DENDROBIOLOGY

2021, vol. 86, 8–18

https://doi.org/10.12657/denbio.086.002

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Predicting parameters of Weibull probability density function for diametric distributions in *A. melanoxylon*, *E. camaldulensis*, and *E. nitens* bioenergy plantation

Received: 12 March 2021; Accepted: 1 July 2021

Abstract: Precise modeling of stand diameter distributions is required to provide accurate estimates of volume per diameter class and unit area. Therefore, it is necessary to obtain the most accurate probability density functions parameters estimates to predict stand diameter distribution in time. We evaluate two methods to estimate the parameters of the Weibull probability density function in the modeling of diameter distributions of bioenergy plantations. The methods considered a direct method of parameter prediction based on regression models (PPRM) and an indirect method of parameter recovery through the determination of percentiles (PRDP). Both methods are considered systems of linear equations and are adjusted through simultaneous estimation of parameters using stand variables. The greatest precision was obtained with PPRM. The PRDP method was not effective in the prediction of diameter distributions due to the high level of truncation of the observed distributions showing an overestimation of the distribution for the largest diameter classes. Estimated parameters of the Weibull PDF are directly related to mean height, quadratic mean diameter, and crop age; and are inversely related to stocking.

Keywords Short-rotation crops, stocking, seemingly unrelated regression, parameters recovery, biomass and bioenergy

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Introduction

The ability to predict the diameter distribution in a stand is essential for silvicultural decision makers to make the best forests management (Cao, 2004). Mathematical modeling is used to predict growth and yield of a given plantation considering stand variables such as stocking, basal area, dominant height and stand diameter distribution (Gove & Patil, 1998). Being the most direct and measurable factor, diameter is often related to other important variables, such as basal area, density, and volume. This makes the diameter distribution model a useful tool to provide more detailed information about the stand without additional inventory costs (Nord-Larsen & Cao, 2006). Diameter distribution over time may be used to determine stand state variables such as basal area, volume or biomass per unit area (Mehtätalo, 2004).

Various theoretical probability density functions (PDF) have been used to describe stand diameter distribution of plantations. However, the Weibull PDF is the most commonly used. Bailey and Dell (1973) were the first to use this function to describe stand diameter distributions of plantations under traditional silviculture. Since then, the Weibull PDF has been frequently used in the field of forestry due to its ability to predict a wide variety of distribution forms. More so, the Weibull PDF has a closed form cumulative distribution function (CDF), which made easy to estimate the proportion of trees in diameter classes.

Precise modeling of stand diameter distributions is required to provide accurate estimates of volume per diameter class and unit area (Cao, 2004; Jiang & Brooks, 2009; Parresol, 2003; Zhang et al., 2003). Therefore, it is necessary to obtain the most accurate PDF parameters estimates in order to predict stand diameter distribution in time. Smalley and Bailey (1974) and Schreuder et al. (1979) were pioneers in the prediction of the Weibull function parameters based on empirical functions using stand variables. Since then, efforts to predict the parameters have been done explicitly, i.e., from linear models using stand attributes as explanatory variables (stocking, mean height, index of site or age).

An indirect technique known as parameter recovery has also been used to determine PDF parameters (Hyink & Moser, 1979; Lohrey & Bailey, 1977). Other authors, such as Zhang and Liu (2006) who using the modified Weibull two-parameter model of Minowa and Hirata (1993), or the procedure proposed by Zhang et al. (2008) who use a combination of Weighted Least Squares Estimation Methods and Monte Carlo experiments to estimate the Weibull parameters, argue that they are a good option to fit the diameter distribution in mature uneven age stands, however these methods are not sufficient when a frequency distribution is multimodal or irregular in shape. According to some authors, the procedure for parameter prediction is not highly precise, although the models used in the predictive process have some biological basis (Cao & Burkhart, 1984; Sun et al., 2019). Studies by those authors have shown low coefficients of determination for the linear models used to predict the parameters of location (a), scale (b), and shape (c) of the Weibull function.

Therefore, in order to improve these estimates, Lohrey and Bailey (1977) and Hyink and Moser (1979) proposed the parameter recovery technique. This method does not directly predict the parameters of the PDF; but rather, the functions estimate parameters that are directly related to the distribution, such as: a) central and non-central moments (Cao et al., 1982; Diamantopoulou et al., 2015; Lynch & Moser, 1986) the methods proposed by Teimouri et al. (2013) and Teimouri et al. (2020) for the estimation for three-parameters Weibull distribution, based on TL-moments and L-moments, who argue that their proposal gives the best overall performance with respect to the maximum likelihood estimation; b) a set of percentiles (Bailey et al., 1989; Baldwin Jr. & Feduccia, 1987). Borders and Patterson (1990) justified the use of this predictive method, indicating that there is a close relationship between the percentiles of the distribution and the plantation attributes. Teimouri et al. (2020) reported that among the estimators investigated for two-parameters Weibull distribution, the method of percentiles outperformed other competitors.

Therefore, the method of parameter recovery came into use replacing the parameter prediction method (Bailey et al., 1989; Brooks et al., 1992; Knowe et al., 1997; Leduc et al., 2001; Lee & Coble, 2006; Lohrey & Bailey, 1977; Sun et al., 2019). Nonetheless, this method is also deficient for estimating the parameters that define a PDF, mainly in truncated distributions and where there are class types without frequency. Several authors have agreed on the limitations of parameter recovery, indicating that it is inefficient in comparison with the parameter prediction method (Borders & Patterson, 1990; Cao, 2004; Jiang & Brooks, 2009; Nepal & Somers, 1992; Vanclay, 1995).

To date, several studies have compared the precision achieved with the methods of parameter prediction and parameter recovery (Cao, 2004; Jiang & Brooks, 2009; Newton & Amponsah, 2005; Nord-Larsen & Cao, 2006; Palahí et al., 2006). However, previous research has only considered data for traditional plantations managed for sawtimber or pulp production in terms of stocking and rotation length. At present, no research has been reported for biomass production plantations for bioenergy considering high stockings and short rotations. There is increased interest in modeling biomass growth and yield for short-rotation crops.

