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Mathematical model-building of relax processes taking place in concrete mix at vibro impact compact deformation

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Summary. In the article is examined the modified cementing system at vibro impact compact method of compression. It is solving the task of system's movement taking into consideration relax processes. It has been considered the physical sense of relax processes taking place at vibro impact compact deformation and it is defined the character of the internal parameters. It has been received complete system of equations describing the course of modified cementing system of concrete mix in the process of vibro impact compact compression.
Key words. Modified concrete mix, vibro impact compact method of compression, relax processes, phenomenological formulas of movement

ACTUALITY OF RESEARCH AND PROBLEM DEFINITION.

The course of modified concrete mix at vibro impact compact deformation in comparison with the course of Newtonian liquids has definite peculiarities [11, 14, 16]. Particularly at a simple move there are normal stresses in a modified concrete mix, which is not observed in Newtonian liquids. At a simple move the viscosity of modified cementing systems, as a rule, is decreasing and a simple extension it is increased and it depends both on applied stresses (or gradients of velocity) and the type of the flow [2, 18]. It is also observed the range of "non classical" effects while considering the non-stationary movement of the system [4, 7, 15, 21, 22, 23].

PURPOSE AND OBJECT, MATERIALS AND RESULTS OF INVESTIGATIONS.

If known internal parameters characterizing the system, the constitutive equation can be defined with a necessary precision. But in general case we cannot define the universal connection between the tensor of stresses and tensor of the gradients of velocity neglecting the internal parameters.

It should be marked the evident properties of a liquid phase of cementing system of zero range which constitutive equation in the first order has the view [15]

$$\sigma_{ik}(\xi, t) = -p\delta_{ik} + \varepsilon_0\xi\delta_{ik} + 2(\varepsilon_1 + \varepsilon_2\xi)\, v_{(ik)}. \qquad (1)$$

As the liquid phase of cementing system has only scalar internal parameters which change with the change of the system density [15] the viscosity is not changing at a shift deformation

$$\frac{d\xi}{dt} = -\frac{1}{\tau}(\xi - \xi_0) + (\alpha_1 + \alpha_2\xi)\, v_{ii}. \qquad (2)$$

Non-Newtonian behavior can be discovered at the other types of deformations [9-11, 15]. Shift viscosity of a liquid phase of cementing system of zero rank of the second order depends on the gradient of the velocity as the shift in the second order leads to the appearance of diagonal components of tensor of stresses and at the same time to the change of the volume; this in its turn leads to the change of indexes of internal parameters defining the measurable value of the viscosity [8, 12, 19-21]. The peculiarity of a liquid phase of a zero rank is its isotropy in the process of the flow.

The peculiarities of a liquid phase of a zero rank do not answer the peculiarities of a modified cementing system because in the process of the flow the cementing system becomes anisotropic and it's necessary to appeal to the systems of higher rank for its description.

Without the analysis of the obtained system of equations for the system of the second rank in the general aspect [15], let's clarify the physical sense of relax process on the ground of the views of building of cementing system taking place at vibro impact compact deformation and define the character of internal variables [5, 6, 13, 14, 16].

While building the theory of the behavior of cementing system of compressed modified concrete mix was used that circumstance that hydrated cement grain is the macrosystem which can be described with the help of phenomenological concerns: free energy, index of friction [1-3, 17, 18].

Let considered system consist of hydrated grains of the cement with the equal density. In the cementing system of a modified concrete mix in the balanced state the hydrated grains of the cement together with the chemical modificator form floccules the sizes of which let's take as the equal to mean squared radius of inertia $\overline{R}^2$. It can be also taken the effective radius of floccules r*, having defined it as the distance from the center of the floccules on which the density in comparison with its index is decreasing twice in the center. The effective radius is connected with the mean squared radius by the ratio:

$$r^* = 0{,}64\left(\overline{R}^2\right)^{1/2}. \quad (3)$$

The sizes of the floccules in the cementing system depend on the character of intermolecular interaction and differ from the sizes of floccules which are in the cement-water paste of a usual concrete mix. But as intermolecular interactions in a modified cementing and general cement-water paste do not differ fundamentally, it can be supposed that in the equilibrium the indicated sizes do not differ considerably i.e. coincide according to the order of an index.

