Application of Generalized Method of Lines for Solving the Problems of Thick Plates Thermoelasticity
Part I. Construction of resolving equations

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Summary. The authors proposed a new version of lowering dimensionality in the application of the method of lines. The basic idea is lowering the dimensionality of input equations per the spatial coordinate by projection method, including the Bubnov-Galerkin method.
Key words: metod of lines, Bubnov-Galerkin-Petrov thermoelasticity method, thick plates, structural mechanics.

INTRODUCTION

The authors proposed a new version of lowering dimensionality in the application of the method of lines. It is greatly expanding the capabilities of method of lines. The proposed generalized method of lines may be used for calculating the plates of variable thickness, and also problems of dynamics. The basic idea of generalized method of lines is lowering the dimensionality of input equations per the spatial coordinate by projection method. The projection method includes the Bubnov-Galerkin method, generalized by Petrov [4].

PURPOSE OF WORK

One of the most effective methods of solving multidimensional problems of structural mechanics is the combination approach. In this approach a problem is solved in two stages:
1) decreasing the dimension of the input equations by one or two coordinates;
2) the reduced problem is solved analytically or numerically.

Traditionally in structural mechanics, lowering dimensionality of input equations is based on certain hypotheses. Accordingly, the first stage of the method was excluded in a separate research: theory of rods, plates and, shells. Applied hypotheses were strong enough but less accurate. It lead to creation of various theories of plates and shells.

Currently, lowering dimensionality is performed using mathematical methods (for example, the theory of shells I.N.Vekua [1]). With the next solution of reduced equations, lowering the dimension creates a combined method for solving problems of mathematical physics. Such methods include Vlasov-Kantorovich's method. These combined methods are alternative, compared to the general numerical methods such as finite element method, finite difference and variation-difference method.

Mathematical methods of lowering dimensionality are associated with the geometrical characteristics of the considered objects. It greatly restricts the geometry of the problems, for which it is possible to use the combined methods. However, limiting the complexity of the geometry allows the application of very efficient numerical methods. It increases the accuracy and stability of numerical calculation. It also significantly reduces computer time using.

One of the known methods of lowering dimensionality input equations is the "method of lines". In this method, the finite difference method is used for one of the two coordinates. This method will be effective, if the input equations are systems of ordinary differential equations. In the case of constant coefficients in these equations, it is possible to use analytical solution of system of equations (Vinokurov [2], Shkelov L.T. [3]). In this regard, the method of lines is used for the solution of static problems for plates and shells of constant thickness.

The authors proposed a new version of lowering dimensionality in the application of the method of lines. It is greatly expanding the capabilities of method of lines. The proposed generalized method of lines may be used for calculating the plates of variable thickness, and also problems of dynamics. The basic idea of generalized method of lines is lowering the dimensionality of input equations per the spatial coordinate by projection method. The projection

method includes the Bubnov-Galerkin method, generalized by Petrov [4].

In the case of thick plates with constant thickness for equations of plate deformations per thickness, locally basic restricted discrete linear functions are chosen (Fig. 1.).

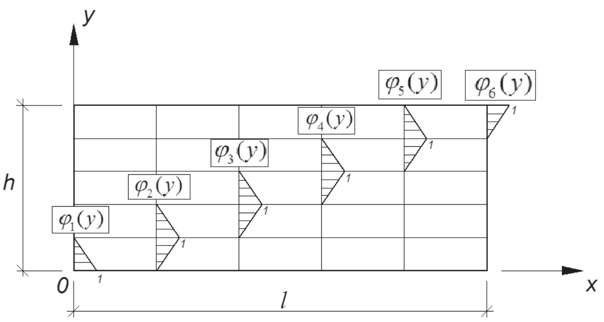

Fig. 1. Basis functions

As in the traditional version of the method of lines, a cross-section of the plate is divided into inflict n lines (including two boundary lines) with an equal range Δ. However, in order to reduce dimensionality, we do not use method of finite differences, but the generalized method of Bubnov-Galerkin-Petrov. By coordinate y the unknown functions $f(x,y)$ is approximated in this manner:

$$f(x,y) \approx f^i(x)\cdot\varphi_i(y) \qquad (1)$$

The constructed algorithm of lowering dimensionality formally resembles algebraic transformations of tensor calculus. In this connection, the generalized method of lines essentially tensor symbols and relevant rules are used. For example, by repeated indexes is assumed summation. Resolving equations according to Bubnov-Galerkin method, after substituting approximate ratios of the form (1) are scalar multiplied in Hilbert space for basic functions $\varphi_i(y)$.