Woody crops for biomass production for bioenergy have only been recently established in experimental areas and potential for establishing these crops on an operational scale and the most adequate strategy for projecting their growth and yield is still unknown. The objective of our study was to analyze the precision of two alternatives for parameter prediction overtime applying the methodology used by Jiang and Brooks (2009) to model diameter distributions of three species (*Acacia melanoxylon* R. Br., *Eucalyptus camaldulensis* Dehnh., and *Eucalyptus nitens* Deane & Maiden) established at three stockings (5000, 7500, 10000 trees ha^{-1}) for bioenergy purposes.

Materials and methods

Site characteristics and establishment of the study

The trial was established in August 2007 in the interior rainfed of the Ñuble Region, Ninhue Township, Chile (36°17'37.86"S 72°22'55.13"W). The site presents nutritional and water limitations and is predominately characterized by low yield forest plantations dedicated to pulp or sawtimber production. The site was previously occupied by a 24-year-old *Pinus radiata* D. Don plantation, had a mean annual rainfall of 1324 mm and minimum, mean, and maximum mean annual temperatures of 0.0 °C, 11.3 °C, and 23.5 °C, respectively. Soils are Cauquenes soil family series derived from granitic rocks and are classified as a mesic Ultic Palexeralfs (Alfisol). Soils are deep (> 100 cm), well drained and well evolved of clay textures throughout the profile (CIREN, 1999). The terrain has a rolling to abrupt topography but the slope in the study area does not exceed 5%.

Prior to beginning of the trial, the site was prepared by extracting the stumps of the previous crop and subsoiled in a grid layout to 80 cm depth using a Caterpillar D8K tractor with 60 cm distance between rows. Weeds were controlled at pre-and post-planting using a chemical mixture containing 4 kg ha^{-1} glyphosate (Roundup Max), 1.5 kg ha^{-1} simazine, and 2.5 kg ha^{-1} atrazine. Post-planting fertilization included 15 g boronatrocalcite, 75 g diammonium phosphate and 25 g sulpomag applied in a circle at 25 cm from the collar of each plant. A 1.2 m height fence buried about 0.3 m deep was built to protect the study area from animals.

The trial was established as a complete randomized block design with three replicates. Blocks were square areas of 75 m at each side (5625 m^2) consisting of nine experimental units of 25 m per side (625 m^2) with 49 measurement trees, and a buffer zone to reduce edge effects. Three species (*A. melanoxylon*, *E. camaldulensis*, and *E. nitens*) were established in each block at three stocking (5000, 7500, and 10000 trees ha^{-1}).

Tree measurements

Individual tree measurements at each experimental unit were made in October and December 2007, and July and December of 2008, 2009, 2010 and July 2011. At each measurement time collar diameter (D) at 0.1 m above the ground, diameter at breast height (DBH) once the trees were taller than 1.3 m, crown diameter, and total height of all the trees were measured for each experimental unit. In this study we used only collar diameter (D) for all analyses. Data from October 2007 measurements was excluded from the analysis because the lack of diameter classes at this stage of stand development.

Methods of parameter prediction

Diameter data, time of measurement and planting density were tabulated into classes of amplitude two for each species.

From the records tabulated in relative cumulative frequencies, the Weibull´s CDF of three parameters was adjusted for each one data grouping, according to the results obtained by Sandoval et al. (2012) (Eq. 1).

$$CDF = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \text{ with } x \geq a,\ a \geq 0,\ b > 0,\ c > 0, \quad [1]$$

where x is the diameter class, a is the parameter of location, b is the parameter of scale, and c is the parameter of shape. As suggested by Frazier (1981), the parameter of location was restricted to predict the minimum value of D in the plantation according to the expression $\hat{a} = 0.5\ x_{\min}$. Direct and indirect methods for the parameters prediction of Weibull function were evaluated.

For the direct method, consisting on prediction of parameters based on regression models (PPRM), we used the equations proposed by Jiang and Brooks (2009) expressed by Eq. 2, Eq. 3, and Eq. 4:

$$\hat{a} = \exp[b_{10} + b_{11}\ln(D_q) + b_{12}\ln(A) + b_{13}\ln(1/H_m)] \quad [2]$$

$$\hat{b} = \exp[b_{20} + b_{21}\ln(D_q) + b_{22}\ln(N) + b_{23}\ln(A) + b_{24}(1/H_m)] \quad [3]$$

$$\hat{c} = \exp[b_{30} + b_{31}\ln(D_q) + b_{32}\ln(N) + b_{33}\ln(A) + b_{34}(1/H_m)] \quad [4]$$

where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are the parameters of location, scale, and shape, respectively, estimated using the method of parameter prediction. D_q is the quadratic mean diameter, N is the number of trees per surface unit, A is the age of the crop expressed in months from the establishment of the plantation, and H_m is the mean height of the crop.

For indirect method, denominated parameter recovery, consisted of the prediction of Weibull distribution parameters based on the determination of percentiles (PRDP). The methodology proposed by Bailey et al. (1989), and recently used to describe diameter distributions in young plantations of *Pinus palustris* Mill. (Jiang & Brooks, 2009), was based on the determination of four percentiles of the distribution of diameters, i.e., x_0, x_{25}, x_{50}, and x_{95}, that is, the value of the diameter that accumulated 0, 25, 50, and 95%, respectively (Eq. 5, Eq. 6 and Eq. 7):

$$\hat{a} = \frac{\mathrm{N}^{1/3} x_0 - x_{50}}{\mathrm{N}^{1/3} - 1} \qquad [5]$$

$$\hat{b} = \frac{\hat{a}\Gamma_1}{\Gamma_2} + \sqrt{\left(\frac{\hat{a}}{\Gamma_2}\right)^2 (\Gamma_1^2 - \Gamma_2^2) + \frac{D_q^2}{\Gamma_2}} \qquad [6]$$

$$\hat{c} = \frac{\ln(-\ln(1 - 0.95)) - \ln(-\ln(1 - 0.25))}{\ln(x_{95} - a) - \ln(x_{25} - a)} \qquad [7]$$

where $\Gamma_1 = \Gamma(1 + 1 / \hat{c})$ and $\Gamma_2 = \Gamma(1 + 2 / \hat{c})$, with Γ being the gamma function. In turn, the four percentiles were estimated based on regression models that used stand variables for the prediction. Jiang and Brooks (2009) proposed the following equations (Eq. 8, Eq. 9, Eq. 10 and Eq. 11):

$$x_0 = \exp[b_{10} + b_{11} \ln(D_q) + b_{12} \ln(A) + b_{13} \ln(1 / H_m)] \qquad [8]$$

$$x_{25} = \exp[b_{20} + b_{21} \ln(x_{50}) + b_{22} \ln(A)] \qquad [9]$$

$$x_{50} = \exp[b_{30} + b_{31} \ln(D_q)] \qquad [10]$$

$$x_{95} = \exp[b_{40} + b_{41} \ln(x_{50}) + b_{42} \ln(A) + b_{43} \ln(1 / H_m)] \qquad [11]$$

For both methods (PPRM and PRDP), the functions were adjusted using simultaneous parameters estimation, as proposed by Borders (1989). The adjustments were done using the option seemingly unrelated regression (SUR) of the procedure model of SAS (SAS Institute Inc., 2018).

