

## Morphogenesis and correction of planar rod constructions with a small amount of free nodes

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**Summary.** This publication is devoted to practical aspects of a new method of form correction of planar rod constructions. This method should be used after the initial shape of the frame construction is already defined. At the same time, suggested method makes it possible to determine the components of the stress-strain state of the construction and has the same mathematical foundation as a method of cutting out of nodes in theoretical mechanics. Also, the article demonstrates the principle of applying the method on the example of correction of form of elementary frame construction with hinge joint of rods. An example illustrates the advantages of this method over methods of numerical simulation, because it does not require changing of instrument base during the transition from automated shaping of construction to determination of efforts in its rods.

**Key words:** geometric modeling, discrete model, rod frame constructions, differential regularities, numerical simulations.

### INTRODUCTION

Most of the tasks of building mechanics involve determination of certain components of the stress-strain state (SSS) of constructions. It is assumed, that the geometrical parameters of structures are predetermined and represent the initial conditions for calculations. In the case, when before the calculations the morphogenesis process is carried out, it is necessary to apply two distinct methods: the first one – to determine the actual form of construction, the second one – to calculate the parameters of SSS.

At the present stage of development of computer aided design systems the most programs intended for calculation of con-

struction for strength and stability involve the ability to import finite data from the specialized software's environment for building of graphic models of these constructions. However, models that are products of graphics programs created directly by the users (architects, engineers, designers, etc.), while automated algorithms for morphogenesis of constructions in the environment of these programs are nearly absent. Should be added, that existing mathematical algorithms of morphogenesis of building structures do not have sufficient variability to describe the features of work of construction elements in the process of loading and operation with required accuracy.

All the above mentioned points to the need to prepare a theoretical and tool base to create a unified method of building structures shape modeling, with possibility of their adjustments and subsequent calculation of component of SSS of their elements.

One of the most pressing areas of this problem is the formation and modeling of the rod building structures' work. Rod constructions have an important place among building structures, because their designing requires from engineers a high level of skills and responsibility. They are used in the design of beam coatings, trusses and covering membranes, bearing and self-supporting frames of buildings, frame structures and many others.

In this publication we will consider the outlined problems in the context of researching and modeling of planar rod or frame structures with hinge joint of rods.

## PURPOSE OF WORK

Basic principles of morphogenesis problem solving and subsequent adjustment of mesh and rod constructions were presented in a series of works [1-5]. The main idea of described in these works method is to apply the fundamental differential patterns between geometric and physical parameters of network structures and external fields for systemic redistribution of interaction forces between their vertices [6]. At the same time, works have the generalizing character and dedicated to the realization of mathematical apparatus of morphogenesis on examples of universal models of network structure (such as discrete surfaces, as in [5]). Obvious, that for the application of the proposed method in the tasks of structural mechanics and mechanics of rod systems it is necessary to adapt it to some extent, taking into account the engineering specifics. Such an adaptation is the main purpose of this work.

## REVIEW OF PREVIOUS RESEARCHES

For a start, we present the basic provisions of the method, summarized in [6], and simplified for two-dimensional case.

Suppose there is some two-dimensional rod system with hinge joint in the free and basic (reference) nodes. We shall assume, that the known topological characteristics of the system (number and order of rods connection) and rigidity (stiffness) parameters of its rods  $\mathbf{n}_{i,j}$ , which are expressed by the ratios of the absolute values of longitudinal efforts in rods  $R_{i,j}$  and their lengths  $\delta_{i,j}$ :

$$\mathbf{n}_{i,j} = R_{i,j} / \delta_{i,j}. \quad (1)$$

It is known, that the equilibrium state of each node can be described, using the principle of cutting out of units by replacing each rod, that connects to the node, on corresponding resistance efforts. Omitting the projection of vectors of forces  $\bar{\mathfrak{S}}_i$ , acting on the node from the outside, and vectors of internal efforts  $\bar{R}_{i,j}$ , which cut off rods, on the coordinate axes, we obtain the following

system of equilibrium equations of arbitrary node (taking into consideration equality (1)):

$$\sum_{j=1}^n (s_j - s_i) \cdot \mathbf{n}_{i,j} + \bar{\mathfrak{S}}_i = 0, \quad (2)$$

where:  $s$  – generalizing designation of coordinates;  $n$  – quantity of uncommitted nodes of construction.

