

Modelling of the 2D convective heat exchange between subjected to freezing and to following defrosting logs and the surrounding environment

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Abstract: *Modelling of the 2D convective heat exchange between subjected to freezing and to following defrosting logs and the surrounding environment.* A 2D mathematical model for the computation of the temperature on the logs' surfaces perpendicular to the radius, t_{sr} , and perpendicular to the frontal side, t_{sp} , and also the non-stationary temperature distribution in the longitudinal sections of subjected to freezing and following defrosting logs at convective exponentially changing boundary conditions has been presented. The model includes mathematical descriptions of the thermal conductivity in radial and longitudinal directions, λ_r and λ_p , the effective specific heat capacity, c_e , and the density, ρ , of the non-frozen and frozen wood, and also of the heat transfer coefficient between the surrounding air environment and the radial and longitudinal directions of horizontally situated logs, α_r and α_p respectively. With the help of the model, as an example, computations have been carried out for the determination of α_r , α_p , t_{sr} , t_{sp} , λ_{sr} , λ_{sp} , and the temperature change in the center of beech log with diameter of 0.24 m, length of 0.48 m, initial temperature 20 °C, and moisture content 0.6 kg·kg⁻¹, during its 50 h freezing and its 50 h following defrosting at an exponentially changing air temperature during freezing from 20 °C to -20 °C and during defrosting from -20 °C to = 20 °C.

Keywords: 2D modelling, beech log, freezing, defrosting, heat transfer coefficient, surface temperature, temperature distribution

INTRODUCTION

It is known that the duration of the thermal treatment of the frozen logs in the winter aiming at their plasticizing for the production of veneer and also the energy consumption needed for this treatment depend on the degree of the logs' icing [1, 2, 9, 10].

In the specialized literature there are very limited reports about the temperature distribution in subjected to defrosting frozen logs [4, 8] and there is no information at all about the temperature distribution in logs during their freezing. That is why the modelling and the multi parameter study of the freezing process of logs are of considerable scientific and practical interest.

The aim of the present work is to present a 2D mathematical model for the computation of the temperature on the cylindrical logs' surfaces and the non-stationary temperature distribution in the longitudinal sections of subjected to freezing logs at convective exponentially changing boundary conditions. For the achieving of this goal, as a base, a model of the heating and cooling processes of logs is used, which has been suggested and modified earlier by the first co-author [3, 4].

MECHANISM OF THE 2D HEAT DISTRIBUTION IN LOGS SUBJECTED TO FREEZING AND FOLLOWING DEFROSTING

The mechanism of the heat distribution in logs during their heating or cooling can be described by the equation of the heat conduction [3, 4]. When the length of the logs does not exceed their diameter by at least 3 ÷ 4 times, then the heat transfer through the frontal sides of the logs can not be neglected, because it influences the change in temperature of their cross sections, which are equally distant from the frontal sides [1, 2]. In such cases, for the calculation of the change in the temperature in the longitudinal sections of the logs (i.e. along

the coordinates r and z of these sections, refer to Fig. 1) during their freezing the following 2D model can be used [4]:

$$c_e \rho \frac{\partial T}{\partial \tau} = \lambda_r \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{\partial \lambda_r}{\partial T} \left(\frac{\partial T}{\partial r} \right)^2 + \lambda_p \frac{\partial^2 T}{\partial z^2} + \frac{\partial \lambda_p}{\partial T} \left(\frac{\partial T}{\partial z} \right)^2 \quad (1)$$

with an initial condition

$$T(r, z, 0) = T_0 \quad (2)$$

and boundary conditions for convective heat transfer:

- along the radial coordinate r on the logs' surface during freezing and defrosting process:

$$\frac{\partial T(r, 0, \tau)}{\partial r} = - \frac{\alpha_p^{\text{fr}}(r, 0, \tau)}{\lambda_{\text{sp}}(r, 0, \tau)} \left[T_{\text{sp}}^{\text{fr}}(\tau) - T_m^{\text{fr}}(\tau) \right] \quad (3)$$

$$\frac{\partial T(r, 0, \tau)}{\partial r} = \frac{\alpha_p^{\text{dffr}}(r, 0, \tau)}{\lambda_{\text{sp}}(r, 0, \tau)} \left[T_{\text{sp}}^{\text{dffr}}(\tau) - T_m^{\text{dffr}}(\tau) \right]. \quad (4)$$