Data analysis

The precision of each one of the parameter prediction methods was evaluated in terms of the relative probability observed for each of the distributions. Thus, the Weibull CDF was expressed as a PDF (Eq. 12):

$$PDF = \left(\frac{c}{b}\right)\left(\frac{x - a}{b}\right)^{c-1} \exp\left[-\left(\frac{x - a}{b}\right)^{c}\right], \qquad [12]$$

The precision of the methods was evaluated through the root mean square error (*RMSE*) and the error index (*EI*) developed by Reynolds et al. (1988). The *RMSE* was used as an indicator of precision for each PDF estimated with each method, and the *EI* was used to determine the sum of absolute weighted differences between observed and estimated distributions. As in Mehtätalo (2004), herein, we used the basal area as a factor of weighting, incorporating the modification done in Sandoval et al. (2012). The *RMSE* (Eq.13) and *EI* were calculated (Eq.14), respectively.

$$RMSE = \sqrt{\sum_{i=1}^{n} \left(F(x_i) - \hat{F}(x_i)\right)^2 / n}, \qquad [13]$$

$$EI = \sum_{i=1}^{n} \frac{gi}{G} \left|F(x_i) - \hat{F}(x_i)\right| \qquad [14]$$

In these criteria, $F(x_i)$ and $\hat{F}(x_i)$ are the relative observed and estimated frequencies, respectively; n is the number of diametric classes; and g_i and G are the basal area of each diameter class and the total basal area of this distribution, respectively.

Results

PPRM method resulted in the greatest precision of all three species studied (Table 1). This result was consistent regardless the indicator used (*RMSE* and *EI*). The *EI* underscored that direct estimation generally resulted in estimates of greater precision for the larger size-classes, whose individuals were of greater interest for commercial purposes. Better results were only found using the PRDP method for *A. melanoxylon* planted at 10000 trees ha^{-1}, in this case larger diameter frequencies were estimated better. In general, overestimation of diameter distributions frequencies with the PRDP method were observed for larger diameter classes.

The precision determined through the PPRM method varied with stocking. For all three species, the greatest precision was obtained for 5000 trees ha^{-1}. In *A. melanoxylon*, *E. camaldulensis*, and *E. nitens*, the estimation errors did not exceed 2.2, 1.9, and 2.3%, respectively. On the other hand, for *A. melanoxylon* and *E. nitens*, the lowest precision was obtained for 7500 trees ha^{-1} (2.7 and 3.7%, respectively), whereas for *E. camaldulensis*, the average error was 2.3% for 10000 trees ha^{-1}. The precision of the adjusted Weibull PDF revealed estimation errors that fluctuated between 1.4 and 3.5%, which was lower than the error obtained with PPRM (between 1.9 and 3.7%). These results showed that PPRM was precise in relation to the results obtained with the fit of the Weibull PDF (Table 1).

Although the precision of PPRM varied between stockings, no clear tendency was observed that showed any relationship between the precision of the estimate and stocking (Table 1). For *A. melanoxylon* and *E. nitens*, we observed greater precision in the estimation of the frequency of the distribution observed consecutively at stockings of 5000, 10000, and 7500 trees ha^{-1},

Table 1. Precision of the parameter prediction methods obtained at 48 months since the establishment of the study

Species	Stocking (trees ha^{-1})		Methods				Weibull PDF	
			PPRM		PRDP			
	Nominal	Actual	RMSE	IE	RMSE	IE	RMSE	IE
A. melanoxylon	5000	3095	0.0221	0.0197	0.0318	0.0266	0.0201	0.0177
	7500	3979	0.0266	0.0166	0.0353	0.0192	0.0195	0.0137
	10000	6734	0.0226	0.0154	0.0279	0.0146	0.0175	0.0100
E. camaldulensis	5000	4591	0.0190	0.0146	0.0245	0.0161	0.0136	0.0098
	7500	6989	0.0203	0.0135	0.0294	0.0170	0.0179	0.0118
	10000	9523	0.0225	0.0160	0.0297	0.0187	0.0151	0.0099
E. nitens	5000	2823	0.0226	0.0167	0.0307	0.0195	0.0195	0.0156
	7500	3061	0.0366	0.0264	0.0455	0.0319	0.0351	0.0264
	10000	4625	0.0270	0.0159	0.0365	0.0184	0.0245	0.0136