If we assume, that the rigidity parameters of rods  $\mathbf{n}_{i,j}$  and external load  $\bar{\mathfrak{S}}_i$  is predetermined, then system of equations of type (2), composed for all free nodes of rod construction, can be solved relative to coordinates of these nodes. In this way, the process of prior morphogenesis of construction can be implemented. Having the coordinates of nodes, and determining the length of rods, it is not difficult to calculate internal forces from formula (1).

To make it possible to determine appropriate rigidity parameters of rods  $\mathbf{n}_{i,j}$  during adjustment of position of nodes, and to be able to calculate internal forces, the system (2) should be supplemented by a system of parametric equations of rod's state. These equations have the following form:

1) for the rods, connecting two free nodes of construction ( $S_a$  and  $S_b$ ):

$$\begin{aligned} & \sum_{i=1}^{m-1} \delta_{a,i}^2 \cdot \mathbf{n}_{a,i} + \chi \cdot \delta_{a,b}^2 \cdot \mathbf{n}_{a,b} + \\ & + \sum_{j=1}^{n-1} \delta_{b,j}^2 \cdot \mathbf{n}_{b,j} - (\varphi_a + \varphi_b) + B_{a,b} = 0; \end{aligned} \quad (3)$$

2) for the rods, that connect one free and one basic nodes ( $S_a$  and  $S_{ref}$ ):

$$\begin{aligned} & \sum_{i=1}^{m-1} \delta_{a,i}^2 \cdot \mathbf{n}_{a,i} + \chi \cdot \delta_{a,ref}^2 \cdot \mathbf{n}_{a,ref} - \varphi_a + \\ & + (R_{x_{ref}} \cdot x_{ref} + R_{y_{ref}} \cdot y_{ref}) + B_{a,ref} = 0, \end{aligned} \quad (4)$$

where:  $m$  and  $n$  – number of nodes adjacent to the  $a$ -th and  $b$ -th (or  $ref$ -th) nodes,  $\chi$  – some non-negative constant;  $\varphi_a$  and  $\varphi_b$  – nodal values of the scalar potential (of the field of objective function),  $R_{ref}$  – values of efforts in the rods that are connected to the rocker bearing,  $B_{a,b}$  and  $B_{a,ref}$  – general oper-

ating constants of integration.

In a matrix form the process of forming and subsequent correction of rod construction shape can be described by following system:

$$\begin{cases} [s^p] = [\mathbf{K}^{p-1}]^{-1} \cdot (-[g^{p-1}] - [\mathfrak{S}^p]), \\ \{\mathbf{K}^p\} = [(\delta^p)^2]^{-1} \cdot (\{\varphi^{p-1}\} - \{\varphi^p\} + \\ + [(\delta^p)^2] \cdot \{\mathbf{K}^{p-1}\}). \end{cases} \quad (5)$$

Here:  $[s]$  – matrix of coordinates (with dimension  $k \times 2$ , where  $k$  – the quantity of nodes of the model),  $[g]$  – matrix of the boundary conditions (with dimension  $k \times 2$ ),  $[\mathfrak{S}]$  – matrix of external influences (with dimension  $k \times 2$ ),  $[\mathbf{K}]$  – matrix of stiffness parameters of rod structure (with dimension  $k \times k$ ),  $\{\mathbf{K}\}$  – column vector of stiffness parameters of rod structure,  $[\delta^2]$  – matrix of geometric parameters of rod structure (with dimension  $h \times h$ , where  $h$  – quantity of model's rods),  $\{\varphi\}$  – column vector of nodal values of the scalar potential,  $\{\varphi'\}$  – column vector of expected nodal parameters of the scalar potential,  $p$  – index corresponding to the current step of the iterative calculation.

Solving the system (5) (if necessary using iterative calculation), we define values of corrected parameters of rods rigidity and coordinate of nodes of the model.