There are forces of different nature between the hydrated grains as it leads to the appearance of labile floccules having final life cycle and connecting all the grains into uniform system in which can be the floccules of different type (which differ by the time of life) connected by a definite type of interaction.

Thus, the cementing system in the process of physical modification can be introduced by effective system with labile floccules the number of which depends on the temperature, pressure and applied stress. Let's consider the law of the change of the number of floccules of one of the types in detail. Let it set K – a maximum possible number of floccules. Let it set the possibility of destruction and possibility of the formation of floccules for the unit of time p^- and p^+. Then the change of the number of floccules in the unit of time can be found as the difference between the number of the formed and destructed nodes:

$$\frac{dz}{dt} = -p^- z + (K - z)\, p^+ . \quad (4)$$

For the stationary case we found:

$$z_0 = \frac{p^+}{p^+ + p^-} K \approx \frac{p^+}{p^-} K . \quad (5)$$

The equation (4) can be written as:

$$\frac{dz}{dt} = -\frac{1}{\tau}(z - z_0), \quad (6)$$

where: $\tau = \dfrac{1}{p^+ + p^-}$ – time of relax depending on the applied gradients of velocity (or stresses).

At small gradients of the velocity the time of relaxation can be decomposed according to the gradients of velocity. Then from the formula (6) with the accuracy to the members of the first order according to the gradients of the velocity we got the equation (2).

Thus, one of the relaxing processes in a physically modified cementing system introduces itself the destruction and formation of floccules. The definition of the possibility of the formation of floccule is connected with the consideration of its nature.

Thus, at the first approach the building of a cementing system is described by the following indexes: mean squared index of radius of inertia $\left\langle R^2 \right\rangle_0$ of hydrated cementing grain; by the type of floccules differed by their mean life cycle; number of floccules taken as a unit of volume. Let's suppose that the considered system consists of cementing grains of an equal diameter and all the floccules have the same nature and, thus, the equal life cycle.

Let's consider the change of sizes and the form of the floccules in the stream at vibro impact compact way of compression of concrete mix.

Let's enumerate all cementing grains from 0 to z. Let set s_i^α – the distance between the neighbor grains with the numbers α–1 and α; r_i^α – the coordinate of α-floccule, thus, $s_i^\alpha = r_i^\alpha - r_i^{\alpha-1}$.

Let's formulate, first of all, the kinetic equation for the function of the possibility of finding coordinates $z+1$ of a floccules in the space at the given index of tensor of the gradients of velocity vik. The spring force acts on each of floccules:

$$2\,T\kappa\left(s_i^{\alpha+1} - s_i^\alpha\right) = -2\,T\kappa A_{\alpha\gamma} r_i^\gamma. \tag{7}$$

And the force of hydrodynamic resistance:

$$X = \zeta\left(v_i^\alpha - w_i^\alpha\right), \tag{8}$$

where: $v_i^\alpha = v_{ik} r_k^\alpha$ – the velocity of the system in the place where is α-floccule on condition that as it would not be there,

w_i^α – medium velocity of the movement of α-floccule,

κ – elastic ratio of the phase between the floccules,

$\zeta = 6\pi\eta_{(s)}\alpha$ – frictional coefficient of the floccules of the radius (α) in the medium with the viscosity $\eta_{(s)}$.

The matrix $A_{\alpha\gamma}$ is given by:

$$\left\| \begin{matrix} 1 & -1 & 0 & . & . & . & 0 \\ -1 & 2 & -1 & . & . & . & 0 \\ 0 & -1 & 2 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \end{matrix} \right\|. \tag{9}$$

The velocity of movement of hydrated grains of cement is defined on condition of the equilibrium of forces acting on a grain moreover, to real forces it should be added the effective force:

$$F_{эф} = -T\frac{\partial \ln W}{\partial r_i^\alpha}. \tag{10}$$

Then continuity equation:

$$\frac{\partial W}{\partial t} + \sum_{\alpha,i}\frac{\partial\left(w_i^\alpha W\right)}{\partial r_i^\alpha} = 0, \tag{11}$$

for the case of many particles leads to the desired equation.

