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Wheel-rail conformal contact modeling

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Summary: Experimental tests have shown, that wheel – rail contact interaction modeling in a rail gauge corner zone with the use of halfspace - based theories can lead to a great inaccuracy in calculations. The most similar to experimental results one can obtain using FEM. However for certain reasons FEM contact solution can't be integrated directly to the railway vehicle dynamics simulation.
In the present paper the technique is developed for wheel – rail conformal contact modeling in a rail gauge corner. The difference in modeling results of presented technique from FEM results is approximately 5%. Thus the technique can be used for wheel – rail contact interaction modeling in a rail gauge corner zone during the railway vehicle dynamics simulation.
Key words: wheel, rail, conformal contact

INTRODUCTION

Numerical solutions based on Hertz theory [8,9,10,16] and Bussinesque – Cerutti solution [1,11,12,15,20] are often used when solving contact problems. The main limitations of these approaches is that characteristic geometrical sizes of contacting bodies are considerably greater then area of contact, and this allows to approximate the surfaces of bodies with half-spaces and separate the contact problem into two more simple problems - normal and tangential. This assumption is correct for the contact of new non - conformal profiles of wheel and rail in areas near the rail symmetry axis. However approximation with half-spaces surfaces of wheel and rail in the area of rail gauge corner is quite debatable, as the contact patch in this case is not settled in single plane and its sizes are near to characteristic sizes of contacting bodies. Moreover, a separation of contact problem into normal and tangential is improper due to the significant coupling of normal and tangential stresses , as will be shown further.

OBJECTS AND PROBLEMS

For a first time the weakness of Hertz theory when modeling conformal contact of two surfaces was theoretically grounded by Steuermann [18]. But his theory, as the Hertz one, is developed for half- spaces and the contact patch is assumed to be located in the single plane. That is not correct according to [6, 13, 17]. Also, the more sophisticated approaches must be used to define the initial separation between surfaces [7].

In paper [21] the methods of contact stresses investigation with the use of tensoresistors are presented. Taking these experiments as the basis, Yakovlev has shown, that when contact is moving to the rail gauge corner, where lateral curvature radius is 15 mm and is same order of magnitude with characteristic sizes of contact patch, the application of Hertz theory and half – space theory introduces a great inaccuracy in the calculation.

In our days the numerical methods for conformal contact modeling are seems to be the most attractive and precise. From the middle of 60 –s the finite elements methods (FEM) is often used when solving elastic problems. FEM modeling allows to obtain more realistic results, than from analytical solutions. The results of FEM modeling for different forms of conformal surfaces [2, 22] have shown, that contact stiffness, depending on the degree of surfaces' conformity and load applied, can be 50% greater than contact stiffness, predicted by Hertz theory.

Solving contact problem directly with FEM when modeling railway vehicle dynamics is very complicated and time-consuming. That's why in paper [19] was introduced the calculation of so – called "influence coefficients" with the use of FEM, as the alternative for the Bussinesque – Cerutti solution for conformal contact problems. A numerical modeling of real wheel and rail surfaces in rail gauge corner zone with curvature radiuses 10.5 and 15 mm have shown that the area of the patch decreases by 30–35%, and the maximum pressures increase correspondingly comparing to the Hertz theory. However with such approach for every wheel and rail profile combination it is necessary to calculate a great number of influence coefficients for every mesh node in advance. It seems to be more effective to use approach introduced in paper [17]. The idea of this approach is to get the most appropriate analytical influence functions by means of comparing them to FEM modeling results.

The aim of this paper is to present relatively robust and accurate technique for conformal contact, that can be painlessly integrated in railway vehicle dynamics simulation program.

When solving conformal contact problem with numerical methods, the following sub-problems have to be solved:

- definition of the spatial form of contact surface,
- definition of initial separation between contact surfaces,
- calculation of proper influence coefficient (functions).

The main difficulty for definition of spatial form of contact surface for conformal contact is that it not known in advance. So some hypnotizes are required to predict it's shape. In paper [13] it is suggested to use as contact surface the surface of one of the bodies in contact. It is motivated by the fact that conformal surfaces are very closed to each other in contact area. But this assuming the chosen body to be rigid and that can lead to some inaccuracies in calculations. So it seems to be more effective to define a contact surface as "intermediate" between surfaces of contacting bodies (see Fig. 1).

