



Heating-times of tungsten filament incandescent lamps

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ABSTRACT

Electricity provides power to incandescent lamps to heat the filament hot enough so that it provides light for illumination. The time taken to achieve a state of full brilliance after being turned on is so fast that it may be presumed to be adiabatic, hence it is deemed adiabatic heating time. In view of the fact that in actual practice simultaneously black-body radiation cools the filament, the General Electric in its bulletin provides a better time scale describing heating and cooling of filament. These are average times required for the filament to warm up to 90 per cent light output after the circuit is closed, or to cool down to 10 per cent after the circuit is opened, respectively. The exercise of estimating luminous flux and heating-times for typical 10, 100, 500 and 1000 W lamps have been undertaken for the first time for the benefit of students. This problem involves three disciplines electricity, optics and heat. The first describes the power needed to heat the filament quickly, optics then helps us in determining the light output and discipline of heat provides methods for solving the heat equation in order to estimate time to light or time to darkness. The study reveals that the supposition of linear configurations for the filaments neither matches luminous flux nor the heating-times. HS Leff's suggestion of introducing a shadow factor which reduces the exposed surface area, as it so happens in the coiled filaments, successfully explains the measured observations.

Keywords: Tungsten filament lamps, linear and coiled configurations, 10-1000 W, steady-state operation, lumen output, and heating-time

1. INTRODUCTION

The switching time of an incandescent lamp was defined [1] as the time to achieve the state of full brilliance after being turned on; in view of the fact that the heating course of

action is quite fast it was presumed the whole process to be adiabatic. However, this is not true in actual practice. At this time the simultaneous phenomenon of black-body radiation from the hot filament cools the filament. This fact modifies the steady-state temperature, slows down the rise of temperature and there is a drop in the light output. A better time scale for filament lamps has been quoted by General Electric [2] and they call it the heating-time and cooling-time for the specific lamp. They define it to be the average times required for the filament to warm up to 90 per cent light output after the circuit is closed, or to cool down to 10 per cent after the circuit is opened. The light output and heating-time data for coiled filament bulbs in the range 6 – 1000 W are reported by General Electric in their catalogue [2]. The objective of this paper is to elucidate these numbers from the point of view of students and teachers of physics and material science demonstrating how the disciplines electricity, optics and heat play roles in accomplishing the objectives; the first three sections are devoted to the roles played by these disciplines. Section 5 will describe Numerical work part. Finally, Section 6 discusses the conclusions.

2. ELECTRICITY

The electricity part is concerned with the Joule heating of the tungsten filament through the electric power operating the lamp. Suppose the resistance of the cold filament of a bulb at room temperature T_0 is R_0 . As soon as the power is turned on, a maximum in-rush current

$$I_{max} = V/R_0 \quad (1)$$

flows in it. Here V is the constant voltage value for the direct current and the root-mean-square value for an alternating current; the inductance effects [3] in coiled filaments are quite small for alternating currents, assuring that the current I and voltage V are approximately in phase and they satisfy the relation (1). Since I_{max} is quite large (typically of the order of 10 A or more) the corresponding Joule heat makes the temperature T of the filament rise very quickly. The physical properties of the metal constituting the filament are expected to be temperature dependent; the resistance R starts amplifying fast enough according to the power-law parameterization [3]

$$R = R_0(T/T_0)^{1.214} \quad (2)$$

The index 1.214 is obtained by making a least-square fit to the available data on the resistivity of tungsten [4] in the range 293–3000 K. Consequently, the Joule heating gets further accelerated. Once the steady-state temperature T_S is reached the filament resistance R_S and the wattage P will satisfy the relation

$$P = \frac{V^2}{R_S} = \frac{V^2}{R_0(T_S/T_0)^{1.214}} \quad (3)$$

The above relation corresponds to adiabatic heating, that is, no heat leaves the system. However, as mentioned in the beginning this is not true in the case of incandescent lamps. At this time the simultaneous phenomenon of black-body radiation from the hot filament

depending on the fourth-power of instant temperature cools the filament. This fact modifies the steady-state temperature, slows down the rise of temperature and there is a drop in the light output; the steady-state temperature T_S will be governed by the following relation for uncoiled filament

$$P = \frac{V^2}{R_0(T_S/T_0)^{1.214}} = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot (T_S^4 - T_W^4) \quad (4)$$

where: σ is the Stefan-Boltzmann constant, A is the surface area, $\varepsilon_{Overall}$ is the average emissivity of the metal tungsten and T_W is the temperature of the walls of the enclosure; the term T_W^4 will not be considered for making the calculation simple. Thus, there are following two options for fixing the operating temperature T_S