- along the longitudinal coordinate z on the logs' surface during freezing and defrosting process:

$$\frac{\partial T(0, z, \tau)}{\partial z} = - \frac{\alpha_r^{\text{fr}}(0, z, \tau)}{\lambda_{\text{sr}}(0, z, \tau)} \left[T_{\text{sr}}^{\text{fr}}(\tau) - T_m^{\text{fr}}(\tau) \right] \quad (5)$$

$$\frac{\partial T(0, z, \tau)}{\partial z} = \frac{\alpha_r^{\text{dffr}}(0, z, \tau)}{\lambda_{\text{sr}}(0, z, \tau)} \left[T_{\text{sr}}^{\text{dffr}}(\tau) - T_m^{\text{dffr}}(\tau) \right]. \quad (6)$$

The mathematical model of the logs' freezing process, which consists of eqs. (1) ÷ (6) can be solved without any simplification with the help of the explicit form of the finite-difference method [2, 4]. For this purpose the calculation mesh can be built on $\frac{1}{4}$ of the longitudinal section of the log due to the circumstance that this $\frac{1}{4}$ is mirror symmetrical towards the remaining $\frac{3}{4}$ of the same section (Figure 1).

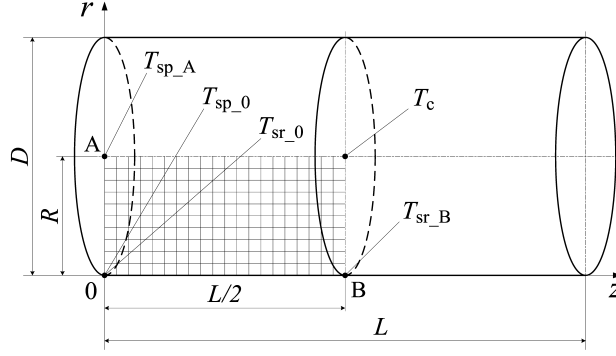


Figure 1. Positioning of the knots of the calculation mesh on $\frac{1}{4}$ of the longitudinal section of a log subjected to freezing and following defrosting

For the solving of the mathematical model (1) ÷ (6) it is needed to have mathematical descriptions of all variables in it. Such descriptions are given and explain below.

MATHEMATICAL DESCRIPTION OF THE TEMPERATURE OF THE FREEZING AND DEFROSTING AIR MEDIUMS

It is possible to have two cases for freezing of different materials in freezers. The first case is when the material is put into a working freezer with constant unchanged temperature in it and, consequently, the freezing medium temperature $T_m^{fr}(\tau) = T_{m0}^{fr} = \text{const}$.

The mathematical model (1) ÷ (6) obtains more complicated boundary conditions in the second case, when the material is put into a non-working freezer and after that the freezer is switched on. In this case, the temperature of the air environment in the freezer T_m^{fr} decreases exponentially with time according to the equation

$$T_m^{fr} = T_{m1}^{fr} + (T_{m0}^{fr} - T_{m1}^{fr}) \exp\left(-\frac{\tau}{\tau_{exp}^{fr}}\right). \quad (7)$$

The defrosting of the frozen materials after opening of the freezer's door is realized at exponential increasing of the air temperature according to the equation

$$T_m^{dfr} = T_{m1}^{dfr} - (T_{m1}^{dfr} - T_{m1}^{fr}) \exp\left(-\frac{\tau - \tau_{fr}}{\tau_{exp}^{dfr}}\right). \quad (8)$$

MATHEMATICAL DESCRIPTION OF THE HEAT TRANSFER COEFFICIENTS BETWEEN THE AIR AND THE LOGS DURING THEIR FREEZING AND DEFROSTING