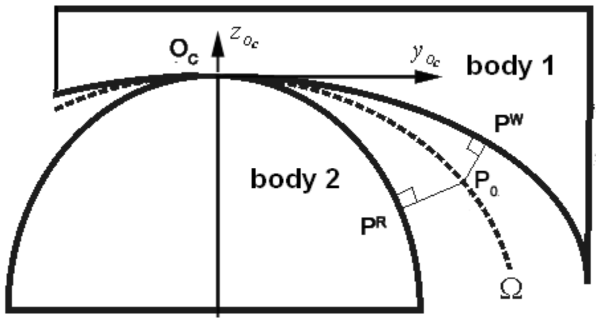


Fig. 1. Contact surface and initial separation calculation scheme

The expression for intermediate contact surface can be written in form:

$$z = f(x,y) = K \cdot f_1(x,y) + (1-K) \cdot f_2(x,y), \quad (1)$$

where: $K = \frac{k_2}{k_1 + k_2}$, $k_i = \frac{1-\nu_i^2}{\pi E_i}$, $i = 1,2$, $K \in [0,1]$, ν_1, ν_2, E_1, E_2 - Poisson coefficients and Young modulus of bodies, f_1 и f_2 - functions, that describes bodies surfaces.

In case when materials of bodies are the same, $K = 1/2$ and $f(x,y) = [f_1(x,y) + f_2(x,y)]/2$.

Next, for the numerical solution, the constructed surfaces can be meshed with grid with desired spacing in x and y directions. For every point P_0, which is grid cell center, its orthogonal projections P^W and P^R on the surfaces of contacting bodies are detected.

Uniqueness of the projections is insured by the regularity of the profile functions. Then the initial separation in point P_0 will be sum of distances from P_0 to points P^W and P^R:

$$d_R = \min(\rho(P_0, P^1)) + \min(\rho(P_0, P^2)), \quad (2)$$

where: $\rho(P_0, P^i)$ stays for the distance from point P_0 to arbitrary point of i –th body ($i = 1,2$). Thus, the problem reduces to finding the minimum of two – variables function, which can be solved with any suitable method of multidimensional optimization, for example with the gradient descent method.

It is common opinion, that friction has negligible influence on normal pressure distribution. The basic premise for that is the next expression for normal elastic displacements difference of contacting bodies' points:

$$w^+ - w^- = K \int_{A_C} \left(\frac{\cos\theta}{R} \tau_{x'z} + \frac{\sin\theta}{R} \tau_{y'z} \right) dx'dy' + \frac{1}{\pi E} \int_{A_C} \frac{1-\nu}{R} \sigma_{zz} dx'dy', \quad (3)$$

where: A_C - contact area,

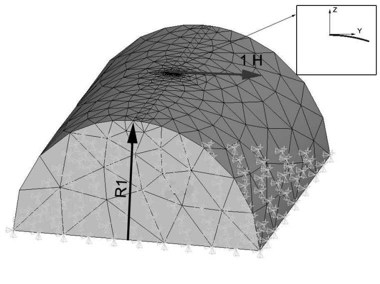
$w^{\pm}$ - normal elastic displacements,

"+","–" - indexes of upper and lower bodies respectively,

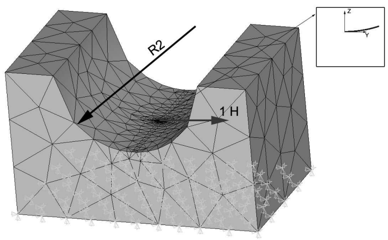
$$K = \frac{1}{4\pi} \left(\frac{1-2\nu^+}{E^+} - \frac{1-2\nu^-}{E^-} \right).$$

If wheel and rail have identical properties, than $K = 0$ and tangential stresses have no any influence on normal pressures, and it is possible to divide contact problem on normal and tangential.

To check the correctness of contact problem division in conformal contact case we carried out the next numerical experiment. In ANSYS program environment parametric FEM models of cylinder with radius $R1$ and cylindrical cavity with radius $R2$ were developed (see Fig. 2). In the upper point of cylinder and lower point of cavity unit tangential loads were applied. The foundations of cylinder and cavity are supposed to be fixed and have corresponding boundary conditions applied.



a



b

Fig. 2. FEM models of: a – cylinder, b – cavity

The experiment was carried out for two pairs of radiuses values: $R1 = 80$, $R2 = 80{,}1$ and $R1 = 15$, $R2 = 15{,}1$. After the solution converged, nodal displacements data was stored into structured file with a specially developed APDL macros . The file was further processed by Microsoft Excel, to calculate projections of displacement vector on surfaces normals (normal displacements). The difference of this projections for cylinder and cavity is shown on Fig. 3.