Considering the viscosity of a cementing system a considerable, we drop the force of inertia and, setting equal to zero the sum of forces (7), (8) and (10), we find the expression for the medium velocity of movement of α-floccule:

$$w_i^\alpha = v_i^\alpha - \frac{2T\kappa}{\zeta}A_{\alpha\gamma} r_i^\alpha - \frac{T}{\zeta}\frac{\partial \ln W}{\partial r_i^\alpha}. \tag{12}$$

From the continuity equation (11) with the usage of (12) we find the kinetic equation for the plastic modified cementing system:

$$\frac{\partial W}{\partial t} = \sum_\alpha\left[\frac{T}{\zeta}\frac{\partial^2 W}{\left(\partial r_i^\alpha\right)^2} - \left(v_{ik} r_k^\alpha - \frac{2T\kappa}{\zeta}A_{\alpha\gamma} r_i^\gamma\right)\frac{\partial W}{\partial r_i^\alpha}\right] + \frac{12T\kappa z}{\zeta}W. \tag{13}$$

By the orthogonal transformation of coordinates $r_i^\alpha = R_{\alpha\gamma}\rho_i^\gamma$ и $\rho_i^\gamma = R_{\beta\gamma} r_i^\beta$ the matrix (9) can be taken to the diagonality with approximate chief indexes $\lambda_\alpha = \dfrac{\pi^2\alpha^2}{z^2}$. In view of properties of transformation it is true the ratio:

$$\sum_\alpha\left\langle \rho_i^\alpha \rho_k^\alpha \right\rangle = \sum_\alpha\left\langle r_i^\alpha r_k^\alpha \right\rangle. \tag{14}$$

Let's copy out the sums of different degrees of characteristic numbers:

$$\sum_\alpha \lambda_\alpha = 2z;\ \sum_\alpha \lambda_\alpha^{-1} = \frac{z^2}{6};\ \sum_\alpha \lambda_\alpha^{-2} = \frac{z^4}{90}.$$

The equation (13) has the following view in the new coordinates:

$$\frac{\partial W}{\partial t} = \sum_\alpha\left[\frac{T}{\zeta}\frac{\partial^2 W}{\left(\partial \rho_i^\alpha\right)^2} - \left(v_{ik}\rho_k^\alpha - \frac{2T\kappa\lambda_\alpha}{\zeta}\rho_i^\alpha\right)\frac{\partial W}{\partial \rho_i^\alpha}\right] + \frac{6T\kappa\lambda_\alpha}{\zeta}W. \tag{15}$$

PURPOSE AND OBJECT, MATERIALS AND RESULTS OF INVESTIGATIONS

Multiplying the equation (15) by $\rho_i^\gamma \rho_k^\gamma$ and integrating to all variables, we find the following system of equations for the moments of distribution function of the second order $\left\langle \rho_i^\alpha \rho_k^\alpha \right\rangle = \int W \rho_i^\alpha \rho_k^\alpha d\rho$:

$$\frac{d\langle \rho_i^\alpha \rho_k^\alpha \rangle}{dt} = -\frac{1}{\tau_\alpha}\left(\langle \rho_i^\alpha \rho_k^\alpha \rangle - \langle \rho_i^\alpha \rho_k^\alpha \rangle_0\right) + v_{ij}\langle \rho_k^\alpha \rho_j^\alpha \rangle + v_{kj}\langle \rho_i^\alpha \rho_j^\alpha \rangle, \quad (16)$$

where: $\tau_\alpha = \frac{\zeta}{4T\kappa\lambda_\alpha}$ – time of relaxation,

$\langle \rho_i^\alpha \rho_k^\alpha \rangle_0 = \frac{1}{2\kappa\lambda_\alpha}\delta_{ik}$ – equilibrium index for the moments of distribution function.