The freezing and defrosting of the wood materials at atmospheric environment or in a freezer takes place in the conditions of free convection. For the calculation of the heat transfer coefficients in such conditions of heating or cooling of horizontally situated logs (refer to Fig. 1) Chudinov [1] suggests the following equations:

$$\alpha_r^{fr}(0, z, \tau) \approx 0.997 \sqrt[4]{\frac{\Delta T}{R}} = 0.997 \sqrt[4]{\frac{T(0, z, \tau) - T_m^{fr}(\tau)}{R}}, \quad (9)$$

$$\alpha_r^{\text{dfr}}(0, z, \tau) = 0.997 \sqrt[4]{\frac{T_m^{\text{dfr}}(\tau) - T(0, z, \tau)}{R}}, \quad (10)$$

$$\alpha_p^{\text{fr}}(r, 0, \tau) \approx 3.49 + 0.093\Delta T = 3.49 + 0.093[T(r, 0, \tau) - T_m^{\text{fr}}(\tau)] \text{ at } \Delta T \leq 10 \text{ K}, \quad (11)$$

$$\alpha_p^{\text{dfr}}(r, 0, \tau) = 3.49 + 0.093[T_m^{\text{dfr}}(\tau) - T(r, 0, \tau)] \text{ at } \Delta T \leq 10 \text{ K}, \quad (12)$$

$$\alpha_p^{\text{fr}}(r, 0, \tau) \approx 2.486 \sqrt[4]{\Delta T} = 2.486[T(r, 0, \tau) - T_m^{\text{fr}}(\tau)] \text{ at } \Delta T > 10 \text{ K}, \quad (13)$$

$$\alpha_p^{\text{dfr}}(r, 0, \tau) = 2.486[T_m^{\text{dfr}}(\tau) - T(r, 0, \tau)] \text{ at } \Delta T > 10 \text{ K}. \quad (14)$$

More precise equations for the determination of the heat transfer coefficients between the freezing and defrosting air mediums and logs' surfaces in radial direction and in direction parallel to the wood fibers can be obtained after suitable experiments have been carried out.

MATHEMATICAL DESCRIPTION OF THE THERMO-PHYSICAL CHARACTERISTICS OF THE LOGS

The solution of the non-linear 2D mathematical model of the logs' freezing and defrosting processes, which is presented by equations (1) ÷ (14), can be realized using the given in [2, 4, 5] mathematical descriptions of the effective heat capacity of the frozen and non-frozen wood, c_e , the density of frozen and non-frozen wood, ρ , and the thermal conductivity of the frozen and non-frozen wood, λ_r and λ_p . With the help of the mathematical description of λ_r and λ_p , the current values of the thermal conductivities on the logs' surfaces $\lambda_{sr}(0, z, \tau)$ and $\lambda_{sp}(r, 0, \tau)$, which participate in eqs. (3) ÷ (6) can be calculated also during the solving of the model.

RESULTS AND DISCUSSION

The suggested above mathematical descriptions of T_m^{fr} , T_m^{dfr} , α_r^{fr} , α_r^{dfr} , α_p^{fr} , and α_p^{dfr} are introduced in the earlier created and later modified by the first co-author non-stationary model of the heating and cooling of cylindrical wood materials [3, 4]. This model is presented in common form by the eqs. (1) ÷ (6). The updated model with the descriptions of T_m^{fr} , T_m^{dfr} , α_r^{fr} , α_r^{dfr} , α_p^{fr} , and α_p^{dfr} has been solved with the help of explicit schemes of the finite difference method in a way, analogous to the one used and described in [4] for the solution of a model of the heating and cooling process of cylindrical wood materials. For the solution of the updated model a software program has been prepared in the calculation environment of Visual Fortran Professional.

With the help of the program as an example computations have been carried out for the determination of the 2D change of the temperature in the longitudinal sections of subjected to 50 hours freezing at $t_{m1}^{\text{fr}} = -20$ °C and following 50 h defrosting at $t_{m2}^{\text{dfr}} = 20$ °C beech (*Fagus Silvatica* L.) log.

The freezing and defrosting processes of beech log with a diameter of $D = 0.24$ m (i.e. with radius of $R = 0.12$ m), length of $L = 0.48$ m, moisture content $u = 0.6$ $\text{kg} \cdot \text{kg}^{-1}$, and initial temperature $t_0 = 20$ $^{\circ}\text{C}$ have been studied. A log with such u contains maximum possible quantity of bound water and contains a significant quantity of free water, too.