### CORRECTING OF THE SHAPE OF ROD CONSTRUCTIONS

Changing the position of nodes, using model systems (5), must be carried out by replacing the current values of the scalar potential  $\varphi_i$  on expected values  $\varphi'_i$ . It is assumed, that external influence  $\mathfrak{S}_i$  is in gradient connection with the current scalar potential. However, in [7] was founded that this relationship is not required. Moreover, there is possible the variant of local correction of construction. In this case the potential  $\varphi_i$  should be corrected only in certain points of the model.

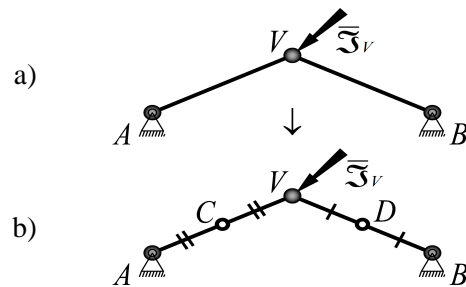
If we need to move the selected set of nodes in the individual order, each of these nodes will have the functions of the scalar potential. Thus, in each node, which exposed by moving, scalar potential value will be represented by its objective function. Obviously, the objective function value must decline to zero under the condition, that the coordinates of the node come near to values set by the engineer. Therefore, objective function of an arbitrary  $i$ -th node  $S_i$  should be presented in the form of its distance from a certain established point  $T$ :

$$\begin{aligned} \varphi_i = \varphi(s_i) = \zeta(x_i, y_i) = \\ = \vartheta \cdot ((x_T - x_i)^2 + (y_T - y_i)^2)^{1/2}. \end{aligned} \quad (6)$$

Here:  $\vartheta$  – coefficient, entered to effect on speed of convergence of the iterative calculation.

The value of the expected node potential will be zero:  $\varphi'_i = 0$ .

Let's consider the example of shape correction of elementary rod construction in the form of a planar frame, which consist of 2 rods and has 2 hinged-fixed reference node ( $A$  and  $B$ ) and only 1 free hinge node  $V$  with the given load  $\mathfrak{S}_V$  (see. Fig. 1,  $a$ ).



**Fig. 1.** The rod frames:  $a$  – frame AVB;  $b$  – frame ACVDB, obtained by adding nodes  $C$  and  $D$  to the frame AVB

This frame is statically undetectable. In addition, it can not be corrected, using the system (5). The fact is, that in order to properly influence to the changes of rigidity parameters of rods, using nodal potentials, in each parametric equation of rods of type (3)

or (4) must be contained various combinations of nodal potentials. In our case, the equations of rods  $AV$  and  $VB$  contain only potentials  $\varphi_V$  of node  $V$ . These equations have the following form:

$$\chi \cdot \delta_{A,V}^2 \cdot \mathfrak{N}_{A,V} + \delta_{V,B}^2 \cdot \mathfrak{N}_{V,B} - \varphi_V + (R_{x_A} \cdot x_A + R_{y_A} \cdot y_A) + B_{A,V} = 0, \quad (7)$$

$$\delta_{A,V}^2 \cdot \mathfrak{N}_{A,V} + \chi \cdot \delta_{V,B}^2 \cdot \mathfrak{N}_{V,B} - \varphi_V + (R_{x_B} \cdot x_B + R_{y_B} \cdot y_B) + B_{V,B} = 0. \quad (8)$$

From equations (7) and (8) we see, that when we change potential  $\varphi_V$ , then parameters  $\mathfrak{N}_{A,V}$  and  $\mathfrak{N}_{V,B}$  will change in direct proportion. In this case, the trajectory of the node  $V$  can not be controlled.

To make control of rigidity parameters of construction possible, we should halve rods  $AV$  and  $VB$  by additional nodes  $C$  and  $D$  respectively. We obtain the following relations:

$$\delta_{A,C} = \delta_{C,V} = \delta_{A,V}/2, \quad (9)$$

$$\delta_{V,D} = \delta_{D,B} = \delta_{V,B}/2. \quad (10)$$

The resulting structure is shown at Fig. 1.b.