- $$\frac{V^2}{R_0(T_S/T_0)^{1.214}} = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot T_S^4 \quad (5)$$

- $$P = \sigma \cdot A \cdot \varepsilon_{Overall} \cdot T_S^4 \quad (6)$$

Both of these methods were attempted. The problem faced with the first method was that usually the T_S estimate's were not consistent with the wattage of the bulb. Therefore the second expression was adopted.

It will be worth devoting one paragraph on the coiling of the filament [3] for the reason that light-bulb filaments do not appear in the linear configurations. Rather, they are coiled once or twice to reduce their effective radiating surface areas; this favours the portable size of light-bulbs. The coiling enforces some of the surface area to radiate inward [3] to the region bounded by the coil resulting in higher filament temperatures. For example, a tightly wound, singly coiled filament would radiate outward only from about half of its surface area. Indeed, the temperature dependence of the resistivity of tungsten wire is not affected by its shape [4] but as the coiling leads to higher temperatures, the resistance of the wire gets amplified (vide 2) which further enhances Joule heating; thus, coiling leads to higher operating temperature [3] and thereby a larger lumen output. The reality that effective radiating surface is reduced due to coiling this feature can be achieved mathematically [3] by incorporating a correction factor δ in the expression of surface area of uncoiled filament. This is called shadow factor [5] having a value unity for linear configuration and a value which is less than one for coiled filaments. Suppose, at room temperature T_0 the uncoiled length of the filament is L_0 and diameter D_0 are known (Table I). Then the expression for the area of uncoiled filament would be

$$A = 2\pi r_0 L_0; \text{ radius of the filament } r_0 = D_0/2 \quad (7)$$

As per suggestion of Leff [3] the equivalent surface area due to coiling would be:

$$A_{Coil} = 2\pi r_0 L_0 \cdot \delta \quad (8)$$

For numerical illustration, first of all a linear configuration for the tungsten filament will be considered and it will be shown that both the luminous fluxes as well as the heating-times fall short of the measured values. This hints that coiling factor has to be brought in. For simplicity the heat losses such as gas losses, end losses and bulb and base losses will be ignored. Next section will take up the optics part to estimate light output from hot filaments.

3. OPTICS

Once the steady-state temperature of the filament is known we proceed to estimate the light output from such a source during its operation. The Planck's law for the electromagnetic radiation from an object at temperature T says

$$I(\lambda, T)d\lambda = \frac{\varepsilon(\lambda, T) \cdot A \cdot 2\pi hc^2 \cdot d\lambda}{\lambda^5 [\exp(hc/\lambda kT) - 1]} \text{ W.} \quad (9)$$

where: $I(\lambda, T)d\lambda$ is the power radiated between the wavelengths (in meters) λ and $\lambda + d\lambda$ from its surface having area A square meters and emissivity ε ; h and k are Planck's constant and Boltzmann's constant, respectively. The expression for the luminous flux [6,7] Q can be written as

$$Q(\lambda_i \rightarrow \lambda_f) = \int_{\lambda_f}^{\lambda_i} \frac{683 \cdot V(\lambda) \cdot \varepsilon_{\text{visible}} \cdot A \cdot 2\pi hc^2 \cdot d\lambda}{\lambda^5 [\exp(hc/\lambda kT) - 1]} \quad (10)$$