If the gradients of velocity disappear suddenly, the moments relax according to the law $\langle \rho_i^\alpha \rho_k^\alpha \rangle - \langle \rho_i^\alpha \rho_k^\alpha \rangle_0 \approx e^{-t/\tau_\alpha}$.

Thus, here the internal variables are tensor moments $\langle \rho_i^\alpha \rho_k^\alpha \rangle$, the law of change of which is defined by the system (16). The system (16) has the same structure as a phenomenological equation [15]:

$$\frac{d\,\xi_{ik}}{d\,t} = -\frac{1}{\tau}\left(\xi_{ik} - \xi_{ik}^0\right) + \beta_1\delta_{ik}v_{ss} + \beta_2 v_{(ik)} + \beta_3\xi_{ss}v_{(ik)} + \beta_4\delta_{ik}\xi_{js}v_{js} + \beta_5\xi_{jj}\delta_{ik}v_{ss} + \beta_6\xi_{ik}v_{ss} + \beta_7\left(\xi_{is}v_{sk} + \xi_{ks}v_{si}\right) + \beta_8\left(\xi_{is}v_{ks} + \xi_{ks}v_{is}\right). \quad (17)$$

However, some members of equation (17) disappear in this case.

Let's find the solution of the system (16) for simple shear flow ($v_{12} = 0$). At this the system (16) is given by (index of a grain for the simplicity of writing is dropped):

$$\begin{cases} \frac{d\langle \rho_1^2 \rangle}{dt} = -\frac{1}{\tau}\left(\langle \rho_1^2 \rangle - \langle \rho_1^2 \rangle_0\right) + 2v_{12}\langle \rho_1\rho_2 \rangle; \\ \frac{d\langle \rho_2^2 \rangle}{dt} = -\frac{1}{\tau}\left(\langle \rho_2^2 \rangle - \langle \rho_2^2 \rangle_0\right); \\ \frac{d\langle \rho_3^2 \rangle}{dt} = -\frac{1}{\tau}\left(\langle \rho_3^2 \rangle - \langle \rho_3^2 \rangle_0\right); \\ \frac{d\langle \rho_1\rho_2 \rangle}{dt} = -\frac{1}{\tau}\langle \rho_1\rho_2 \rangle + v_{12}\langle \rho_2^2 \rangle; \\ \frac{d\langle \rho_1\rho_3 \rangle}{dt} = -\frac{1}{\tau}\langle \rho_1\rho_3 \rangle + v_{12}\langle \rho_2\rho_3 \rangle; \\ \frac{d\langle \rho_2\rho_3 \rangle}{dt} = -\frac{1}{\tau}\langle \rho_2\rho_3 \rangle. \end{cases} \quad (18)$$

The solution of introduced system (18) is given by:

$$\langle \rho_1^2 \rangle = \langle \rho_1^2 \rangle_0\left[1 + 2\tau^2 v^2{}_{12}\left(1 - e^{-t/\tau} - \frac{t}{\tau}e^{-t/\tau}\right)\right],$$

$$\langle \rho_1\rho_2 \rangle = \langle \rho_1^2 \rangle_0 \tau v_{12}\left(1 - e^{-t/\tau}\right), \quad (19)$$

$$\langle \rho_2^2 \rangle = \langle \rho_2^3 \rangle = \langle \rho_2^2 \rangle_0 = \langle \rho_3^2 \rangle_0 ; \ \langle \rho_1\rho_3 \rangle = \langle \rho_2\rho_3 \rangle = 0 .$$

The expression for $\langle \rho_i^\alpha \rho_k^\alpha \rangle$ can be calculated directly after the definition of distribution function. The final result for the stationary case accurate to the members of the second order in accordance with the gradients of velocity:

$$\langle \rho_I^\alpha \rho_K^\alpha \rangle = \frac{1}{2\kappa\lambda_\alpha}\left[\delta_{IK} + 2\tau_\alpha v_{(ik)} + 2\tau_\alpha^2\left(2v_{(si)}v_{(sk)} - v_{(si)}v_{[sk]} - v_{(sk)}v_{[si]}\right) + \ldots\right] \quad (20)$$