The decreasing of t_m^{fr} from the value of $t_{m0}^{\text{fr}} = t_0 = 20$ $^{\circ}\text{C}$ to $t_{m1}^{\text{fr}} = -20$ $^{\circ}\text{C} = \text{const}$ and the following increasing of t_m^{dfr} from $t_{m1}^{\text{fr}} = -20$ $^{\circ}\text{C} = \text{const}$ to $t_{m1}^{\text{dfr}} = 20$ $^{\circ}\text{C} = \text{const}$ go exponentially with time constants $\tau_{\text{exp}}^{\text{fr}} = \tau_{\text{exp}}^{\text{dfr}} = 3600$ s. The calculated according to eqs. (7) and (8) exponential change of t_m^{fr} and t_m^{dfr} with these time constants can be seen on the Figure 2 for the curve of t_m .

The calculations have been done with average values of basic density of beech wood $\rho_b = 560$ $\text{kg} \cdot \text{m}^{-3}$ and fiber saturation point at 293.15 K (i.e. at 20 $^{\circ}\text{C}$) of this wood $u_{\text{fsp}}^{293.15} = 0.31$ $\text{kg} \cdot \text{kg}^{-1}$ [7].

For the computations of the log's freezing and defrosting processes the mathematical descriptions of the thermal conductivity, the effective heat capacity and the density of the subjected to defrosting wood have been used [2, 4, 5]. The not large difference (so named hysteresis) between these thermo-physical characteristics during freezing and defrosting of the wood [1] needs to be additionally studied, mathematically described, and input into the updated model.

On Figure 2 the computed change in the freezing and defrosting medium temperatures, t_m^{fr} and t_m^{dfr} (see curve t_m), in the surface temperatures at points 0, A and B (refer to Fig.1) of the log (see curves t_{sp_A} , t_{sr_B} and $t_{\text{sp}_0} = t_{\text{sr}_0}$) and also in the temperature in the central point of the log, t_c , during the freezing and defrosting, is shown. On Figure 3 the computed change in the heat transfer coefficients between the log's surfaces in longitudinal and radial directions and also the change in the thermal conductivities in longitudinal and radial wood directions at points 0, A, and B on the log's surfaces and in the log's center (refer to Fig. 1), during the freezing and defrosting processes, is shown.

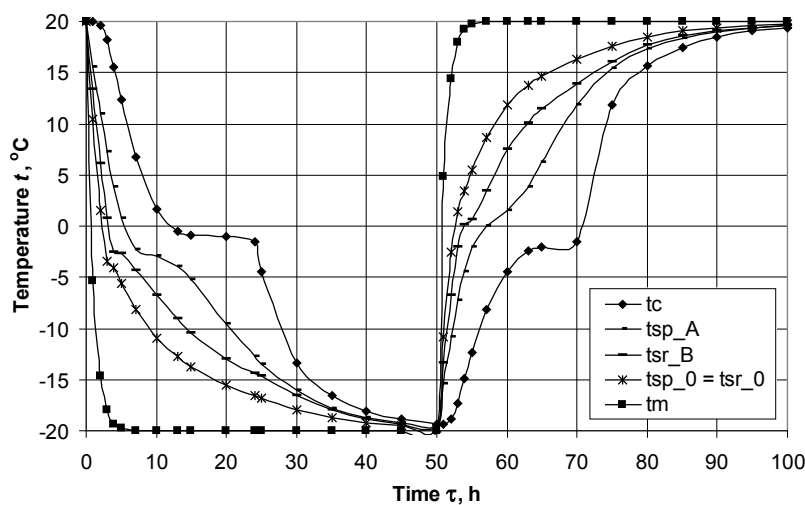


Figure 2. Change in t_m , t_{sr} , t_{sp} , and t_c of beech log with $D = 0.24$ m, $L = 0.48$ m, $u = 0.6$ $\text{kg} \cdot \text{kg}^{-1}$, and $t_0 = 20$ $^{\circ}\text{C}$ during its 50 h freezing at -20 $^{\circ}\text{C}$ and during its 50 h following defrosting at 20 $^{\circ}\text{C}$, depending on τ