Obviously, if any external forces will not act the additional nodes  $C$  and  $D$ , then internal forces in a rods  $AC$  and  $CV$ ,  $VD$  and  $DB$  will be equal and in fact will remain in solid segments  $AV$  and  $VB$ . That is:

$$R_{A,C} = R_{C,V} = R_{A,V}, \quad (11)$$

$$R_{V,D} = R_{D,B} = R_{V,B}. \quad (12)$$

Considering the equation (9) – (12), we write the relations between the stiffness parameters of rods  $AC$ ,  $CV$  and  $AV$ , as well as  $VD$ ,  $DB$  and  $VB$ :

$$\mathfrak{N}_{A,C} = \mathfrak{N}_{C,V} = 2 \cdot \mathfrak{N}_{A,V}, \quad (13)$$

$$\mathfrak{N}_{V,D} = \mathfrak{N}_{D,B} = 2 \cdot \mathfrak{N}_{V,B}. \quad (14)$$

Let's compose the equilibrium equations of type (2) for all free nodes ( $C$ ,  $V$  and  $D$ ):

$$\mathfrak{N}_{A,C} \cdot s_A + \mathfrak{N}_{C,V} \cdot s_V - (\mathfrak{N}_{A,C} + \mathfrak{N}_{C,V}) \cdot s_C = 0, \quad (15)$$

$$\mathfrak{N}_{C,V} \cdot s_C + \mathfrak{N}_{V,D} \cdot s_D - (\mathfrak{N}_{C,V} + \mathfrak{N}_{V,D}) \cdot s_V + \mathfrak{I}_{s_V} = 0, \quad (16)$$

$$\mathfrak{N}_{V,D} \cdot s_V + \mathfrak{N}_{D,B} \cdot s_B - (\mathfrak{N}_{V,D} + \mathfrak{N}_{D,B}) \cdot s_D = 0. \quad (17)$$

Rewrite equation (15) – (17), taking into account the identities (13) and (14) and all possible simplifications:

$$s_A - 2 \cdot s_C + s_V = 0, \quad (18)$$

$$\mathfrak{N}_{A,V} \cdot s_C + \mathfrak{N}_{V,B} \cdot s_D - (\mathfrak{N}_{A,V} + \mathfrak{N}_{V,B}) \cdot s_V + \mathfrak{I}_{s_V}/2 = 0, \quad (19)$$

$$s_V - 2 \cdot s_D + s_B = 0. \quad (20)$$

Now we compose system of parametric equations of state for rods  $AC$ ,  $CV$ ,  $VD$  and  $DB$ :

$$\chi \cdot \delta_{A,C}^2 \cdot \mathfrak{N}_{A,C} + \delta_{C,V}^2 \cdot \mathfrak{N}_{C,V} - \varphi_C + (R_{x_A} \cdot x_A + R_{y_A} \cdot y_A) + B_{A,C} = 0, \quad (21)$$

$$\delta_{A,C}^2 \cdot \mathfrak{N}_{A,C} + \chi \cdot \delta_{C,V}^2 \cdot \mathfrak{N}_{C,V} + \delta_{V,D}^2 \cdot \mathfrak{N}_{V,D} - (\varphi_C + \varphi_V) = 0, \quad (22)$$

$$\delta_{C,V}^2 \cdot \mathfrak{N}_{C,V} + \chi \cdot \delta_{V,D}^2 \cdot \mathfrak{N}_{V,D} + \delta_{D,B}^2 \cdot \mathfrak{N}_{D,B} - (\varphi_V + \varphi_D) = 0, \quad (23)$$

$$\delta_{V,D}^2 \cdot \mathfrak{N}_{V,D} + \chi \cdot \delta_{D,B}^2 \cdot \mathfrak{N}_{D,B} - \varphi_D + (R_{x_B} \cdot x_B + R_{y_B} \cdot y_B) + B_{D,B} = 0. \quad (24)$$

We shall add pairs of equations (21) with (22) and (23) with (24), given the identity (9), (10), (13) and (14), setting the coefficient  $\chi = 2$  (according to [2], since as quantity of model's nodes is higher than the quantity of rods), and performing all possible simplification:

$$3 \cdot \delta_{A,V}^2 \cdot \mathfrak{N}_{A,V} + (1/2) \cdot \delta_{V,B}^2 \cdot \mathfrak{N}_{V,B} - (2 \cdot \varphi_C + \varphi_V) + (R_{x_A} \cdot x_A + R_{y_A} \cdot y_A) + B_{A,C} = 0, \quad (25)$$

$$(1/2) \cdot \delta_{A,V}^2 \cdot \mathfrak{N}_{A,V} + 3 \cdot \delta_{V,B}^2 \cdot \mathfrak{N}_{V,B} - (\varphi_V + 2 \cdot \varphi_D) + (R_{x_B} \cdot x_B + R_{y_B} \cdot y_B) + B_{D,B} = 0. \quad (26)$$