It should be noted that in Bubnov-Galerkin method, the basis functions must satisfy the homogeneous boundary conditions per coordinate y. These basis functions do not satisfy such conditions. However, according to the generalization of Petrov [4] it is enough that these functions satisfy natural boundary conditions. It should be noted that in the construction of reduced equations for intensive unknowns (displacement in the theory of elasticity) and extensive unknowns (stresses) transformation of corresponding components is performed differently. Herewith we get two basic matrices – G and B, which are recorded in an index form as: $g_{ij}=(\varphi_i,\varphi_j)$, $b_{ij}=(\varphi_i,\frac{d\varphi_j}{dy})$. This is the scalar product of two functions:

$$(\varphi_i,\varphi_j)=\int_0^h \varphi_i(y)\cdot\varphi_j(y)\cdot dy\,. \qquad (2)$$

Conversion of components with derivative y of the function n-type displacement and stresses-type functions is formed in different ways. This is the use of lowering dimension of a plane problem using the theory of elasticity (3, 4).

The peculiarity of this functional basis is that this basis is not orthogonal, and thus there exist two types of index values, f^i and f_i. These magnitudes are different by rules of conversion at transition on another basis. Contravariant magnitudes denoted by upper index and covariant magnitudes – lower index. Accordingly, $\{g_{ij}\}$ – two indexes magnitude is twice covariant metric tensor and the inverse matrix $\{g_{ij}\}^{-1}=\{g^{ij}\}$ is twice contravariant metric tensor. Metric tensor provides a transition from covariant to contravariant components and vice versa:

$$f_i=g_{ij}\cdot f^j\,,\ f^i=g^{ij}\cdot f_j\,. \qquad (5)$$

The scalar product in this case is the integral of multiplication functional factors. Therefore in mathematics covariant and contravariant function magnitudes have an identified name. Because the covariant magnitudes appear in the decomposition by basis (fig. 1), they are called coefficients. Covariant magnitudes appear as a scalar product of the basis elements:

$$f(x,y)=(f(x,y),\varphi_i(y))=\int_0^h f(x,y)\cdot\varphi_i(y)dy\,,$$

they are called – moments.

Therefore, reduced equations can be written in four ways:

– In moments, if displacement and stresses in the moments;
– In coefficients, if all unknowns are written in coefficients;
– Two versions of combined record: displacement in the moments, stresses in the coefficients or displacement in coefficient, stresses in the moments.

$$\begin{aligned}(\frac{\partial u(x,y)}{\partial y},\varphi_i(y))&=\int_0^h\frac{\partial u(x,y)}{\partial y}\cdot\varphi_i(y)dy=\int_0^h\frac{\partial(u^j(x)\varphi_j(y))}{\partial y}\cdot\varphi_i(y)dy=\\&=u^j(x)\int_0^h\varphi_i(y)\varphi_j'(y)dy=b_{ij}u^j(x),\end{aligned} \qquad (3)$$

$$\begin{aligned}&(\frac{\partial\sigma_x(x,y)}{\partial y},\varphi_i(y))=\int_0^h\frac{\partial\sigma_x(x,y)}{\partial y}\cdot\varphi_i(y)dy=\sigma_x(x,y)\varphi_i(y)\Big|_0^h-\int_0^h\sigma_x(x,y)\cdot\varphi_i'(y)dy=\\&(\sigma_x(x,h)\cdot\varphi_n(h)-\sigma_x(x,0)\cdot\varphi_1(0))-\int_0^h{\sigma_x}^j(x)\cdot\varphi_j(y)\cdot\varphi_i'(y)dy=\\&(\sigma_x(x,h)\cdot\varphi_n(h)-\sigma_x(x,0)\cdot\varphi_1(0))-b_{ji}{\sigma_x}^j(x)\end{aligned} \qquad (4)$$

After formulating the constructing equations, we need to formulate the reduced boundary value and initial – boundary value problem in index form.

The described technique can be applied to solve the problem of thermal stresses in a rod of rectangular cross-section (Fig. 2.), which occupies a three-dimensional region: $[0 \le x \le l] \times [0 \le y \le h_y] \times [0 \le z \le h_z]$.

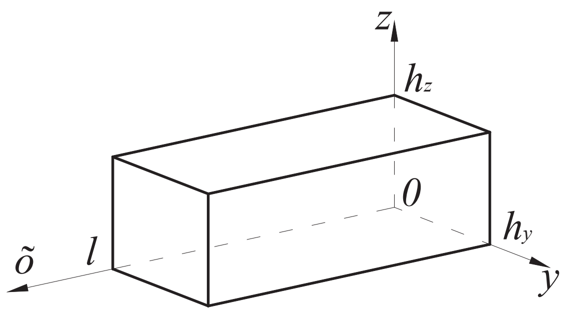

Fig. 2. Beam of rectangular cross section

The problem of thermal stresses is considered within limits of an important partition of the theory of elasticity – thermoelasticity [5, 6]. In this problem we consider two physical fields – thermal and mechanical.