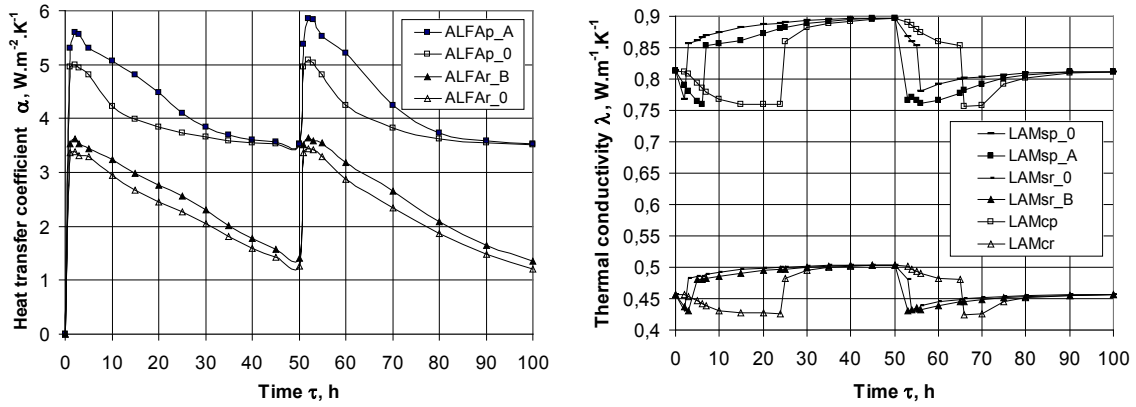


Figure 3. Change in α_r and α_p (left) and in λ_{sp} , λ_{sr} , λ_{cp} , and λ_{cr} (right) of beech log with $D = 0.24$ m, $L = 0.48$ m, $u = 0.6$ kg·kg⁻¹, and $t_0 = 20$ °C during its 50 h freezing at -20 °C and during its 50 h following defrosting at 20 °C, depending on τ

The obtained results lead to the following conclusions:

1. On the curve of situated on the log's centre characteristic point on Fig. 2 the specific almost horizontal section of retention of the temperature t_c for a long period of time in the range from -1 °C to -2 °C can be seen, while at this point a complete freezing of the free water in the wood occurs and also after that while at this point a complete defrosting of the frozen free water occurs. Such retention of the temperature on the logs' axis has been observed in wide experimental studies during the defrosting process of pine logs containing ice from the free water [8].

2. The character of the change in the heat transfer coefficients, α_p and α_r is almost identical during the studied freezing and defrosting processes of log with a given value of the wood moisture content (Fig. 3-left). According to eqs. (9) ÷ (14) the current values of these coefficients depend mainly on the current differences ΔT between the freezing or defrosting medium temperature and the temperature in the concrete point on the log's surface. With the increase of the duration of the freezing processes α_p and α_r decrease because of the decreasing of ΔT .

An important point of note is that at $\Delta T = 0$ in eqs. (11) and (12) $\alpha_p^{fr}(r,0,\tau) = \alpha_p^{dfr} = 3.49$ (refer to Fig. 2-left). This is in contradiction to the physical law of thermal exchange between the log's surface and the air environment because at $\Delta T = 0$ this exchange must be equal to 0 such as in eq. (9), (10), (13), and (14). This means that eqs. (11) and (12) need further clarification.

3. The character of the change in the wood thermal conductivity on the log's surfaces, λ_{sp} and λ_{sr} , and in the log's centre, λ_{cp} and λ_{cr} , is very complex (Fig. 3-right). The current values of these conductivities depend not only on the wood moisture content and on the current temperature at the respective log's points, but also on the momentous aggregate condition of the water at these points [5]. The larger values of λ_{sp} , λ_{sr} , λ_{cp} and λ_{cr} on Fig. 3-right related to the frozen log's surface or central point, and the lower values of λ_{sp} , λ_{sr} , λ_{cp} and λ_{cr} related to the log's points with non-frozen free water in them at respective moments.