Using equations (18)-(20), we write the components of the first expression of system (5). They will have the following form:

1. The matrix of coordinates  $[s]$ :

$$[s] = [X \quad Y], \quad (27)$$

where:  $\{X\}$  and  $\{Y\}$  – column vectors of coordinates of nodes, which have the form:

$$\{X\}^T = [x_C \quad x_V \quad x_D], \quad (28)$$

$$\{Y\}^T = [y_C \quad y_V \quad y_D]. \quad (29)$$

2. The matrix of boundary conditions  $[g]$ :

$$[g] = [g_x \quad g_y], \quad (30)$$

where:  $\{g_x\}$  and  $\{g_y\}$  – column vectors of boundary conditions, which have the form:

$$\{g_x\}^T = [-I \cdot x_A \quad 0 \quad -I \cdot x_B], \quad (31)$$

$$\{g_y\}^T = [-I \cdot y_A \quad 0 \quad -I \cdot y_B]. \quad (32)$$

3. The matrix of external forces  $[\mathfrak{S}]$ :

$$[\mathfrak{S}] = [\mathfrak{S}_x \quad \mathfrak{S}_y], \quad (33)$$

where:  $\{\mathfrak{S}_x\}$  and  $\{\mathfrak{S}_y\}$  – column vectors of external forces, which have the form:

$$\{\mathfrak{S}_x\}^T = [0 \quad \mathfrak{S}_{x_V}/2 \quad 0], \quad (34)$$

$$\{\mathfrak{S}_y\}^T = [0 \quad \mathfrak{S}_{y_V}/2 \quad 0]. \quad (35)$$

4. The matrix of stiffness parameters  $[\mathfrak{N}]$ :

$$[\mathfrak{N}] = \begin{bmatrix} -2 & I & 0 \\ \mathfrak{N}_{A,V} & -(\mathfrak{N}_{A,V} + \mathfrak{N}_{V,B}) & \mathfrak{N}_{V,B} \\ 0 & I & -2 \end{bmatrix}. \quad (36)$$

Now, on the basis of equations (25) and (26) we can write the components of the second expression of system (5). They will have the following form:

1. The column vector of stiffness parameters of rod construction  $\{\mathfrak{N}\}$ :

$$\{\mathfrak{N}\}^T = [\mathfrak{N}_{A,V} \quad \mathfrak{N}_{V,B}]. \quad (37)$$

2. The matrix of geometric parameters of rod structure  $[\delta^2]$ :

$$[\delta^2] = \begin{bmatrix} 3 \cdot \delta_{A,V}^2 & (1/2) \cdot \delta_{V,B}^2 \\ (1/2) \cdot \delta_{A,V}^2 & 3 \cdot \delta_{V,B}^2 \end{bmatrix}. \quad (38)$$

3. The column vector of expected nodal potentials  $\{\varphi'\}$ :

$$\{\varphi'\}^T = [\varphi'_{A,V} \quad \varphi'_{V,B}] = [0 \quad 0]. \quad (39)$$

4. The column vector of current nodal potentials  $\{\varphi\}$ :

$$\{\varphi\}^T = [\varphi_{A,V} \quad \varphi_{V,B}] = [2 \cdot \varphi_C + \varphi_V \quad \varphi_V + 2 \cdot \varphi_D]. \quad (40)$$

Call attention to the elements of the column vector  $\{\varphi\}$ . If one follows an algorithm of application of the system (5) and choose as an objective functions the distances of type (6), then in case of the unsuccessful selection of coefficient  $\vartheta$  and low value of calculation error iterative calculation may be divergent. This can happen, because at the stage close to achieving nodes of their planned coordinates, displacement step of one of the nodes will exceed a distance to the point of his appointment. At the same time algorithm (5) will try to shorten the distance of a particular node to the established point by subsequent replacement of potentials (objective functions) "without realizing a miss". This will only lead to further distancing of the node from its destination.