The factor $V(\lambda)$ takes care of the fact that the electromagnetic waves in the region $\lambda_i = 380 \text{ nm}$ to $\lambda_f = 760 \text{ nm}$ are perceptible to our eyes; it is best at $\lambda_m = 555 \text{ nm}$ and becomes vanishingly small outside this interval. This fact is represented by

$$V(\lambda) \cong \exp(-az^2 + bz^3); z \equiv \lambda/\lambda_m; \lambda_m = 555 \text{ nm} \quad (11)$$

$$a = 88.90, b = 112.95$$

The factor 683 occurs for the reason that at $\lambda_m = 555 \text{ nm}$ the electromagnetic radiation of one watt provides a luminous flux of 683 lumens. The expression for the effective surface area of the coil (vide 8) modifies the relation (10) to

$$Q(\lambda_i \rightarrow \lambda_f) = \int_{\lambda_f}^{\lambda_i} \frac{683 V(\lambda) \cdot \varepsilon_{\text{visible}} \cdot 2\pi \cdot r_0 \cdot L_0 \cdot \delta \cdot 2\pi hc^2 \cdot d\lambda}{\lambda^5 \cdot [\exp(hc/\lambda kT) - 1]} \quad (12)$$

The above integral will provide the luminous flux emitted by a bulb during its steady-state operation to be evaluated through Simpson-rule; for simplicity, thermal expansion of filament has been ignored. It is well known that emissivity of tungsten is substantially large in the visible wavelength region [8] and decreases with rise in the temperature of the filament. In contrast, the average value of emissivity over the entire wavelength spectrum is rather small and increases with the rise of temperature of the filament [4]; in view of the pedagogic nature of this article

$$\epsilon_{Visible} = 0.44 \text{ (vide ref. 8) and } \epsilon_{Overall} = 0.0000689T^{1.0748} \quad (13)$$

would be adopted; $\epsilon_{Overall}$ has been obtained through a least-square fit to the data in the temperature range 293-3000 K reported by Jones and Langmuir [4].

The estimated operating temperatures vide (6) and the corresponding luminous fluxes vide (12) for couple of typical lamps viz. 10, 100, 500, and 1000 W are listed in Table IIa for filaments if assumed to have linear configurations ($\delta = 1$); luminous fluxes fall short of the analogous reported values by General Electric. This suggests that the coiling factor has to be brought in. For this purpose, δ was decreased in the steps of 0.01 until the estimated flux matched with the observed value; these delta values and the particular temperatures of the hot filaments are reported in Table IIb. The last sentence needs to be further elaborated. Once the shadow factor δ is reduced the area of the filament also gets condensed through (8), the steady-state temperature rises (vide 6), and there is an enhancement of lumen output via the integral (12).

Having successfully reproduced the light outputs of lamps, the steady-state temperature as well as the shadow factor for each lamp is known to us. Next, we move to the calculations of their heating-times; as per definition of the heating-time the hot temperature T_{Hot} at which the light output achieves 90% brilliance after the circuit is closed has to be ascertained; in other words while the temperature rises from T_0 to T_{Hot} the associated lumen increases from 0% to 90% during the heating process.

For this reason, a program was developed in GW-BASIC to evaluate the integral (12) for the luminous flux by Simpson-rule for each one degree Kelvin rise of temperature of the filament starting from T_0 until the lumen output reached 90% mark. This exercise was carried out for each lamp. The temperature T_{Hot} of the filament was recorded both for linear and coiled configurations and these are mentioned in Tables IIa and IIb, respectively. The next section will be devoted to evaluation of heating-times for the same couple of typical light-bulbs.

4. HEAT

Estimating the heating-time as per definition. Let us write down heat equation in which the amount of energy produced adiabatically per unit time by Joule heating goes wholly into raising the temperature of the filament after the power supply is turned on

$$MC \frac{dT}{dt} = \frac{V^2}{R} = \frac{V^2}{R_0(T/T_0)^{1.214}} \quad (14)$$

where: M is the mass of the filament in kilogram, C is the specific heat in Joule per kilogram per degree Kelvin and t is the time variable in seconds. The specific heat of tungsten metal as reported by metallurgists [9] in the range 0 – 3000 °C has the following expression

$$C = 3R_g(1 - \theta_D^2/20 T^2) + 2aT + 4bT^3 \text{ J kg}^{-1}\text{K}^{-1}. \quad (15)$$

where: T is in Kelvin, $R_g = 45.2268 \text{ J kg}^{-1}\text{K}^{-1}$ is gas constant for tungsten, $\theta_D = 310 \text{ K}$ is a constant called the Debye temperature for tungsten at room temperature, $a = 4.5549 \cdot 10^{-3} \text{ J kg}^{-1}\text{K}^{-2}$ and $b = 5.77874 \cdot 10^{-10} \text{ J kg}^{-1}\text{K}^{-4}$.