CONCLUSIONS

This paper presents 2D mathematical model for the computation of the temperature on the logs' surfaces and the non-stationary temperature distribution in the longitudinal section of subjected to freezing and to following defrosting logs at convective exponentially changing boundary conditions. As a base, a model of the heating and cooling processes of logs is used,

which has been created and modified earlier by the first co-author. The mechanism of the heat distribution in the logs during their freezing and defrosting is described by the 2D partial differential equation of heat conduction. For the numerical solution of the model a software program has been prepared in the calculation environment of Visual Fortran Professional. With the help of the program computations have been carried out for the determination of the 2D change in the temperature of the longitudinal section of beech log with $D = 0.24$ m, $L = 0.48$ m, $u = 0.6$ kg·kg⁻¹, and $t_0 = 20$ °C during its 50 hours freezing at exponentially decreasing air temperature until reaching of -20 °C and during the following 50 h defrosting at exponentially increasing air temperature until reaching of 20 °C. The results presented on the figures in this paper show that the procedures for calculation of the non-stationary 2D temperature change in the prepared software program functions well for the mutually connected processes of the freezing and the defrosting of the logs at convective boundary conditions. The obtained results show the complex character of the change in the temperature on the logs' surfaces and in the longitudinal logs' section, and also of the heat transfer coefficient between the logs' surfaces and the processing freezing or defrosting air environment. Also the change in the wood thermal conductivity on the logs' surfaces and at the separate points in the logs, especially strong depending on the aggregate condition of the water at each point at every moment of the studied processes, has a very complex character.

The created model can be used for a science-based determination and automatic control [4, 6] of the duration of the logs' freezing processes at different initial and boundary conditions.

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SYMBOLS

c = specific heat capacity (J·kg⁻¹·K⁻¹)

D = diameter (m)

exp = exponent

L = length (m)

r = radial coordinate (m): $0 \leq r \leq R$

R = radius (m)

t = temperature (°C): $t = T - 273.15$

T = temperature (K): $T = t + 273.15$

u = moisture content (kg·kg⁻¹): $u = W/100$

W = moisture content (%): $W = 100u$

z = longitudinal coordinate (m): $0 \leq z \leq L/2$

α = heat transfer coefficient between the log's surface and the surrounding air environment

(W·m⁻²·K⁻¹)

Δ = difference (for the temperature)

λ = thermal conductivity (W·m⁻¹·K⁻¹)

ρ = density (kg·m⁻³)

τ = time or time constant of the exponent (s)

SUBSCRIPTS AND SUPERSSCRIPTS

A = point on the logs' surface and longitudinal axe (refer to Fig. 1)

b = basic (for density, based on dry mass divided to green volume)

B = point on the logs' surface and radial axe (refer to Fig. 1)

c = center (for the temperature or the thermal conductivity on the logs' center)
 dfr = defrosting
 e = effective (for the specific heat capacity of the frozen and non-frozen wood)
 exp = exponent (for the time constant of the exponentially change in the air temperature)
 fr = freezing (for the temperature or for the heat transfer coefficient of the processing medium)
 fsp = fiber saturation point of the wood
 m = medium (for the temperature of the freezing air)
 m0 = initial (for the medium temperature at the beginning of the logs' freezing or defrosting)
 m1 = end (for the medium temperature at the end of the logs' freezing or defrosting)
 p = parallel to the wood fibers
 r = radial direction
 sp = surface on the direction parallel to the wood fibers
 sr = surface on the radial direction
 0 = initial (for the radial, longitudinal, and time coordinates or for the average mass temperature of the logs at the beginning of the freezing process)
 293.15 = at 293.15 K, i.e. at 20 °C (for the value of the fiber saturation point of wood species)

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Streszczenie: *Modelowanie konwekcyjnej wymiany ciepła pomiędzy kłódami w procesie zamrażania i odmrażania, a otoczeniem.* Zaproponowano dwuwymiarowy model matematyczny rozkładu temperatur na przekrojach kłód prostopadłych do ich pobocznic i czół, w warunkach konwekcyjnej wymiany ciepła z otoczeniem przy ich zamrażaniu i odmrażaniu. Model zawiera opisy matematyczne przewodności cieplnej w kierunkach wzdłużnym i poprzecznym, pojemności cieplnej i gęstości drewna zamrożonego i niezamrożonego, oraz konwekcyjnej wymiany ciepła z otoczeniem w tych dwóch kierunkach.
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