Thermal field in solids is described by the thermal conductivity equation. In the most general form, thermal field depends not only on three spatial coordinates but also on time coordinates. The corresponding problem in determining of the component of thermal field is described by the equations of non-stationary thermal conductivity. Components depend on the time coordinate. As a system of differential equations in partial derivatives of the first order in the spatial and time coordinates, these equations are written in the form:

$$\begin{cases} \rho c \dfrac{\partial T}{\partial t} = \dfrac{\partial q_x}{\partial x} + \dfrac{\partial q_y}{\partial y} + \dfrac{\partial q_z}{\partial z} + Q \\ q_x = -\lambda_T \dfrac{\partial T}{\partial x} \\ q_y = -\lambda_T \dfrac{\partial T}{\partial y} \\ q_z = -\lambda_T \dfrac{\partial T}{\partial z} \end{cases}, \tag{6}$$

where $T = T(x, y, z)$ – temperature function, q_x, q_y, q_z – components of the heat flux $\vec{q}(x, y, z)$, ρ – density of the material, c – specific heat, λ_T – coefficient of thermal conductivity. Q – the quantity of heat generated by internal heat sources.

To ensure unity of solution of the system (6) we need to specify the initial and boundary conditions. The initial conditions are in the form:

$t = 0$, $T(x, y, z) = T_0(x, y, z)$,

where:

$T_0(x, y, z)$ – temperature distribution throughout the volume of the body at the initial time.

The boundary conditions of the problem will be set as conditions of convective heat transfer.

when $x = 0$:

$$q_x(0, y, z, t) = \alpha_{xT}^0 (T_{xC}^0 - T_x^0) - q_{xC}(0, y, z, t),$$

when: $x = l$,

$$q_x(l, y, z, t) = \alpha_{xT}^l (T_x^l - T_{xC}^l) + q_{xC}(l, y, z, t). \tag{7}$$

The temperatures and heat flows of external environment from the side of relevant part of boundary surface of beam are marked as “C”:

when: $y = 0$,

$$q_y(x, 0, z, t) = -\alpha_{yT}^0 (T_y(x, 0, z, t) - T_{yC}(x, 0, z, t)) - q_{yC}(x, 0, z, t),$$

when: $y = h_y$, (8)

$$q_y(x, h_y, z, t) = \alpha_{yT}^{h_y} (T_y(x, h_y, z, t) - T_{yC}(x, h_y, z, t)) + q_{yC}(x, h_y, z, t),$$

when: $z = 0$,

$$q_z(x, y, 0, t) = -\alpha_{zT}^0 (T_z(x, y, 0, t) - T_{zC}(x, y, 0, t)) - q_{zC}(x, y, 0, t),$$

when $z = h_z$: (9)

$$q_z(x, y, h_z, t) = \alpha_{zT}^{h_z} (T_z(x, y, h_z, t) - T_{zC}(x, y, h_z, t)) + q_{zC}(x, y, h_z, t).$$

In the next numerical calculations, such form of boundary conditions allows to take into account the relevant part of surface boundary conditions of first order $\alpha_T \to \infty$ and second order $\alpha_T \to 0$.

Changing temperature of solid body in time practically does not cause dynamic effects. Therefore, mechanical fields (displacement, stress and strain fields) are stationary and are described by static equations.

These equations are written as equations in partial derivatives of first order:

$$\frac{\partial \sigma_x}{\partial x} = -\frac{\partial \tau_{xy}}{\partial y} - \frac{\partial \tau_{xz}}{\partial z} - X, \tag{10}$$

$$\frac{\partial \tau_{xy}}{\partial x} = -\frac{\partial \sigma_y}{\partial y} - \frac{\partial \tau_{yz}}{\partial z} - Y, \tag{11}$$

$$\frac{\partial \tau_{xz}}{\partial x} = -\frac{\partial \tau_{yz}}{\partial y} - \frac{\partial \sigma_z}{\partial z} - Z, \tag{12}$$

$$\frac{\partial u^*}{\partial x} = \frac{\mu}{(\lambda + 2\mu)} \sigma_x - \frac{\lambda}{(\lambda + 2\mu)} \frac{\partial v^*}{\partial y} - \frac{\lambda}{(\lambda + 2\mu)} \frac{\partial w^*}{\partial z} + \frac{(3\lambda + 2\mu)}{(\lambda + 2\mu)} \alpha_T (T - T_0), \tag{13}$$

$$\frac{\partial v^*}{\partial x} = \tau_{xy} - \frac{\partial u^*}{\partial y}, \tag{14}$$

$$\frac{\partial w^*}{\partial x} = \tau_{xz} - \frac{\partial u^*}{\partial z}. \tag{15}$$

The equations can be separately considered:

$$\sigma_y = \frac{\lambda}{\mu}\frac{\partial u^*}{\partial x} + \frac{(\lambda+2\mu)}{\mu}\frac{\partial v^*}{\partial y} + \frac{\lambda}{\mu}\frac{\partial w^*}{\partial z} - \frac{(3\lambda+2\mu)}{\mu}\alpha_T(T-T_0), \tag{16}$$

$$\sigma_z = \frac{\lambda}{\mu}\frac{\partial u^*}{\partial x} + \frac{\lambda}{\mu}\frac{\partial v^*}{\partial y} + \frac{(\lambda+2\mu)}{\mu}\frac{\partial w^*}{\partial z} - \frac{(3\lambda+2\mu)}{\mu}\alpha_T(T-T_0), \tag{17}$$

$$\tau_{yz} = \left(\frac{\partial w^*}{\partial y} + \frac{\partial v^*}{\partial z}\right), \tag{18}$$

where: $f^* = \mu \cdot f$.