As it can be seen from Fig. 3, in the second case nodal normal displacement difference is significantly higher than in the first one. It is also can be seen that in the vicinity of load application point (< 0.1 mm) can be observed intermittent nodal displacements. That is common fact for FEM solutions. However when the distance from load application point increases the solution becomes more stable.

Thus for conformal contact problem solution in the rail gauge corner zone, where curvature radius is equal to 15 mm, it is necessary to use different from (3) expressions for normal elastic displacements.

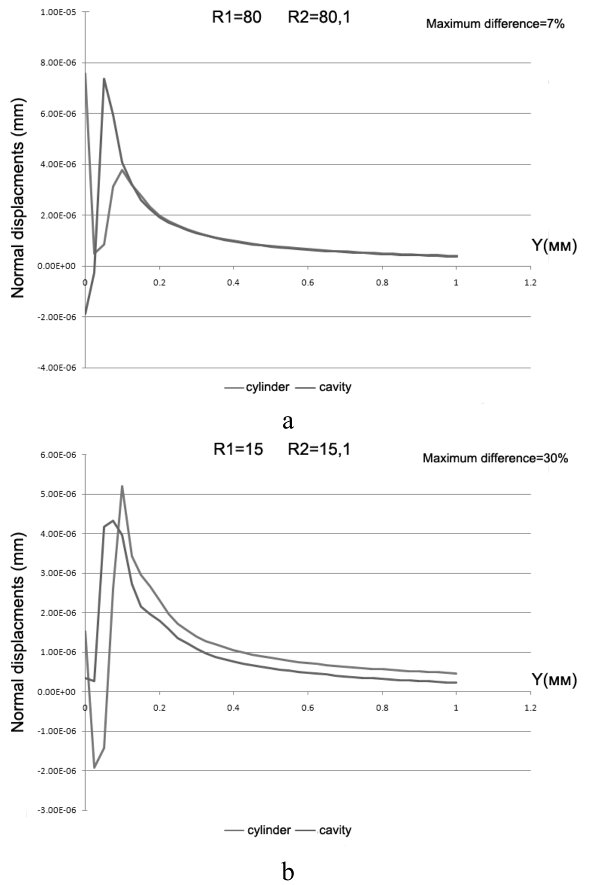


Fig. 3. Normal nodal displacements for $R1 = 80$, $R2 = 80{,}1$ (a) and $R1 = 15$, $R2 = 15{,}1$ (b)

To obtain this expressions the numerical experiment described above was continued with different shapes of conformal surfaces. For a search of suitable semi - empirical function different modification of Kelvin fundamental solution were considered, similar to [17]. We derived a next expressions for normal elastic displacements of points on body surfaces, caused by unit load (the most correlated with FEM):

$$w_{ij}^{z} = \frac{1}{\rho_{ij}}\left[\frac{3k_f}{4} + \frac{k_d (z_i - z_j)^2}{\rho_{ij}^2}\right],$$

$$w_{ij}^{x} = k_d \frac{(1 + 0.25 sign(k))(x_i - x_j)}{\rho_{ij}^2}, \qquad (4)$$

$$w_{ij}^{y} = k_d \frac{(1 + 0.25 sign(k))(y_i - y_j)}{\rho_{ij}^2},$$

where: x, y, z – direction of load application, i – considered point index, j – load application point index, k – body index (1 – upper body (concave), -2 – lower body (convex)), ρ_{ij} – distance between points,

$$k_d = \frac{1}{8\pi(1-\nu)G}, \; k_f = (3 - 4\nu)k_d.$$

These expressions were used in specially developed program VDEUNU CONTACT (based on mathematical models [3, 4, 5]) for calculation of compliance matrix coefficients in conformal contacts.