Mass of the filament can be written as

$$M = \text{Volume of filament} \cdot \text{Density of tungsten} \\ = \pi \cdot r_0^2 \cdot L_0 \cdot 1.93 \cdot 10^4 \text{ Kg.} \quad (16)$$

We can find the expression for heating-time while the process is adiabatic for which we may rearrange (14) as follows

$$dt = \frac{R_0 M C}{V^2} \left(\frac{T}{T_0}\right)^{1.214} \cdot dT \\ = \frac{R_0 M [3R_g(1 - \theta_D^2/20T^2) + 2aT + 4bT^3] T^{1.214}}{V^2 \cdot T_0^{1.214}} \cdot dT \quad (17)$$

Following the definition of heating-time, when the filament has warmed up to 90% light output after closing the circuit the adiabatic heating-time $\tau_{heating}^{adiabatic}$ would be

$$\tau_{heating}^{adiabatic} = \int_{T_0}^{T_{Hot}} \frac{R_0 M [3R_g(1 - \theta_D^2/20T^2) + 2aT + 4bT^3] T^{1.214}}{V^2 \cdot T_0^{1.214}} \cdot dT \\ = \frac{M}{\rho T_{Hot}^{1.214}} \left\{ 3R_g \frac{T_{Hot}^{2.214} - T_0^{2.214}}{2.214} - \frac{3R_g \theta_D^2}{20} \frac{T_{Hot}^{0.214} - T_0^{0.214}}{0.214} + 2a \frac{T_{Hot}^{3.214} - T_0^{3.214}}{3.214} + \right. \\ \left. 4b \frac{T_{Hot}^{5.214} - T_0^{5.214}}{5.214} \right\} \text{ seconds} \quad (18)$$

This expression will provide heating-time while the process happens adiabatically, but as mentioned in the beginning, simultaneously the black-body radiation from the hot filament cools it; this modifies the heat equation (14) as

$$MC \frac{dT}{dt} = \frac{V^2}{R_0(T/T_0)^{1.214}} - \sigma \cdot 2\pi r_0 L_0 \cdot \delta \cdot \epsilon_{Overall} \cdot T^4 \quad (19)$$

One can use this heat equation to estimate the heating-time $\tau_{Heating}$ by rearranging it and integrating from the limits T_0 to T_{Hot} , we get

$$\tau_{Heating} = \int_{T_0}^{T_{Hot}} \frac{M [3R_g(1 - \theta_D^2/20T^2) + 2aT + 4bT^3]}{\frac{V^2}{R_0(T/T_0)^{1.214}} - \sigma \cdot 2\pi r_0 L_0 \cdot \delta \cdot \epsilon_{Overall} \cdot T^4} \cdot dT \quad (20)$$

This integral has to be evaluated through Simpson-rule. If one looks at the denominator of the integrand the first term will correspond to heating-time while the process is adiabatic but it gets corrected due to fact that simultaneously cooling is taking place by black-body radiation; this reduces the magnitude of denominator and, in turn, the heating-time increases. This expression has been used to estimate heating-times for 10, 100, 500, and 1000 W lamps and the corresponding values are listed in Table IIa for linear configuration and in Table IIb for coiled filaments.