The characteristic of a floccule of deformed cementing system as the whole can be served diad:

$$\langle R_i R_k \rangle = \frac{1}{z}\sum_\alpha \langle r_i^\alpha r_k^\alpha \rangle, \quad (21)$$

which is after (14) and (20), is written by the gradients of velocity in the stationary case by the following way:

$$\langle R_i R_k \rangle = \frac{z}{12\kappa}\left[\delta_{ik} + \frac{\zeta z^2}{30\kappa T}v_{(ik)} + \frac{\zeta^2 z^4}{1260T^2\kappa^2}\left(2v_{(si)}v_{(sk)} - v_{(si)}v_{[sk]} - v_{(sk)}v_{[si]}\right) + \ldots\right]. \quad (22)$$

The distribution function of density of cementing system, obviously, answers the conditions $\int\rho(r)\,dr = m\int\rho(r)\,r_i r_k dr = m\langle R_i R_k \rangle$, where m – the weight of hydrated cementing particle. Approximating ρ (r) as Gaussian function, from the formulas (22) in the system of coordinates where diad $\langle R_i R_k \rangle$ is diagonal, we get:

$$\rho(r) = \left(\frac{1}{2\pi}\right)^{3/2}\frac{m}{\left(\langle R_1^2 \rangle\langle R_2^2 \rangle\langle R_3^2 \rangle\right)^{1/2}}e^{-\frac{1}{2}\sum_{i=1}^{3}\frac{r_i^2}{\langle R_i^2 \rangle}}. \quad (23)$$

It is obvious that the distribution function of density can be approximated by triaxial ellipsoid. Defining the effective sizes of ellipsoid as per the decrease of density in two times, we find the semi-axes $a_{(i)} = 1{,}10\langle R_i^2 \rangle^{1/2}$ and effective volume of hydrated cementing grain $\Omega = 5{,}6\left(\langle R_1^2 \rangle\langle R_2^2 \rangle\langle R_3^2 \rangle\right)^{1/2}$, which depends on the gradients of velocity. The decomposition of effective volume in the gradients of velocity do not contain the members of the first order as $v_{ik} = 0$. It follows that at the smallest gradients of velocity the flocculе is deformed in the flow without any change in the volume, and then the volume of a floccules is increasing with the increase of the gradient of the velocity.

If the gradients of velocity are absent, all non-diagonal components $\langle R_i R_k \rangle$ are equal to zero and the diagonal ones become equal one to each other. At this the distribution function of density transforms into spherically symmetric function:

$$\rho(r) = m\left(\frac{3}{2\pi\langle R^2\rangle_0}\right)^{3/2} e^{-\frac{3}{2}\frac{r^2}{\langle R^2\rangle_0}}. \quad (24)$$

The additional dissipation of energy in deformed cementing system takes place because of the connection of punctiform centers of friction into unified whole. This means the presence of additional punctiform forces in the liquid phase distributed in volume.

The averaged tensor of stresses of a system σ_{ik} can be calculated by the average index of a tensor of the flow of impulse [4] $\sigma_{ik} = \rho v_i v_k - \Pi_{ik}$. All the indexes are average here. We define non-averaged ones by the stroke.