RESULTS AND DISCUSSIONS

To check the correctness of the presented approach in VDEUNU CONTACT program and in the ANSYS environment was modeled a conformal contact of ДМеТИ wheel and worn P-65 rail (ГОСТ Р51685-2000) with 125kN vertical load. Two different approaches were implemented in VDEUNU CONTACT(see Fig. 4):

- contact patch is located in single plane, initial separation is calculated by normal to plane, Bussinesque – Cerutti influence coefficients are used (Model A),
- contact patch is located on surface (1), initial separation is calculated according to expression (4),.influence coefficients (4) are used (Model B).

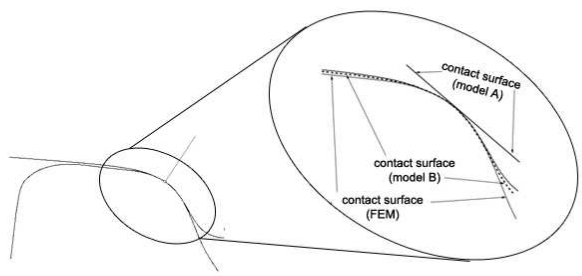
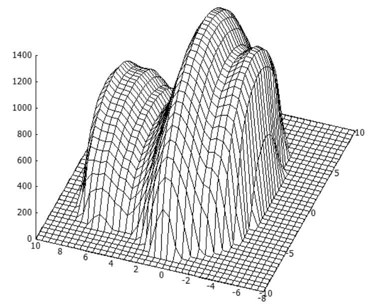


Fig. 4. Wheel – rail conformal contact with different model

Modeling results are shown on Fig. 5 and Table1. A significant difference can be seen in the modeling results between Model A and ANSYS (25% in contact area and 43% in maximal contact pressures). At the same time for Model B this difference is reduced to 5% approximately.



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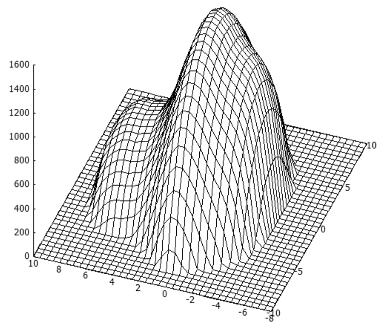

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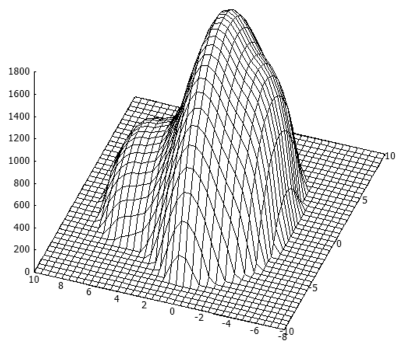

c

Fig. 5. Contact pressure distribution: a – VDEUNU CONTACT, Model A; b – VDEUNU CONTACT, Model B; c – ANSYS

Table1. Contact patch characteristics

Model	Contact area (mm^2)	Maximal contact pressure (MPa)
Model A	189	1244
Model B	160	1688
ANSYS	152	1787

CONCLUSIONS

1. The basics of numerical solutions of conformal contacts is presented.
2. It is shown, that conformal contact modeling results, based on half - space theory, can significantly differ from the FEM results.
3. The modeling results with the use of presented method differ from FEM results only by 5% approximately. Thus the technique can be used for wheel – rail contact interaction modeling in railway vehicle dynamics simulation.

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МОДЕЛИРОВАНИЕ КОНТАКТА СОГЛАСОВАННЫХ ПОВЕРХНОСТЕЙ КОЛЕСА И РЕЛЬСА

Александр Голубенко, Александр Костюкевич, Илья Цыгановский

Аннотация: Экспериментальная проверка показала, что моделирование контактного взаимодействия колеса и рельса в зоне выкружки рельса при помощи теории упругих полупространств может приводить к большим неточностям в вычислениях. В этом случае наиболее близкие к экспериментальным результаты можно получить при помощи МКЭ. Однако решение контактной задачи с помощью МКЭ по объективным причинам не может напрямую использоваться при моделировании динамики рельсовых экипажей.
В статье предложена методика моделирования контакта согласованных поверхностей колеса и рельса в зоне выкружки рельса. Полученная разница между результатами моделирования с помощью предложенной методики и результатами МКЭ составляет около 5%. Таким образом, данная методика может быть использоватна для решения контактной задачи в районе выкружки рельса при моделировании динамики рельсовых экипажей.
Ключевые слова: колесо, контакт, согласованные поверхности