5. NUMERICAL WORK

It will be worth recapitulating the numerical part described at various stages so far. Based on expression (6) the steady-state temperatures for typical lamps of 10, 100, 500, and 1000 W operational on 120 V were estimated followed by their light outputs values vide (12) for filaments supposed to have linear configurations. The lumen values so determined fell short of the reported ones (Table IIa). This guided us to take up the shadow factor suggested by HS Leff. The shadow factor δ , which is unity for linear configuration and a value less than one for coiled shape, was decreased in the steps of 0.01 and through a program in GW-BASIC the integral (12) was evaluated by Simpson-rule for luminous flux; this program took care of the fact that once the shadow factor δ is reduced the area of the filament also gets condensed through (8), the steady-state temperature rises (vide 6), and there is an enhancement of lumen output via the integral (12). This exercise was carried out for each lamp until the lumen output matched with the consequent measured ones (Tables IIb). This provided us the steady-state temperature T_S and shadow factor δ which had reproduced the observed light output. Our next concern is the heating-time for each lamp; this requires the prior knowledge of the hot temperature T_{Hot} . As mentioned above when the power is turned on the temperature and consequently light output rises and we have to locate the temperature where light output has gone up to 90 percent. Once again the program mentioned above was run to evaluate the integral (12) for the luminous flux by Simpson- rule for each one degree Kelvin rise of temperature of the filament starting from the T_0 until the objective is achieved (Table IIb). Finally, the expression (20) was employed to estimate heating-time while the temperature moves in the range $T_0 \rightarrow T_{Hot}$ both for linear configuration and actual coiled case. Next, we focus on the conclusions.

6. CONCLUSIONS AND DISCUSSIONS

The incandescent coiled tungsten filament lamps are in the process of being phased out because of their poor efficiency, nevertheless, they will continue to be source of illumination to the minds of physics students as evident from a large number of publications on this subject in the last three decades; the topics covered therein are the temperature and colour of the filament [10], efficiency and efficacy of the lamp [6], mortality statistics and life of the bulb [11,12], switching time [1], exponent – rules [13] and so on. On the other hand, no attention was paid to the topic heating-times of these lamps and this has been accomplished here.

Of course, there are couple of methods to arrive at the operating temperature of a coiled tungsten filament lamp which consumes electric power for heating the filament to

incandescence so quickly. For our purpose the temperature which yields the wattage of the bulb through Stefan-Boltzmann radiation is perfect. Under the supposition of linear configuration the light output during steady-state operation do not match for typical bulbs of 10, 100, 500, and 1000 W but rather fall short of the quoted values for each lamp. This finding supported the inclusion of coiling factor suggested by HS Leff [3].

The coiling not only reduces the radiating surface area, it enforces some of the area to radiate inward to the region bounded by the coil as well, resulting in higher filament temperatures. This aspect is achieved by multiplying the area of uncoiled filament by the shadow factor, which is equal to one for linear configuration and a value less than one for the coiled case. The shadow factor was decreased in the steps of 0.01 until the estimated luminous flux matched with consequent observed ones; a fall in the shadow factor results in higher coil temperature as well as the associated light output. In the next step the luminous flux output for one degree rise in temperature of heating filament was evaluated starting from the room temperature 293 K. This ascertained the hot temperature at which light output had ascended 90% from the room temperature once the circuit is closed. Lastly, the expression for heating-time from temperature T_0 to T_{Hot} , when the lamp is turned on, was derived. Based on this, heating-times were estimated for both the configurations viz. linear and coiled ones. The salient findings may be concluded as follows.

- The supposition of linear configuration yields both the luminous flux and heating-time falling short of reported ones for typical lamps viz. 10, 100, 500, and 1000 W (Table IIa).
- These findings favoured the role of coiling which reduces the exposed surface area so that the steady-state temperature as well as light outputs are having higher values as desired by us.
- The introduction of shadow factor reproduced the luminous fluxes for each lamp and the estimated heating-times based on these shadow factors did come close to those reported ones for 10, 100, 500, and 1000 W lamps (Table IIb).
- The matching could not be achieved better than what is reported in Table IIb is due to uncertainties in the value of resistance of the filament which plays very essential role in the estimation of heating-time vide (20) because of three reasons observed by Jones and Langmuir [14]; these are (i) *the resistance increases relatively slowly with the temperature as compared with most other properties used for temperature estimation*, (ii) *the resistance and its temperature coefficient are very sensitive to traces of impurities (carbon)*, and (iii) *at low temperatures the resistance of the filament is often so low that uncertainties in the lead and contact resistances are apt to play a large part*.
- Furthermore, if one examines the integral (20) an interesting finding can be derived; in this integral let us observe the denominator having two terms. If we just keep first term which represents power generated by the electric current, it will yield the adiabatic heating time. However, the second term delays it due to simultaneous cooling effect of black body radiation; its value will be close to the cooling-time of a lamp defined as:

$$\tau_{Cooling} = \int_{T_S}^{T_{Cold}} \frac{M[3R_g(1-\theta_D^2/20T^2)+2aT+4bT^3]}{-\sigma \cdot 2\pi r_0 L_0 \cdot \delta \cdot \epsilon_{Overall} \cdot T^4} \cdot dT \quad (21)$$