To avoid the described effect, should use as a coefficient  $\vartheta$  not a constant but logical operator. The operator  $\vartheta$  must analyze the differences  $\Delta\varphi$  between elements of the col-

umn vector of potentials  $\{\varphi\}$  on the current and previous steps of iterative calculation:

$$\Delta\varphi_{i,j}^p = \varphi_{i,j}^p - \varphi_{i,j}^{p-1}. \quad (41)$$

Thus, the value of operator  $\vartheta$  should be determined by the expression:

$$\vartheta_{i,j}^p = \zeta(\Delta\varphi_{i,j}^p) = \begin{cases} 1 & \text{if } \Delta\varphi_{i,j}^p > 0, \\ 0 & \text{if } \Delta\varphi_{i,j}^p = 0, \\ -1 & \text{if } \Delta\varphi_{i,j}^p < 0. \end{cases} \quad (42)$$

This character of dependence of function from the values of the argument can be described by hyperbolic tangent of the argument (such as logical operators used in neural modeling [11-13]). Such continuous function will look like:

$$\begin{aligned} \vartheta_{i,j}^p &= \zeta(\Delta\varphi_{i,j}^p) = \tanh(\alpha \cdot \Delta\varphi_{i,j}^p / 2) = \\ &= \frac{1 - \exp(-\alpha \cdot \Delta\varphi_{i,j}^p)}{1 + \exp(-\alpha \cdot \Delta\varphi_{i,j}^p)}, \end{aligned} \quad (43)$$

where:  $\alpha$  – coefficient, whose value determines the sharpness of the character changing of the function (43) during the transition from  $-1$  to  $1$  (through  $0$ ).

Therefore, the column vector of the current potentials  $\{\varphi^p\}$ , which will take into account the values of each component at the previous step of calculation, will have the following form:

$$\begin{aligned} \{\varphi^p\}^T &= [\vartheta_{A,V}^p \cdot \varphi_{A,V}^p \quad \vartheta_{V,B}^p \cdot \varphi_{V,B}^p] = \\ &= [\vartheta_{A,V}^p \cdot (2 \cdot \varphi_C^p + \varphi_V^p) \quad \vartheta_{V,B}^p \cdot (\varphi_V^p + 2 \cdot \varphi_D^p)]. \end{aligned} \quad (44)$$

Let us consider the objective functions that make up the column vector  $\{\varphi^p\}$ .

Obviously, for the node  $V$  the function  $\varphi_V$  is defined as a distance to a certain established point  $T$  (using formula (6)):

$$\varphi_V = ((x_T - x_V)^2 + (y_T - y_V)^2)^{1/2}. \quad (45)$$

Here, we don't have to use the coefficient  $\vartheta$ , because it is already counted as a logical

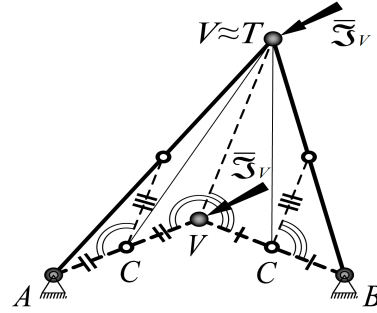
operator in the expression (44).

For nodes  $C$  and  $D$  objective functions can no longer be defined as the distances to the centers of segments  $AT$  and  $TB$ , because these distances are always equal (see. Fig. 2).

For the selection of objective functions of scalar potential in nodes  $C$  and  $D$  we will use the following fact. Obviously, in case of coincidence of node  $V$  with point  $T$ , vertices of triangles  $ACT$  and  $BDT$  will be placed on straight lengths  $AT(V)$  and  $BT(V)$  respectively. Then will be valid the following equations:

$$AT = AC + CT, \quad (46)$$

$$BT = AD + DT. \quad (47)$$



**Fig. 2.** The rod frame  $ACVDB$ :

- 1) at the moment of formation (dashed);
- 2) in corrected state (solid)