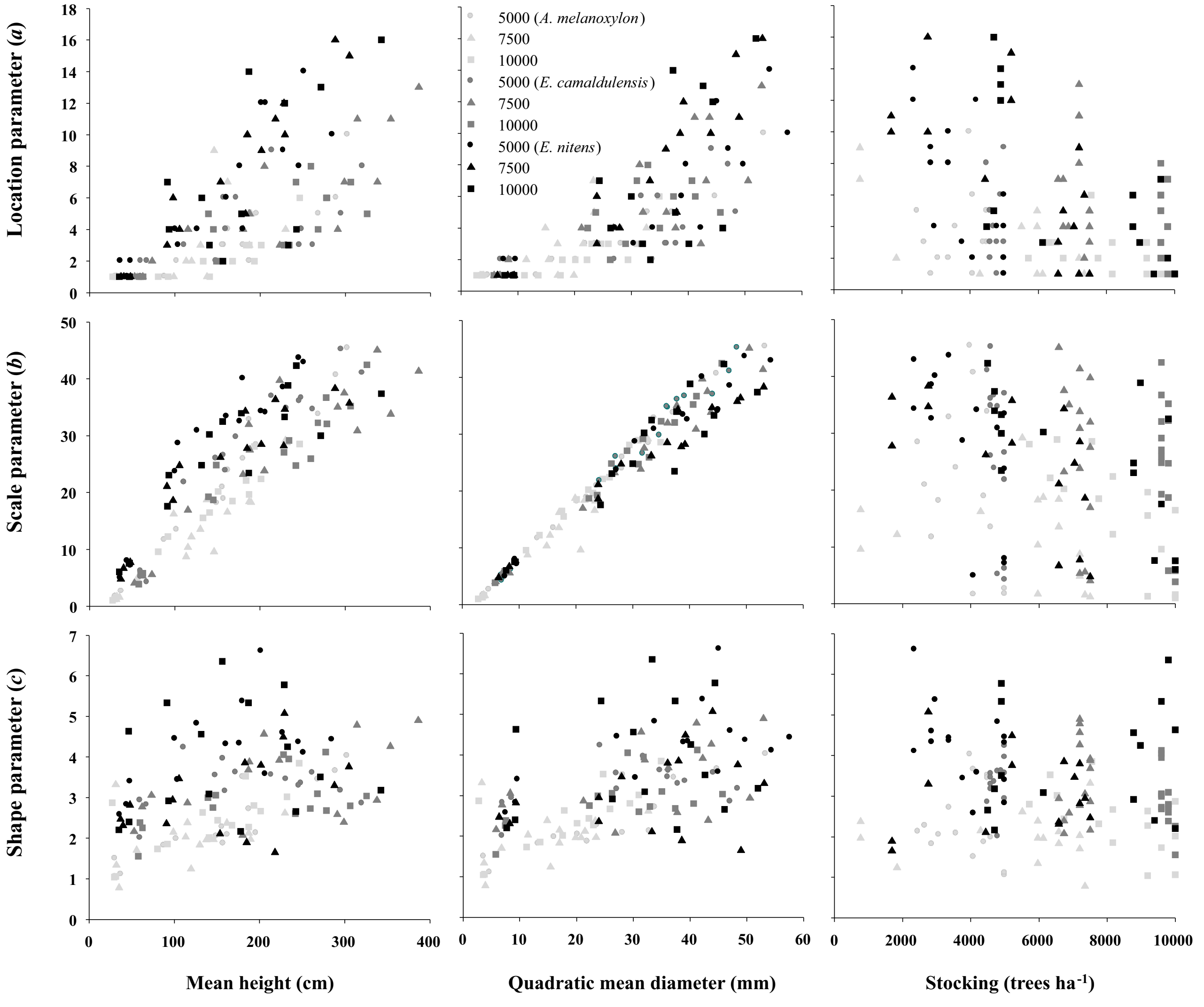

Fig. 1. Relationship between parameters of the Weibull PDF and stand variables for the three species at the three stocking

Table 2. Estimation of parameters in the system of equations for PPRM method

Parameter	Stocking	*A. melanoxylon*					RMSE	*E. camaldulensis*					RMSE	*E. nitens*					RMSE
$\hat{a}$		b_{10}	b_{11}	b_{12}	b_{13}			b_{10}	b_{11}	b_{12}	b_{13}			b_{10}	b_{11}	b_{12}	b_{13}		
	5000	−4.022*	0.774*	0.081ns	−19.286*		0.950	−9.846*	1.509ns	−0.261*	268.940*		1.305	−4.999*	0.867ns	0.098*	52.302*		2.343
	7500	−5.779*	1.178ns	−0.349*	117.387*		2.102	2.000*	0.091ns	0.037*	−198.378*		2.210	−1.016*	0.427ns	0.136*	−55.280*		1.440
	10000	1.359*	0.845*	−1.441*	−250.873*		0.585	7.423*	−0.879ns	0.557*	−292.471*		1.582	5.014*	−0.684ns	0.972*	−172.214*		4.021
$\hat{b}$		b_{20}	b_{21}	b_{22}	b_{23}	b_{24}		b_{20}	b_{21}	b_{22}	b_{23}	b_{24}		b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	
	5000	0.230*	0.416*	−0.043ns	−0.006ns	−22.314*	1.258	−0.157ns	0.365*	0.143ns	0.017*	−49.020*	1.464	0.559ns	0.478*	−0.027ns	−0.097*	−11.986*	2.688
	7500	−0.631ns	0.375*	0.138*	−0.095*	−37.097*	2.206	−0.968ns	0.601*	−0.001ns	−0.025*	32.103*	2.456	−0.846ns	0.567*	0.053*	−0.120*	4.858*	1.306
	10000	1.072*	0.350*	−0.109*	0.259*	−7.922*	0.818	−10.229ns	0.755*	0.917ns	−0.103*	43.172*	1.589	−1.544ns	0.752ns	0.022ns	−0.266*	22.364*	4.327
$\hat{c}$		b_{30}	b_{31}	b_{32}	b_{33}	b_{34}		b_{30}	b_{31}	b_{32}	b_{33}	b_{34}		b_{30}	b_{31}	b_{32}	b_{33}	b_{34}	
	5000	−2.049*	0.599*	−0.034*	−0.518*	35.165*	0.242	−24.547*	0.230*	2.971*	−0.325*	−12.044*	0.290	13.150*	−0.510ns	−0.709ns	−0.434*	−138.740*	0.728
	7500	−0.823*	0.126ns	0.007*	0.182ns	26.668*	0.646	−37.378*	0.355ns	4.183*	−0.346ns	0.651*	0.651	−0.001ns	−0.264ns	0.337ns	0.199*	−55.300*	0.925
	10000	−2.597*	−0.071ns	−0.262*	−0.550ns	2.022*	0.618	20.616ns	−0.023ns	−2.113*	0.059ns	−22.381*	0.546	−4.645*	−1.524*	1.469*	1.706*	−147.555*	1.110

*: Indicates significance of the parameter ($\alpha<0.05$); ns: denotes non-significance of the parameter ($\alpha\geq0.05$).