Here T_{Cold} is the temperature of the filament when the light output has dropped to 10 percent after turning off the power supply. Thus, the heating-time under discussion must be approximately equal to the sum of adiabatic heating time and the cooling-time [15] (Table IIc).

- It will be a good exercise for students having interest in theoretical physics to repeat the calculations for the lamps of 2000, 5000, and 10000 W whose characteristics data [2] are mentioned in Table III; luminous flux values are available whereas heating-times data are not available.
- Those inclined towards experiments should design experiments for measuring the heating-times of the above mentioned higher wattage lamps for validating the theory.

Table I. Operating data [2] at room temperature 293 K for standard lamps designed for 120 V reported by General Electric (USA).

Power P watt	Length L_0 meters	Diameter D_0 meters	Gas Loss P_{Gas} %	End Loss P_{End} %	Bulb & Base Loss P_{Bulb} %
10*	0.43180	0.001626	–	1.5	5.0
100**	0.47498	0.006096	11.5	1.3	5.2
500**	0.87376	0.018034	8.8	1.8	7.1
1000**	1.016	0.02794	6.0	1.9	4.7

*Vacuum-single coiled **Gas filled-coiled-coil

Table IIa. Estimates of operating temperature and the associated luminous flux, observed luminous flux [2], estimated temperature of hot filament when the luminous flux achieves 90% output, and estimated and observed heating-times for filaments having linear configurations.

Power P watt	Estimated Steady-state temperature T_S in Kelvin	Estimated luminous flux in lumens ($\delta = 1$)	Observed luminous flux in lumen	Temperature at which luminous flux achieves 90% ($\delta = 1$) after circuit is closed	Estimated heating- time for range $T_0 \rightarrow T_{Hot}$ for $\delta = 1$	Observed heating- time
10	2305	74	80	2282	0.049	0.06
100	2744	1736	1920	2712	0.088	0.13

500	2699	8528	10850	2668	0.300	0.38
1000	2755	17638	23740	2723	0.422	0.67

Table IIb. Estimated operating temperature, shadow factor, observed luminous flux, estimate of hot temperature when the luminous flux achieves 90% after the circuit is closed, calculated heating-time and the observed heating-time

Power P watt	Estimated steady-state temperature T_S in Kelvin	Estimated shadow factor δ which reproduces luminous flux of column 4	Observed luminous flux in lumen	Temperature at which luminous flux achieves 90% after closing the circuit	Estimated heating time for range $T_0 \rightarrow T_{Hot}$ in seconds vide (20)	Observed heating time in seconds
10	2338	0.93	80	2314	0.053	0.06
100	2814	0.88	1920	2782	0.100	0.13
500	2903	0.68	10850	2869	0.454	0.38
1000	2990	0.66	23740	2953	0.738	0.67

Table IIc. Estimated adiabatic heating-time, observed cooling-time, their sum are comparable with the observed heating-time as noted above.

Power watt	Estimated adiabatic heating-time $\tau_{heating}^{adiabatic}$ seconds	Observed cooling-time $\tau_{Cooling}$ seconds	Sum of $\tau_{heating}^{adiabatic}$ and $\tau_{Cooling}$ seconds	Observed heating-time $\tau_{Heating}$ seconds
10	0.03	0.02	0.05	0.06
100	0.06	0.06	0.12	0.13
500	0.20	0.19	0.39	0.38
1000	0.29	0.30	0.59	0.67

Table III. Primary data for gas filled single coiled higher wattage lamps designed for 120 V at room temperature 293 K.

Power watt	Length of filament in meters	Diameter of filament in meters	Approximate initial Lumens
2000	1.41478	0.0004572	58000
5000	1.12775	0.0007366	145000
10000	1.3843	0.0011684	335000

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