In the linear as per the velocity (stokes) approximation:

$$\sigma_{ik} = \frac{1}{V}\int \sigma'_{ik} dV. \quad (25)$$

Let the density of volumetric forces acting on deformed cementing system $\sigma_i(x)$, then the law of conservation of momentum is given by:

$$\frac{\partial(\rho v'_i)}{\partial t} + \frac{\partial \Pi'_{il}}{\partial x_l} = \sigma_i(x). \quad (26)$$

The substitution of the expression for the tensor of flow of impulse leads to the formula:

$$\frac{\partial(\rho v'_i)}{\partial t} + \frac{\partial(\rho v'_i v'_l)}{\partial x_l} = \frac{\partial \sigma'_{ij}}{\partial x_i} + \sigma_{i(x)}. \quad (27)$$

Neglecting in the equation (27) inertial members, we obtain:

$$\frac{\partial \sigma'_{il}}{\partial x_l} = -\sigma_{i(x)}. \quad (28)$$

To find the averaged value of tensor of stresses, we multiply both parts of the equation (28) by x_k, integrate over the volume $\int \frac{\partial \sigma'_{il}}{\partial x_l} x_k dV = -\int \sigma_i x_k dV$ and describe the left part of the last ratio

$$\int \frac{\partial(\sigma'_{il} x_k)}{\partial x_l} dV - \int \sigma'_{ik} dV = -\int \sigma_i x_k dV. \quad (29)$$

The first integral in the left part of the equation (29) we convert into the integral over the surface of the frame and obtain $\oint \sigma'_{il} x_k df_l - \int \sigma'_{ik} dV = -\int \sigma_i x_k dV$.

Thus, we find the average value of tensor of stresses:

$$\sigma_{ik} = \frac{1}{V}\int \sigma'_{ik} dV = \frac{1}{V}\oint P_i x_k df + \frac{1}{V}\int \sigma_i x_k dV. \quad (30)$$

where: P – the force acting on the unit surface area of volume.

The first integral which is to the right in the equation (30) and which is the tensor of stresses connected only with the surface forces and, thus, the expression (30) can be rewritten as:

$$\sigma_{ik} = \sigma^0_{(ik)} + \frac{1}{V}\int \sigma_i x_k dV, \quad (31)$$

where: $\sigma^{(0)}_{ik}$ – tensor of stresses at the absence of volumetric forces and the last member in the expression (31) describes additional stresses appearing while adding volumetric forces with the density σ_i.

The forces acting on a liquid phase in the considering case of a cementing system can be defined as punctiform. Thus, the density of forces:

$$\sigma_i(x) = \sum_{a_\alpha} \psi_i^{a_\alpha} \delta\left(q_k^a + r_k^{a_\alpha} - x_k\right), \quad (32)$$

where: q_k^a – coordinate of center of masses of a-floccule of a cementing system,

$r_k^{a_\alpha}$ – coordinate of a_α-floccule,

$\psi_i^{a_\alpha}$ – a force acting on α-hydrated grain of a cement of a_α-floccule.

Calculating with the help of (32) the integral in the expression (31), we obtain:

$$\int \sigma_i x_k dV = \sum_a q_k^a \sum_\alpha \psi_i^{a_\alpha} + \sum_{a_\alpha} \psi_i^{a,\alpha} r_k^\alpha. \quad (33)$$

In the frames of one floccule $\sum_\alpha \psi_i^{a_\alpha} = 0$. Further, as all the floccules we consider the equal ones and in the last summand the summation by the floccules can be substituted by the multiplication by the number of floccules and, thus:

$$\frac{1}{V}\int \sigma_i x_k dV = n\sum_\alpha \psi_i^\alpha r_k^\alpha, \quad (34)$$

where: n – the number of floccules in the unit of volume.

The obtained expression describes the additional stresses caused by the presence of connected centers of friction at their momentary location.

For the connected floccules with the total volume φ the volumetric forces disappear when the velocities of floccules and the velocity of the flow in the point where is the floccule coincide. This will happen only then when the connections between the floccules disappear, the system thixotropic thins down, and the tensor of stresses of which can be defined and, thus, for such cementing system:

$$\sigma_{ik}^{(0)} = -p\delta_{ik} + 2\eta_{(s)}(1+1{,}5\varphi)v_{(ik)}. \qquad (35)$$