Table 3. Estimation of parameters in the system of equations for the PRDP method

Percentile	Stocking	*A. melanoxylon*				RMSE	*E. camaldulensis*				RMSE	*E. nitens*				RMSE
x_0		b_{10}	b_{11}	b_{12}	b_{13}		b_{10}	b_{11}	b_{12}	b_{13}		b_{10}	b_{11}	b_{12}	b_{13}	
	5000	−3.035ns	0.731ns	0.024ns	−0.006ns	1.865	1.851ns	0.265ns	1.094ns	0.890ns	2.867	−1.513ns	0.156ns	0.718ns	−0.185ns	4.694
	7500	−3.730ns	−0.051ns	0.245ns	−1.053ns	4.366	−4.034*	0.209ns	−0.364ns	−1.112ns	4.192	−2.329*	−0.156ns	0.822*	−0.741ns	2.321
	10000	−7.609*	1.400ns	−1.642*	−0.991ns	1.341	−2.490ns	0.022ns	0.062ns	−0.808ns	3.068	−2.320ns	−1.004ns	1.231ns	−1.631ns	7.006
x_{25}		b_{20}	b_{21}	b_{22}			b_{20}	b_{21}	b_{22}			b_{20}	b_{21}	b_{22}		
	5000	−0.792*	1.372*	−0.269*		1.306	0.038ns	0.912*	0.028ns		1.240	−0.206ns	0.963*	0.063ns		1.541
	7500	−0.640ns	1.031*	0.079ns		1.753	−0.328ns	0.985*	0.060ns		2.868	−0.732ns	1.258*	−0.155ns		3.877
	10000	−1.003*	1.194*	0.045ns		2.020	−0.341ns	1.054*	−0.016ns		1.367	−0.249ns	1.061*	−0.050ns		2.824
x_{50}		b_{30}	b_{31}				b_{30}	b_{31}				b_{30}	b_{31}			
	5000	−0.200ns	0.524*			1.090	−0.030ns	0.501*			0.496	−0.015ns	0.501*			0.842
	7500	−0.125ns	0.512*			1.003	−0.150ns	0.518*			0.757	−0.034ns	0.503*			0.845
	10000	−0.292ns	0.536*			1.217	0.026ns	0.491*			1.294	0.063ns	0.489*			0.982
x_{95}		b_{40}	b_{41}	b_{42}	b_{43}		b_{40}	b_{41}	b_{42}	b_{43}		b_{40}	b_{41}	b_{42}	b_{43}	
	5000	0.955*	0.706*	0.145*	−0.0002ns	2.580	0.269ns	0.970*	−0.017ns	−0.042ns	2.112	0.532ns	0.965*	0.068ns	0.060ns	3.189
	7500	−0.423ns	0.515*	0.008ns	−0.457ns	3.652	0.997*	0.958*	0.057ns	0.122ns	3.496	1.204*	1.171*	0.241ns	0.421ns	4.099
	10000	−0.916ns	0.252ns	0.072ns	−0.680ns	3.293	0.217ns	0.963*	−0.029ns	−0.068ns	3.528	0.503*	0.944*	0.091ns	0.052ns	5.781

*: Denotes significance of the parameter ($\alpha<0.05$); ns: denotes non-significance of the parameter ($\alpha\geq0.05$).

whereas for *E. camaldulensis*, greater precision was observed consecutively at 5000, 7500, and 10000 trees ha^{-1}. Apparently, the precision in the estimation of the relative frequency in diameter distributions was related to the degree of truncation of the observed distribution; here, we could see that *A. melanoxylon*, *E. camaldulensis*, and *E. nitens*, established at densities of 10000, 7500, and 10000 trees ha^{-1}, respectively; showed greater degrees of truncation. These results agreed with the higher errors obtained (Table 1).

The results obtained show that, for the three species analyzed, some stand variables were strongly related to the parameters estimated for the Weibull PDF (Fig. 1). It is observed that the location parameters (*a*) and scale (*b*) have a direct relationship with the mean height and mean square diameter, however, they showed weak relationship with the stocking and exhibit a tendency to decrease as stocking increased. The third parameter – shape (*c*) – also exhibits little relationship with the stand variables. The three parameters increased slightly along with the mean height and the mean square diameter, despite this, the number of trees did not seem to explain the shape parameter. Although some independent variables alone did not denote a strong relationship with the estimated parameters of the Weibull PDF, these variables in equations [1], [2] and [3] achieved high precision in the PPRM method (Table 1).

Results of equations used to estimate parameter values with PPRM and percentile values with PRDP are presented in Tables 2 and 3, respectively. Simultaneous parameter estimations showed a higher number of significant estimations PPRM than for PRDP methods. This probably caused lack of precision for PRDP. In general, PPRM showed better precision of equations estimated by parameters $\hat{a}$, $\hat{b}$, $\hat{c}$ and that was achieved for *A. melanoxylon* (equations [1], [2], and [3]). However, for PRDP, the equations predicting the 0, 25, 50, and 95 distribution percentiles ([7], [8], [9], and [10]) obtained, in general, better *RMSE* values for *E. camaldulensis*.

Discussion

In this study, all predictive equations for parameters of the Weibull PDF were adjusted using the procedure of simultaneous estimation of parameters proposed by Borders (1989). Several authors that have used this procedure agree that consideration of endogenous dependent variables and random errors correlated among the equations improve the fit and generate consistent and unbiased estimators (Borders, 1989; Bullock & Burkhart, 2005; Cao, 2004; Jiang & Brooks, 2009; Qin et al., 2006).

Results of earlier studies analyzing the precision of PPRM and PRDP methods differed. Whereas some results highlight the predictive capacity of the PRDP method , others highlight the precision and parsimony of the PPRM method. Jiang and Brooks (2009) studied both methods of parameter prediction in plantations of *Pinus palustris* Mill. using trees from 3 to 20 years of age and stockings ranging from 273 to 857 trees ha^{-1}. These authors found *EI* values of 20.9 (PPRM) and 11.8 (PRDP), disagreeing with our results, which show that PPRM is the more precise method. Cao (2004) tested six methods for predicting Weibull PDF parameters, including the PRDP method which was found to be deficient in terms of precision compared with the methods tested in this research. That author also developed an algorithm to estimate the Weibull PDF parameters through the maximum likelihood method evaluating the PPRM method; the *EI* showed more precise results than the PRDP method. Leduc et al. (2001) analyzed two similar procedures to PPRM and PRDP, obtaining similar results for both.