Boundary conditions of stress-strain state in general are written by the analogy of work [7] (19)-(24).

For construction of the boundary conditions (19)-(24) we write the sum of projections of all power factors that act on the boundary contour, on corresponding axis. In (Fig. 3.) on the plane $x0y$, the magnitudes which act on area $x=0$ are shown.

The first index —signifies the number of the axis which is perpendicular to the area. The second index shows the direction of displacement or stress.

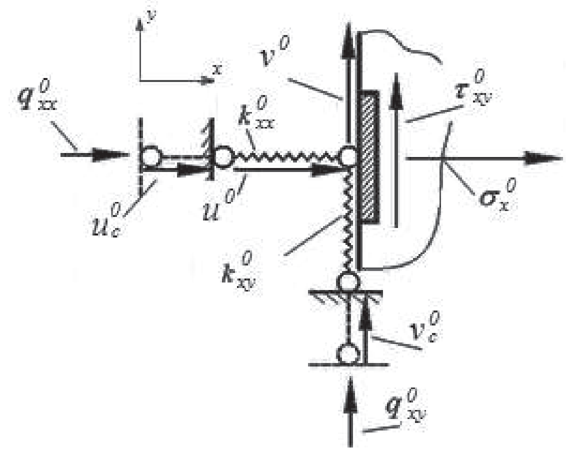

Fig. 3. Modeling of the boundary conditions

u_c, v_c — displacement of points of the external environment,
u, v — horizontal and vertical displacement of points of object,
q_{xx}, q_{xy} — external load on object,
σ_x, τ_{xy} — normal and tangential (shear) stresses along the contour inside the object,
k_{xx}, k_{xy} — spring stiffness.

when: $x=0$,

$$\frac{1}{\sqrt{1+(k^0_{xx})^2}}\sigma^0_x - \frac{k^0_{xx}}{\sqrt{1+(k^0_{xx})^2}}u^0 + \frac{1}{\sqrt{1+(k^0_{xx})^2}}q^0_{xx} + \frac{k^0_{xx}}{\sqrt{1+(k^0_{xx})^2}}u^0_c = 0, \tag{19}$$

$$\frac{1}{\sqrt{1+(k^0_{xy})^2}}\tau^0_{xy} - \frac{k^0_{xy}}{\sqrt{1+(k^0_{xy})^2}}v^0 + \frac{1}{\sqrt{1+(k^0_{xy})^2}}q^0_{xy} + \frac{k^0_{xy}}{\sqrt{1+(k^0_{xy})^2}}v^0_c = 0, \tag{20}$$

$$\frac{1}{\sqrt{1+(k^0_{xz})^2}}\tau^0_{xz} - \frac{k^0_{xz}}{\sqrt{1+(k^0_{xz})^2}}w^0 + \frac{1}{\sqrt{1+(k^0_{xz})^2}}q^0_{xz} + \frac{k^0_{xz}}{\sqrt{1+(k^0_{xz})^2}}w^0_c = 0, \tag{21}$$

when: $x=l$,

$$-\frac{1}{\sqrt{1+(k^l_{xx})^2}}\sigma^l_x - \frac{k^l_{xx}}{\sqrt{1+(k^l_{xx})^2}}u^l + \frac{1}{\sqrt{1+(k^l_{xx})^2}}q^l_{xx} + \frac{k^l_{xx}}{\sqrt{1+(k^l_{xx})^2}}u^l_c = 0, \tag{22}$$

$$-\frac{1}{\sqrt{1+(k^l_{xy})^2}}\tau^l_{xy} - \frac{k^l_{xy}}{\sqrt{1+(k^l_{xy})^2}}v^l + \frac{1}{\sqrt{1+(k^l_{xy})^2}}q^l_{xy} + \frac{k^l_{xy}}{\sqrt{1+(k^l_{xy})^2}}v^l_c = 0, \tag{23}$$

$$-\frac{1}{\sqrt{1+(k^l_{xz})^2}}\tau^l_{xz} - \frac{k^l_{xz}}{\sqrt{1+(k^l_{xz})^2}}w^l + \frac{1}{\sqrt{1+(k^l_{xz})^2}}q^l_{xz} + \frac{k^l_{xz}}{\sqrt{1+(k^l_{xz})^2}}w^l_c = 0. \tag{24}$$

where: $f^0 = f(0,y,z)$, $\quad f^l = f(l,y,z)$.