However, until the equalities (46) and (47) are not valid, iterative calculation should be continued. The values, that will characterize proximity to the completion of the calculation, will be objective function in the nodes  $C$  and  $D$ . The following differences will be served by them:

$$\begin{aligned} \varphi_C &= AT - (AC + CT) = \\ &= ((x_T - x_A)^2 + (y_T - y_A)^2)^{1/2} - \\ &- ((x_C - x_A)^2 + (y_C - y_A)^2)^{1/2} - \\ &- ((x_T - x_C)^2 + (y_T - y_C)^2)^{1/2}, \end{aligned} \quad (48)$$

$$\begin{aligned} \varphi_D &= BT - (BD + DT) = \\ &= ((x_T - x_B)^2 + (y_T - y_B)^2)^{1/2} - \\ &- ((x_D - x_B)^2 + (y_D - y_B)^2)^{1/2} - \\ &- ((x_T - x_D)^2 + (y_T - y_D)^2)^{1/2}. \end{aligned} \quad (49)$$

Should be added, that algorithmic implementation of the system (5) can be somewhat simplified by excluding the coordinates of fictitious nodes  $C$  and  $D$  from the calculating process. To do this we have to make two steps:

1) to bring the system (18) – (20) to the static equations describing the equilibrium of node  $V$  only:

$$\begin{aligned} \mathbf{n}_{A,V} \cdot s_A + \mathbf{n}_{V,B} \cdot s_B - \\ - (\mathbf{n}_{A,V} + \mathbf{n}_{V,B}) \cdot s_V + \mathfrak{S}_{sV} = 0, \end{aligned} \quad (50)$$

from where we can easily determine the coordinates of node  $V$ .

2) to rewrite the formulas (48) and (49), taking into account equations (18) and (20):

$$\begin{aligned} \varphi_C &= ((x_T - x_A)^2 + (y_T - y_A)^2)^{1/2} - \\ &- \left( \left( \frac{x_A + x_V}{2} - x_A \right)^2 + \left( \frac{y_A + y_V}{2} - y_A \right)^2 \right)^{1/2} - \end{aligned} \quad (51)$$

$$\begin{aligned} - \left( \left( x_T - \frac{x_A + x_V}{2} \right)^2 + \left( y_T - \frac{y_A + y_V}{2} \right)^2 \right)^{1/2}, \end{aligned}$$

$$\begin{aligned} \varphi_D &= ((x_T - x_B)^2 + (y_T - y_B)^2)^{1/2} - \\ &- \left( \left( \frac{x_V + x_B}{2} - x_B \right)^2 + \left( \frac{y_V + y_B}{2} - y_B \right)^2 \right)^{1/2} - \end{aligned} \quad (52)$$

$$\begin{aligned} - \left( \left( x_T - \frac{x_V + x_B}{2} \right)^2 + \left( y_T - \frac{y_V + y_B}{2} \right)^2 \right)^{1/2}. \end{aligned}$$

The values of internal forces in the rods of construction can be determined from the formula (1):

$$R_{i,j} = \mathbf{n}_{i,j} \cdot \delta_{i,j}. \quad (53)$$

Let's consider a few options of determining the components of SSS of rod frame  $ACVDB$ .

The values of the load vector, which act-

ing on node  $V$ , the value the initial rigidity parameters of the frame, as well as rigidity parameters, nodes coordinates and internal efforts after adjustment of position of node  $V$ , are shown in Table 1. The initial and corrected frames are shown in Figure 3.