However, the system of linear equations with stand variables used to directly predict the Weibull PDF parameters showed better results than the method of parameters recovery evaluated using the index of fit. Several authors have mentioned that the estimation of the Weibull PDF parameters based on the percentile method leads to highly biased estimators (Lumbres & Jin Lee, 2014; Nanang, 1998;

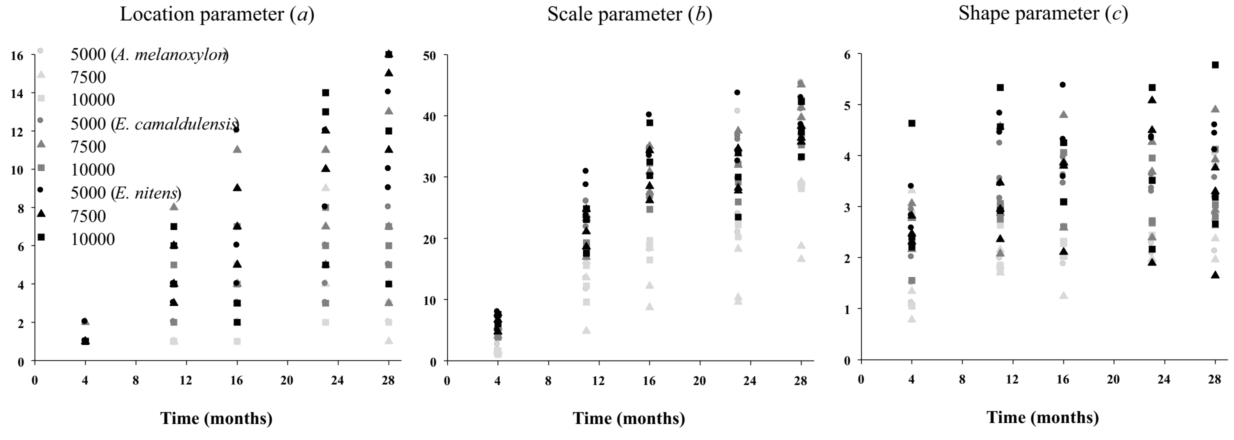

Fig. 2. Evolution of the parameters estimated with the Weibull PDF over time for the three species at the three stockings

Zarnoch & Dell, 1985; Zhang et al., 2003). In general, the PRDP method was efficient for the parameters estimation of the Weibull PDF; however, its precision was conditioned by the level of truncation of the diameter distribution (Borders & Patterson, 1990; Cao, 2004; Jiang & Brooks, 2009; Nepal & Somers, 1992; Vanclay, 1995). Therefore, given the high variability of the diameter distributions of our bioenergy crops established at high stockings, its use is not recommended (Fig. 3).

We observed for the three species a direct relationship between the estimated Weibull PDF parameters and the age of the plantation (Fig. 2). Bullock and Burkhart (2005) studying diameter distributions in juvenile *Pinus taeda* L. plantations obtained the same trends. These authors recorded that both the scale parameter (*b*) and shape (*c*) increased as the age of the plantation increased; although the shape parameter (*c*) showed a lower slope, like our results. However, Chen et al. (2019) argue that when the correlation between parameter estimation and the whole stand characteristics was weak, machine learning algorithm can well simulate nonlinear relations. In this research area, artificial neural network (ANN) models have been considered as an alternative to traditional tree diameter Weibull distribution models (Cai et al., 2010; Diamantopoulou et al., 2015; Leduc et al., 2001).

Conclusions

In this study we used the SUR method of simultaneous parameter estimation for the fit of the system of equations of both methods of parameter prediction. For PPRM the precision of these equations, evaluated based on *RMSE*, showed similar results in relation to other studies. However, with PRDP the