By changing stiffness we can specify any standard conditions of interaction of the object with the external environment.

For equations (6)-(24) the procedure of lowering dimension is performed by coordinates y, z. At first — reduction by y. Basic functions are applied $\{\varphi_i = \varphi_i(y)\}$, $i = 1,...,n$, where the following rules are taken into account:

$$f(x,y,z,t) = f^i(x,z,t) \cdot \varphi_i(y),$$

$$(f(x,y,z,t), \varphi_i(y)) = \int_0^{h_y} f(x,y,z,t) \cdot \varphi_i(y) dy =$$

$$= f_i(x,z,t);$$

$$\left(\frac{\partial f(x,y,z,t)}{\partial x}, \varphi_i(y) \right) = \int_0^{h_y} \frac{\partial f(x,y,z,t)}{\partial x} \varphi_i(y) dy =$$

$$\frac{\partial}{\partial x} \int_0^{h_y} f(x,y,z,t) \cdot \varphi_i(y) dy = \frac{\partial}{\partial x} f_i(x,z,t).$$

Where f is a factor of stress, then

$$\left(\frac{\partial f(x,y,z,t)}{\partial y}, \varphi_i(y) \right) = \int_0^{h_y} \frac{\partial f(x,y,z,t)}{\partial y} \varphi_i(y) dy =$$

$$= f(x,y,z,t)\varphi_i(y) \big|_{y=0}^{y=h_y} - \int_0^{h_y} f(x,y,z,t)\varphi_i'(y) dy =$$

$$= f(x,h_y,z,t)\varphi_i(h_y) - f(x,0,z,t)\varphi_i(0) -$$

$$- \int_0^{h_y} f^j(x,z,t)\varphi_j(y)\varphi_i'(y) dy =$$

$$= f^i(x,h_y,z,t)\delta_{i\cdot}^{\cdot n} - f^i(x,0,z,t)\delta_{i\cdot}^{\cdot 1} -$$

$$- b_{ji} g^{j\alpha} f_\alpha(x,z,t).$$

When: f is a factor of temperature and displacement (T, u, v, w), then:

$$\left(\frac{\partial f(x,y,z,t)}{\partial y}, \varphi_i(y) \right) = \int_0^{h_y} \frac{\partial f(x,y,z,t)}{\partial y} \varphi_i(y) dy =$$

$$= \int_0^{h_y} \frac{\partial (f^j(x,z,t)\varphi_j(y))}{\partial y} \varphi_i(y) dy =$$

$$= \int_0^{h_y} \varphi_i(y)\varphi_j'(y) f^j(x,z,t) dy = b_{ij} g^{j\alpha} f_\alpha(x,z,t).$$

The first equation of system (6) is multiplied as a scalar by $\{\varphi_i = \varphi_i(y)\}$ where $i = 1,...,n$, and than integrated by coordinate y:

$$\left(\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} - \rho c \frac{\partial T}{\partial t} + Q(x,y,z,t)\right) = 0, \varphi_i(y)\right),$$

$$\frac{\partial q_{xi}}{\partial x} + \delta_{\cdot i}^{n\cdot} \cdot q_y{}^i(x,z,t) - \delta_{\cdot i}^{1\cdot} \cdot q_y{}^i(x,z,t) - \tag{25}$$

$$- b_{ji} g^{j\alpha} q_{y\alpha}(x,z,t) + \frac{\partial q_{zi}}{\partial z} - \rho c \frac{\partial T_i}{\partial t} + Q_i(x,z,t) = 0.$$

In the next step we perform the reduction by z of equation (20):

$$\Big(\frac{\partial q_{xi}}{\partial x} + \delta_{\cdot i}^{n\cdot} \cdot q_y{}^i(x,z,t) - \delta_{\cdot i}^{1\cdot} \cdot q_y{}^i(x,z,t) -$$

$$b_{ji} g^{j\alpha} q_{y\alpha}(x,z,t) + \frac{\partial q_{zi}}{\partial z} - \rho c \frac{\partial T_i}{\partial t} +$$

$$+ Q_i(x,z,t) = 0, \varphi_k(z)\Big),$$

$$\frac{\partial q_{xik}}{\partial x} + \left[\delta_{\cdot i}^{n\cdot} \cdot q_{y\cdot k}^{\;i\cdot}(x,t) - \delta_{\cdot i}^{1\cdot} \cdot q_{y\cdot k}^{\;i\cdot}(x,t)\right] -$$

$$- b_{ji} g^{j\alpha} q_{y\alpha k}(x,t) + \left[\delta_{\cdot k}^{m\cdot} \cdot q_{zi\cdot}^{\;\;\cdot k}(x,t) - \delta_{\cdot k}^{1\cdot} \cdot q_{zi\cdot}^{\;\;\cdot k}(x,t)\right] - \tag{26}$$

$$b_{pk} g^{ps} q_{zis}(x,t) - \rho c \frac{\partial T_{ik}}{\partial t} + Q_{ik}(x,t) = 0,$$