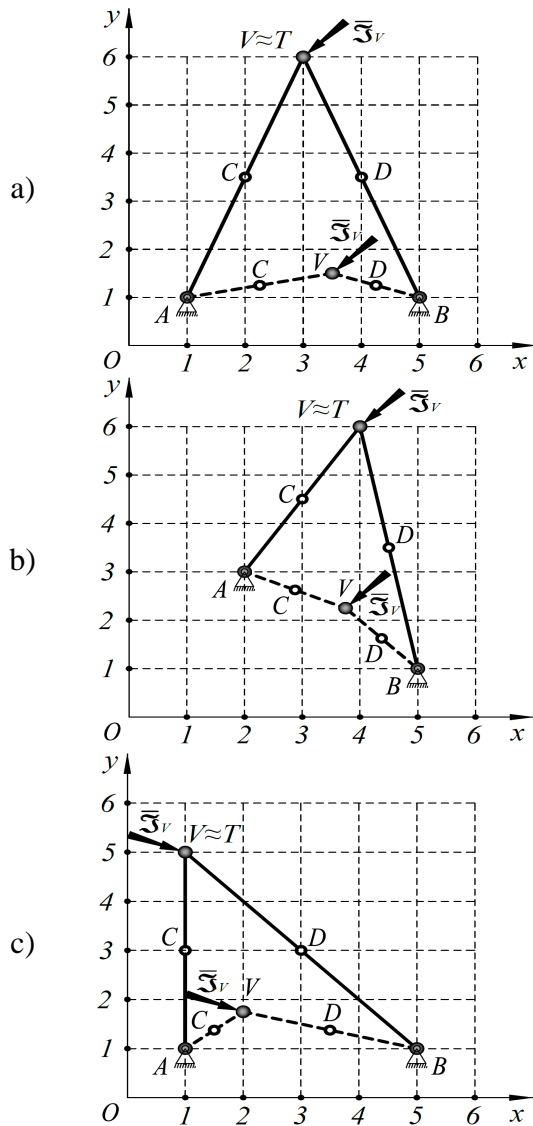
**Table 1**

Parameters of SSS of the frame		The load in the node V		
		$\mathfrak{S}_{xV} = -1$	$\mathfrak{S}_{yV} = -1$	$\mathfrak{S}_{sV} = 2$
Variant →		a)	b)	c)
Incoming	$x_A$	1	2	1
	$x_B$	5	5	5
	$x_T$	3	4	1
	$y_A$	1	3	1
	$y_B$	1	1	1
	$y_T$	6	6	5
	$\mathbf{n}_{A,V}$	-1	-2	-1
	$\mathbf{n}_{V,B}$	-1	-2	-1
Iterations		150	250	150
Calculated	$x_C$	2.00001	3.00216	1.00159
	$x_V$	3.00003	4.00433	1.00318
	$x_D$	4.00001	4.50216	3.00159
	$y_C$	3.49998	4.4961	2.99946
	$y_V$	5.99995	5.99219	4.99893
	$y_D$	3.49998	3.4961	2.99946
	$\mathbf{n}_{A,V}$	-0.35	-0.46085	0.25007
	$\mathbf{n}_{V,B}$	0.15	0.07594	-0.50023
	$R_{A,V}$	-1.8848	-1.65975	1.00002
$R_{V,B}$	0.80776	0.38657	-2.82822	

## CONCLUSIONS

Demonstrated method allows not only to correct the shape of pre-formed structures, but also to determine its internal efforts in the rods. At the same time shown approach to choice potential objective functions can be greatly varied, giving scope for the ingenuity of engineers and researchers. In addition, it is possible to use logical operators in correcting the position of nodes of constructions.

All of these provides for the possibility of application of the suggested method not only in tasks of theoretical and structural mechanics, but also in other fields of science and technology.



**Fig. 3.** Variants of frames ACVDB, formed under given in Table 1 input parameters and after correction

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ФОРМООБРАЗОВАНИЕ И КОРРЕКЦИЯ  
ПЛОСКИХ СТЕРЖНЕВЫХ КОНСТРУКЦИЙ  
С НЕБОЛЬШИМ КОЛИЧЕСТВОМ  
СВОБОДНЫХ УЗЛОВ

**Аннотация.** Публикация освещает практические аспекты метода корректировки формы плоских стержневых конструкций, который следует применять после их предварительного формообразования. Метод позволяет определять компоненты напряженно-деформированного состояния конструкции и имеет ту же математическую основу, что и метод вырезания узлов теоретической меха-

ики. Также, в статье продемонстрирован принцип использования метода на примере корректировки формы элементарной конструкции с шарнирным соединением стержней. Пример показывает преимущества данного метода над методами численного моделирования, так как не требует смены инструментальной базы при переходе от формообразования конструкции к определению усилий в её стержнях.

**Ключевые слова:** геометрическое моделирование, дискретная модель, стержневые рамные конструкции, дифференциальные закономерности, численное моделирование.