating devices with no mechanical support. Kharkov, East-European Journal of Advanced Technologies, # 5 (17), 82-85. (in Russian).
25. **Yeroshin S., Golubenko A., 2008.:** Concept of developing machines and devices with the direct rotation of rotor without mechanical supports. TEKA Commission of Motorization and Power Industry in Agriculture, Vol. 10 A, 37-46.
26. **Yeroshin S., Miroshnik S., 2011.:** Improving the stability rotation ring rotor without mechanical supports. TEKA Commission of Motorization and Power Industry in Agriculture, Vol. 11 A., 282-289.
27. **Yeroshin S.S., Nevzlin B.I., Breshev V.E., 2004.:** Generation of free rotor – cutter tool drive. Odessa, Electric machine building and electric equipment, # 62, 84-87. (in Russian).
28. **Yeroshin S.S., Tarashchanskiy M.T., Miroshnik S.A., 2009.:** Frictional forces effect on disk induction motor circular rotor motion stability. Works of Luhansk department of International Academy for Information System Development, V. Dahl EUNU Press, Luhansk # 2(19), 58-61. (in Russian).
29. **Yeroshin S.S., Tarashchanskiy M.T., Miroshnik S.A., 2008.:** The research of stability criterion for asynchronous end motor circular rotor. Resource-saving technologies of production and pressure shaping of materials in machine building. Collection of scientific works, V. Dahl EUNU Press, Luhansk, 229-234. (in Russian).
30. **Yeroshin S.S., Tarashchanskiy M.T., Miroshnik S.A., Ivanov E.S., 2008.:** The research of asynchronous end motor circular rotor motion in no-load conditions. The works of Luhansk department of International Academy for Information System Development, V. Dahl EUNU Press, Luhansk, #1 (16), 44-49. (in Russian).

КРИТЕРИЙ УСТОЙЧИВОСТИ СВОБОДНОГО РОТОРА ДИСКОВОГО АСИНХРОННОГО ДВИГАТЕЛЯ

Сергей Ерошин, Сергей Мирошник

Аннотация. Усовершенствована математическая модель движения свободного ротора дискового асинхронного двигателя, которая учитывает действие движущих электромагнитных сил и сил, противодействующих движению, таких как сила вязкого трения и сила трения скольжения. На основании новых уравнений движения свободного ротора уточнен критерий его устойчивого движения с учетом влияния комплекса конструкторско-технологических параметров электрической машины.

Ключевые слова. Свободный кольцевой ротор, критерий устойчивости.