Here $$\left[\delta_{\cdot i}^{n\cdot} \cdot q_{y\cdot k}^{\;i\cdot} - \delta_{\cdot i}^{1\cdot} \cdot q_{y\cdot k}^{\;i\cdot}\right] = \begin{bmatrix} -q_{y\cdot k}^{\;1\cdot} \\ 0 \\ \vdots \\ 0 \\ q_{y\cdot k}^{\;n\cdot} \end{bmatrix},$$

$$\left[\delta_{\cdot k}^{m\cdot} \cdot q_{zi\cdot}^{\;\;\cdot k} - \delta_{\cdot k}^{1\cdot} \cdot q_{zi\cdot}^{\;\;\cdot k}\right] = \begin{bmatrix} -q_{zi\cdot}^{\;\;\cdot 1} \\ 0 \\ \vdots \\ 0 \\ q_{zi\cdot}^{\;\;\cdot m} \end{bmatrix},$$

n – number of lines along the axis y, m - number of lines along the axis z.

Indexes $i, j, \alpha, \beta, \gamma$ - are related to reduction by coordinate y; indexes $k, p, s, \phi, \varepsilon$ - reduction by coordinate z.

Taking into account the boundary conditions (8)-(9):

$$\left[\delta_{\cdot i}^{n\cdot}\cdot q_{y\cdot k}^{\ i\cdot}-\delta_{\cdot i}^{1\cdot}\cdot q_{y\cdot k}^{\ i\cdot}\right]=\begin{bmatrix}\alpha_{yT}^{0}\cdot T_{y\cdot k}^{1\cdot}\\0\\\vdots\\0\\\alpha_{yT}^{h_y}\cdot T_{y\cdot k}^{n\cdot}\end{bmatrix}-\begin{bmatrix}\alpha_{yT}^{0}\cdot T_{yC\cdot k}^{\ 1\cdot}\\0\\\vdots\\0\\\alpha_{yT}^{h_y}\cdot T_{yC\cdot k}^{\ n\cdot}\end{bmatrix}+\begin{bmatrix}q_{yC\cdot k}^{\ 1\cdot}\\0\\\vdots\\0\\-q_{yC\cdot k}^{\ n\cdot}\end{bmatrix}$$

$$\left[\delta_{i\cdot}^{\cdot m}\cdot q_{zi\cdot}^{\ \cdot k}-\delta_{i\cdot}^{\cdot 1}\cdot q_{zi\cdot}^{\ \cdot k}\right]=\begin{bmatrix}\alpha_{zT}^{0}\cdot T_{zi\cdot}^{\cdot 1}\\0\\\vdots\\0\\\alpha_{yT}^{h_z}\cdot T_{zi\cdot}^{\cdot m}\end{bmatrix}-\begin{bmatrix}\alpha_{yT}^{0}\cdot T_{zCi\cdot}^{\ \cdot 1}\\0\\\vdots\\0\\\alpha_{yT}^{h_z}\cdot T_{zCi\cdot}^{\ \cdot m}\end{bmatrix}+\begin{bmatrix}q_{zCi\cdot}^{\ \cdot 1}\\0\\\vdots\\0\\-q_{zCi\cdot}^{\ \cdot m}\end{bmatrix}$$

The substitution:

$$\begin{bmatrix}\alpha_{yT}^{0}\cdot T_{y\cdot k}^{1\cdot}\\0\\\vdots\\0\\\alpha_{yT}^{h_y}\cdot T_{y\cdot k}^{n\cdot}\end{bmatrix}=T\alpha_{y\cdot k}^{\ i\cdot},\quad\begin{bmatrix}\alpha_{yT}^{0}\cdot T_{yC\cdot k}^{\ 1\cdot}\\0\\\vdots\\0\\\alpha_{yT}^{h_y}\cdot T_{yC\cdot k}^{\ n\cdot}\end{bmatrix}=TC_{y\cdot k}^{\ i\cdot},\quad\begin{bmatrix}q_{yC\cdot k}^{\ 1\cdot}\\0\\\vdots\\0\\-q_{yC\cdot k}^{\ n\cdot}\end{bmatrix}=q_{yC\cdot k}^{\ i\cdot},\tag{27}$$

$$\begin{bmatrix}\alpha_{zT}^{0}\cdot T_{zi\cdot}^{\cdot 1}\\0\\\vdots\\0\\\alpha_{zT}^{h_z}\cdot T_{zi\cdot}^{\cdot m}\end{bmatrix}=T\alpha_{zi\cdot}^{\ \cdot k},\quad\begin{bmatrix}\alpha_{zT}^{0}\cdot T_{zCi\cdot}^{\ \cdot 1}\\0\\\vdots\\0\\\alpha_{zT}^{h_z}\cdot T_{zCi\cdot}^{\ \cdot m}\end{bmatrix}=TC_{zi\cdot}^{\ \cdot k},\quad\begin{bmatrix}q_{zCi\cdot}^{\ \cdot 1}\\0\\\vdots\\0\\-q_{zCi\cdot}^{\ \cdot m}\end{bmatrix}=q_{zCi\cdot}^{\ \cdot k}.\tag{28}$$