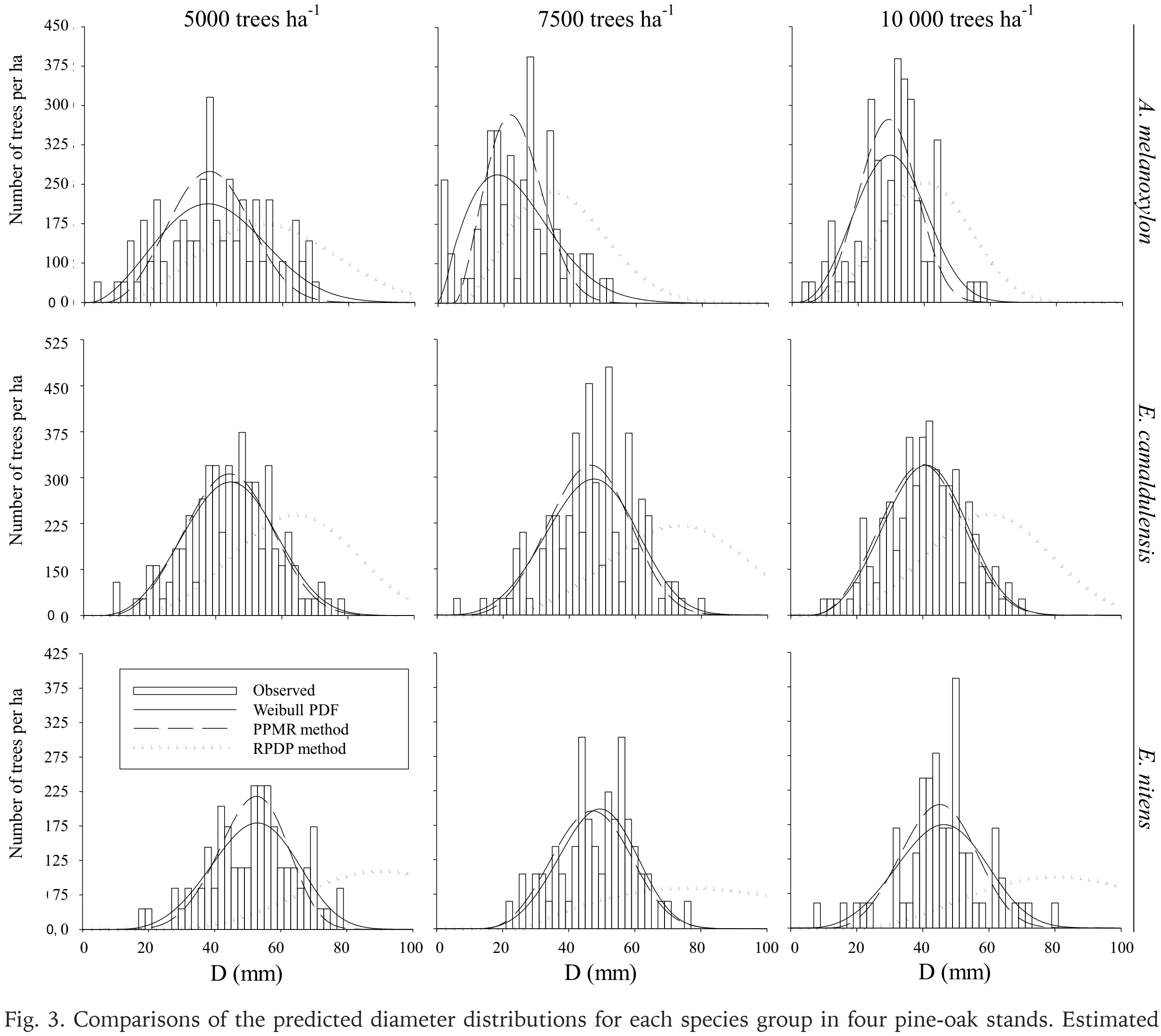

Fig. 3. Comparisons of the predicted diameter distributions for each species group in four pine-oak stands. Estimated Weibull distributions from the four methods for plot 41

system of equations was not able to explain with enough precision the distribution percentiles.

Weibull PDF parameters can be easily estimated from stand variables and these evolved in direct relation to the age of the plantation. Useful variables to predict the parameters of the Weibull PDF were mean height and mean square diameter. However, if we consider the simultaneous estimation method, the number of trees per hectare did not seem to be a useful variable to predict these parameters. To reduce the system of equations, both in PPRM and in PRDP, each parameter-variable relationship should be evaluated prior to its incorporation into the systems; therefore, improving the simultaneous fits in terms of parsimony.

The PPRM method is more appropriate for predicting the parameters estimated of the Weibull PDF. According to both, the *RMSE* and the *EI*, PPRM obtains greater precision than the PRDP method. Apparently, in the early stages of growth, diameter distributions are highly variable and truncated, therefore PRDP is not able to explain with precision the Weibull PDF parameters. For bioenergy crops established at high stockings (e.g greater than 5000 trees ha^{-1}), we recommend using PPRM to estimate the parameters of the diameter distribution using the Weibull PDF. This methodology could be useful in those cases in which part of the harvested crop may be considered for pulpwood or structural use.

Contributions of the co-authors

Conceptualization, J.C. and S.S.; formal analysis, S.S., and J.C.; investigation, S.S., E.A., R.R. and J.C.; writing-original draft preparation, S.S.; writing-review and editing, S.S. and E.A.; visualization, S.S. and J.C.; supervision, J.C.; Project administration, E.A. All authors have read and agreed to the published version of the manuscript.

Acknowledgements

The authors would like to acknowledge the technical and financial support from the forestry enterprise Masisa Forestal S.A.

Funding

This work was supported by project Innova Bio Bio N° 06-PC S1-33. Elaborations of protocols for the biomass production of short rotation woody species for the generation of bioenergy.

Declaration on conflicts of interest

The authors declare that they have no conflict of interest.

References

Bailey RL, Burgan TM & Jokela EJ (1989) Fertilized midrotation-aged slash pine plantations--stand structure and yield prediction models. Southern Journal of Applied Forestry 13: 76–80. doi:https://doi.org/10.1093/sjaf/13.2.76.

Bailey RL & Dell TR (1973) Quantifying diameter distributions with the Weibull function. Forest Science 19: 97–104. doi:10.1093/forestscience/19.2.97.

Baldwin Jr. VC & Feduccia DP (1987) Loblolly pine growth and yield prediction for managed West Gulf plantations. U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station. Research Paper SO-236, New Orleans, LA.

Borders BE (1989) Systems of equations in forest stand modeling. Forest Science 35: 548–556. doi:10.1093/forestscience/35.2.5487.

Borders BE & Patterson WD (1990) Projecting stand tables: a comparison of the Weibull diameter distribution method, a percentile-based projection method, and a basal area growth projection method. Forest Science 36: 413–424.

Brooks JR, Borders BE & Bailey RL (1992) Predicting diameter distributions for site-prepared loblolly and slash pine plantations. Southern Journal of Applied Forestry 16: 130–133.

Bullock BP & Burkhart HE (2005) Juvenile diameter distributions of loblolly pine characterized by the two-parameter Weibull function. New Forests 29: 233–244.

Cai S, Kang X, Zhang X, Gong Z, Qin L & Chen P (2010) A model for tree diameter distribution in stands based on artificial neural network: Proceedings – 2010 International Symposium on Intelligence Information Processing and Trusted Computing, IPTC 2010 (ed. by HB Liu & RH Deng) IEEE Computer Society, Conference Publishing Services (CPS) Huanggang, China, pp. 332–336.

Cao QV (2004) Predicting parameters of a Weibull function for modeling diameter distribution. Forest Science 50: 682–685. doi:10.1093/forestscience/50.5.682.

Cao QV & Burkhart HE (1984) A segmented distribution approach for modeling diameter frequency data. Forest Science 30: 129–137.

Cao QV, Burkhart HE & Lemin Jr RC (1982) Diameter distributions and yields of thinned loblolly pine plantations. School of Forestry and Wildlife Resources Publication No. FWS-1-82. Virginia Polytechnic Institute and State University, Blacksburg, VA.

Chen Y, Wu B & Min Z (2019) Stand diameter distribution modeling and prediction based on maximum entropy principle. Forests 10: 859. doi:10.3390/f10100859.

CIREN (1999) Estudio Agrológico VIII Región. Descripciones de Suelos, Materiales y Símbolos. Publicación Nº 121. Tomos I y II. Centro de Investigación de Recursos Naturales (CIREN), Santiago, Chile.

Diamantopoulou MJ, Özçelik R, Crecente-Campo F & Eler Ü (2015) Estimation of Weibull function parameters for modelling tree diameter distribution using least squares and artificial neural networks methods. Biosystems Engineering 133: 33–45. doi:10.1016/j.biosystemseng.2015.02.013.

Frazier JR (1981) Compatible whole-stand and diameter distribuition models for loblolly pine plantations, Vol. Ph.D. in Forestry: Graduate Faculty of the Virginia Polytechnic Institute and State University, Blackburg.