The force acting on a liquid phase is equal in value and reciprocal in sign of force with which the liquid phase acts on the grain of the cement:

$$\psi_i^\alpha = -\zeta\left(v_i^\alpha - w_i^\alpha\right) \qquad (36)$$

And, thus, the extra term after averaging-out is given by:

$$-n\zeta\sum_\alpha \left\langle \left(v_i^\alpha - w_i^\alpha\right) r_k^\alpha \right\rangle, \qquad (37)$$

where: the brackets as previously define averaging-out in coordinates of all floccules with the help of the distribution function. Using the expressions (12) and (15), we calculate the value (35) and write the tensor of stresses for deformed cementing system as:

$$\sigma_{ik} = -p\delta_{ik} + 2(1+1{,}5\varphi)\eta_{(s)}v_{(ik)} + \\ + \frac{1}{2}n\zeta\sum_\alpha \frac{1}{\tau_\alpha}\left(\left\langle \rho_i^\alpha \rho_k^\alpha \right\rangle - \left\langle \rho_i^\alpha \rho_k^\alpha \right\rangle_0\right). \qquad (38)$$

The obtained constitutive equation is a private case of general expression (1) with tensor internal parameter.

$$\begin{aligned} \sigma_{ik}(\xi_{js}, t) &= -p\delta_{ik} + q\xi_{jj}\delta_{ik} + \mu\left(\xi_{ik} - \xi_{ik}^0\right) + \\ &+ \zeta_1\delta_{ik}v_{ss} + \zeta_2 v_{(ik)} + \zeta_3\xi_{ss}v_{(ik)} + \\ &+ \zeta_4\delta_{ik}\xi_{js}v_{js} + \zeta_5\xi_{jj}\delta_{ik}v_{ss} + \zeta_6\xi_{ik}v_{ss} + \\ &+ \zeta_7\left(\xi_{is}v_{sk} + \xi_{ks}v_{si}\right) + \zeta_8\left(\xi_{is}v_{ks} + \xi_{ks}v_{is}\right) + \\ &+ \zeta_9\left(\xi_{is}v_{sk} - \xi_{ks}v_{si}\right) + \zeta_{10}\left(\xi_{is}v_{ks} - \xi_{ks}v_{is}\right) - \\ &- \tfrac{2}{3}\left(\zeta_9 + \zeta_{10}\right)\xi_{ss}v_{[ik]}. \end{aligned} \qquad (39)$$

The ratio (16) and (36) together with the equations of movement and continuity form complete system of equations describing the flow of modified cementing system of concrete mix in the process of vibro impact compact compression.

CONCLUSIONS

On the ground of the results of the experimental theoretical investigations:

1. It has been defined the physical sense of relaxing processes and the character of internal variables in the process of vibro impact compact deformation.

2. It is built the theory of the behavior of cementing system of compressed modified concrete mix where the hydrated cementing grain is the macrosystem which can e described with the help of phenomenological terms: free energy and index of friction. The cementing system of concrete mix in the process of physical modification is introduced as the effective system with labile floccules, the number of which depend on the temperature, pressure and applied stress.

3. It has been obtained the constitutive equation which is a private case of a general expression with tensor internal parameter. The obtained equations together with the equations of movement and continuity form complete system of equations describing the flow of modified cementing system of concrete mix in the process of vibro impact compact compression.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ РЕЛАКСАЦИОННЫХ ПРОЦЕССОВ, ПРОИСХОДЯЩИХ В БЕТОННОЙ СМЕСИ ПРИ ВИБРО-УДАРНОИМПУЛЬСНОЙ ДЕФОРМАЦИИ

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Аннотация. В статье рассматривается модифицированная цементная система при вибро-ударноимпульсном способе её уплотнения. Решается задача движения системы с учётом релаксационных процессов. Рассмотрен физический смысл релаксационных процессов, происходящих при вибро-ударноимпульсной деформации, и определён характер внутренних переменных. Получена полная система уравнений, описывающая течение модифицируемой цементной системы бетонной смеси в процессе вибро-ударноимпульсного уплотнения.

Ключевые слова. Модифицированная бетонная смесь, вибро-ударноимпульсный способ уплотнения, релаксационные процессы, феноменологические уравнения движения.