Taking into account the boundary conditions (27)-(28) in equation (20):

$$\Big(\frac{\partial q_{xik}}{\partial x}+\left[g^{i\alpha}\cdot T\alpha_{y\alpha k}-g_{k\varepsilon}TC_{y}^{\ i\varepsilon}+g^{i\alpha}\cdot q_{yC\alpha k}\right]-$$
$$-b_{ji}g^{j\alpha}q_{y\alpha k}+$$
$$\left[g^{k\varepsilon}\cdot T\alpha_{zi\varepsilon}-g_{i\alpha}TC_{z}^{\ \alpha k}+g^{k\varepsilon}\cdot q_{yCi\varepsilon}\right]-$$
$$-b_{pk}g^{ps}q_{zis}-\rho c\frac{\partial T_{ik}}{\partial t}+Q_{ik}=0\tag{29}$$

Similarly, the reduction of remaining equations of system (6) is done:

$$q_{xik}=-\lambda_T\frac{\partial T_{ik}}{\partial x},\tag{30}$$

$$q_{yik}=-\lambda_T b_{ij}g^{j\alpha}T_{\alpha k}(x,t),\tag{31}$$

$$q_{zik}=-\lambda_T b_{kp}g^{ps}T_{is}(x,t).\tag{32}$$

Substituting (30)-(32) in (29), equation (33) is formed:

$$-\lambda_T\frac{\partial^2 T_{ik}}{\partial x^2}+\left[g^{i\alpha}\cdot T\alpha_{y\alpha k}-g_{k\varepsilon}TC_{y}^{\ i\varepsilon}+g^{i\alpha}\cdot q_{yC\alpha k}\right]+$$
$$+\lambda_T b_{ji}g^{j\alpha}b_{\alpha\beta}g^{\beta\gamma}T_{\gamma k}+$$
$$\left[g^{k\varepsilon}\cdot T\alpha_{zi\varepsilon}-g_{i\alpha}TC_{z}^{\ \alpha k}+g^{k\varepsilon}\cdot q_{yCi\varepsilon}\right]+$$
$$+\lambda_T b_{pk}g^{ps}b_{s\phi}g^{\phi\varepsilon}T_{i\varepsilon}-\rho c\frac{\partial T_{ik}}{\partial t}+Q_{ik}=0\tag{33}$$

The reduced initial conditions will look like $T_{ik}(x,0)=T_{0ik}(x)$.

The next step is the reduction of equations of system (10)-(14) In this system, the process of reduction expressions are substituted for $\sigma_y,\sigma_z,\tau_{yz}$ (16)-(18), using the above mentioned operations:

$$\frac{du^*_{ik}}{dx}=\frac{\mu}{(\lambda+2\mu)}\sigma_{xik}-\frac{\lambda}{(\lambda+2\mu)}b_{ij}g^{j\alpha}v^*_{\alpha k}-\frac{\lambda}{(\lambda+2\mu)}b_{kp}g^{ps}w^*_{is}+\frac{(3\lambda+2\mu)}{(\lambda+2\mu)}\alpha_T(T_{ik}-T_{0ik}), \quad (34)$$

$$\frac{dv^*_{ik}}{dx}=\tau_{xyik}-b_{ij}g^{j\alpha}u^*_{\alpha k}, \quad (35)$$

$$\frac{dw^*_{ik}}{dx}=\tau_{xzik}-b_{kp}g^{ps}u^*_{is}, \quad (36)$$

$$\frac{d\sigma_{xik}}{dx}=-b_{ji}g^{j\alpha}\tau_{xy\alpha k}+b_{pk}g^{ps}\tau_{xzis}-\left[\delta^{n\cdot}_{\cdot i}\tau_{xy\cdot k}^{\;\;i\cdot}-\delta^{1\cdot}_{\cdot i}\tau_{xy\cdot k}^{\;\;i\cdot}\right]-\left[\delta^{m\cdot}_{\cdot k}\tau_{xy\,i\cdot}^{\;\;\cdot k}-\delta^{1\cdot}_{\cdot k}\tau_{xy\,i\cdot}^{\;\;\cdot k}\right]-X_{ik}, \quad (37)$$