Gove JH & Patil GP (1998) Modeling the basal area-size distribution of forest stands: A compatible approach. Forest Science 44: 285–297. doi:10.1093/forestscience/44.2.285.

Hyink DM & Moser JW (1979) Application of diameter distributions for yield projection in uneven-aged forests: Forest Resource Inventories. Proceeding of SAF/IUFRO Workshop, Colorado State University (ed. by WE Frayer) Department of Forest and Wood Sciences, Colorado State University, FortCollins, pp. 906–916.

Jiang L & Brooks JR (2009) Predicting diameter distributions for young longleaf pine plantations in Southwest Georgia. Southern Journal of Applied Forestry 33: 25–28. doi:10.1093/sjaf/33.1.25.

Knowe SA, Ahrens GR & DeBell DS (1997) Comparison of diameter-distribution-prediction, stand-table-projection, and individual-tree-growth modeling approaches for young red alder plantations. Forest Ecology and Management 98: 49–60.

Leduc DJ, Matney TG, Belli KL & Baldwin Jr VC (2001) Predicting diameter distributions of longleaf pine plantations: A comparison between artificial neural networks and other accepted methodologies. U.S. Department of Agriculture, Forest Service, Southern Research Station, Research Paper SRS-25, Asheville, NC.

Lee YJ & Coble DW (2006) A new diameter distribution model for unmanaged loblolly pine plantations in East Texas. Southern Journal of Applied Forestry 30: 13–20.

Lohrey RE & Bailey RL (1977) Yield tables and stand structure for unthinned longleaf pine plantations in Louisiana and Texas. U. S. Department of Agriculture, Forest Service, Southern Forest Experiment Station, Research Paper SO-133, New Orleans, LA.

Lumbres RIC & Jin Lee Y (2014) Percentile-based Weibull diameter distribution model for *Pinus kesiyastands* in Benguet province, Philippines. Southern Forests: a Journal of Forest Science 76: 117–123. doi:10.2989/20702620.2014.918689.

Lynch TB & Moser JW (1986) A growth model for mixed species stands. Forest Science 32: 697–706.

Mehtätalo L (2004) An algorithm for ensuring compatibility between estimated percentiles of diameter distribution and measured stand variables. Forest Science 50: 20–32. doi:10.1093/forestscience/50.1.20.

Minowa M & Hirata Y (1993) A modified exponential distribution for describing the stand structure of Uneven-aged forests. Journal of The Japanese Forestry Society 75: 449–451.

Nanang DM (1998) Suitability of the normal, log-normal and Weibull distributions for fitting diameter distributions of neem plantations in Northern Ghana. Forest Ecology and Management 103: 1–7.

Nepal SK & Somers GL (1992) A generalized approach to stand table projection. Forest Science 38: 120–133.

Newton P & Amponsah I (2005) Evaluation of Weibull-based parameter prediction equation systems for black spruce and jack pine stand types within the context of developing structural stand density management diagrams. Canadian Journal of Forest Research 35: 2996–3010.

Nord-Larsen T & Cao QV (2006) A diameter distribution model for even-aged beech in Denmark. Forest Ecology and Management 231: 218–225. doi:10.1016/j.foreco.2006.05.054.

Palahí M, Pukkala T & Trasobares A (2006) Modelling the diameter distribution of *Pinus sylvestris*, *Pinus nigra* and *Pinus halepensis* forest stands in Catalonia using the truncated Weibull function. Forestry 79: 553–562. doi:10.1093/forestry/cpl037.

Parresol BR (2003) Recovering parameters of Johnson's SB distribution. U.S. Department of Agriculture, Forest Service, Southern Research Station. Research Paper SRS-31, Asheville, NC.

Qin J, Cao QV & Blouin DC (2006) Projection of a diameter distribution through time. Canadian Journal of Forest Research 37: 188–194.

Reynolds MR, Burk TE & Huang WC (1988) Goodness-of-fit tests and model selection procedures for diameter distribution models. Forest Science 34: 373–399.

Sandoval S, Cancino J, Rubilar R, Esquivel E, Acuña E, Muñoz F & Espinosa M (2012) Probability distributions in high-density dendroenergy plantations. Forest Science 58: 663–672. doi:10.5849/forsci.11-028.

SAS Institute Inc. (2018) SAS/STAT®15.1 User's Guide. SAS Institute Inc, Cary, NC.

Schreuder H, Hafley W & Bennett F (1979) Yield prediction for unthinned natural slash pine stands. Forest Science 25: 25–30.

Smalley GW & Bailey RL (1974) Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama, and Georgia highlands. U.S. Department of Agriculture, Forest Service, Southern Forest Experiment Station, Research Paper SO-96, New Orleans, LA.

Sun S, Cao QV & Cao T (2019) Characterizing diameter distributions for uneven-aged pine-oak mixed forests in the Qinling Mountains of China. Forests 10. doi:10.3390/f10070596.

Teimouri M, Abdolahnezhad K & Ghalandarayeshi S (2020) Evaluation of estimation methods for parameters of the probability functions in tree diameter distribution modeling. Environmental Resources Research 8: 25–40.

Teimouri M, Hoseini SM & Nadarajah S (2013) Comparison of estimation methods for the Weibull distribution. Statistics 47: 93–109. doi:10.1080/02331888.2011.559657.

Vanclay JK (1995) Synthesis: Growth models for tropical forests: A synthesis of models and methods. Forest Science 41: 7–42. doi:10.1093/forestscience/41.1.7.

Zarnoch SJ & Dell TR (1985) An evaluation of percentile and maximum likelihood estimators of Weibull parameters. Forest Science 31: 260–268.

Zhang L & Liu C (2006) Fitting irregular diameter distributions of forest stands by Weibull, modified Weibull, and mixture Weibull models. Journal of Forest Research 11: 369–372. doi:10.1007/s10310-006-0218-7.

Zhang L, Packard KC & Liu C (2003) A comparison of estimation methods for fitting Weibull and Johnson's SB distributions to mixed spruce fir stands in northeastern North America. Canadian Journal of Forest Research 33: 1340–1347.

Zhang LF, Xie M & Tang LC (2008) On weighted least squares estimation for the parameters of Weibull distribution: Recent advances in reliability and quality in design (ed. by H Pham) Springer-Verlag, London, UK, pp. 57–84.