$$\begin{aligned}\frac{d\tau_{xyik}}{dx}&=\frac{\lambda}{\lambda+2\mu}b_{ji}g^{j\alpha}\sigma_{x\alpha k}+\frac{4(\lambda+\mu)}{\lambda+2\mu}b_{ji}g^{j\alpha}b_{\alpha\beta}g^{\beta\gamma}v^*_{x\gamma k}+\\&+\frac{2\lambda}{\lambda+2\mu}\left[b_{ji}g^{j\alpha}\right]\cdot\left[b_{kp}g^{ps}\right]w^*_{\alpha s}-\frac{2(3\lambda+2\mu)}{\lambda+2\mu}b_{ji}g^{j\alpha}\alpha_T(T_{\alpha k}-T_{0\alpha k})+\\&+b_{pk}g^{ps}b_{s\phi}g^{\phi\varepsilon}v^*_{i\varepsilon}+\left[b_{pk}g^{ps}\right]\cdot\left[b_{ij}g^{j\alpha}\right]w^*_{\alpha s}-\left[\delta^{n\cdot}_{\cdot i}\sigma_{y\cdot k}^{\;\;i\cdot}-\delta^{1\cdot}_{\cdot i}\sigma_{y\cdot k}^{\;\;i\cdot}\right]-\left[\delta^{m\cdot}_{\cdot k}\tau_{xz\,i\cdot}^{\;\;\cdot k}-\delta^{1\cdot}_{\cdot k}\tau_{xz\,i\cdot}^{\;\;\cdot k}\right]-Y_{ik},\end{aligned} \quad (38)$$

$$\begin{aligned}\frac{d\tau_{xzik}}{dx}&=\frac{\lambda}{\lambda+\mu}b_{pk}g^{ps}\sigma_{xis}+\frac{4(\lambda+\mu)}{\lambda+2\mu}b_{pk}g^{ps}b_{s\phi}g^{\phi\varepsilon}w^*_{xi\varepsilon}+\frac{2\lambda}{\lambda+2\mu}\left[b_{pk}g^{ps}\right]\cdot\left[b_{ij}g^{j\alpha}\right]v^*_{\alpha s}-\\&-\frac{2(3\lambda+2\mu)}{\lambda+2\mu}\alpha_T b_{pk}g^{ps}(T_{is}-T_{0is})+\left[b_{ji}g^{j\alpha}\right]\cdot\left[b_{kp}g^{ps}\right]v^*_{\alpha\varepsilon}+b_{ji}g^{j\alpha}b_{\alpha\beta}g^{\beta\gamma}w^*_{\gamma k}-\\&-\left[\delta^{n\cdot}_{\cdot i}\tau_{yz\cdot k}^{\;\;i\cdot}-\delta^{1\cdot}_{\cdot i}\tau_{yz\cdot k}^{\;\;i\cdot}\right]-\left[\delta^{m\cdot}_{\cdot k}\sigma_{z\,i\cdot}^{\;\;\cdot k}-\delta^{1\cdot}_{\cdot k}\sigma_{z\,i\cdot}^{\;\;\cdot k}\right]-Z_{ik}\end{aligned} \quad (39)$$

The reduced boundary conditions of stress-strain state will look like:

$$\begin{aligned}&\frac{\mu}{\sqrt{1+(k^0_{xx})^2}}\sigma^0_{xik}-\frac{k^0_{xx}}{\sqrt{1+(k^0_{xx})^2}}u^{*0}_{ik}+\frac{\mu}{\sqrt{1+(k^{*0}_{xx})^2}}q^0_{xxik}+\frac{k^0_{xx}}{\sqrt{1+(k^0_{xx})^2}}u^{*0}_{cik}=0,\\&\frac{\mu}{\sqrt{1+(k^0_{xy})^2}}\tau^0_{xyik}-\frac{k^0_{xy}}{\sqrt{1+(k^0_{xy})^2}}v^{*0}_{ik}+\frac{\mu}{\sqrt{1+(k^0_{xy})^2}}q^0_{xyik}+\frac{k^0_{xy}}{\sqrt{1+(k^0_{xy})^2}}v^{*0}_{cik}=0,\\&\frac{\mu}{\sqrt{1+(k^0_{xz})^2}}\tau^0_{xzik}-\frac{k^0_{xz}}{\sqrt{1+(k^0_{xz})^2}}w^{*0}_{ik}+\frac{\mu}{\sqrt{1+(k^0_{xz})^2}}q^0_{xzik}+\frac{k^0_{xz}}{\sqrt{1+(k^0_{xz})^2}}w^{*0}_{cik}=0,\\&-\frac{\mu}{\sqrt{1+(k^l_{xx})^2}}\sigma^l_{xik}-\frac{k^l_{xx}}{\sqrt{1+(k^l_{xx})^2}}u^{*l}_{ik}+\frac{\mu}{\sqrt{1+(k^l_{xx})^2}}q^l_{xxik}+\frac{k^l_{xx}}{\sqrt{1+(k^l_{xx})^2}}u^{*l}_{cik}=0,\\&-\frac{\mu}{\sqrt{1+(k^l_{xy})^2}}\tau^l_{xyik}-\frac{k^l_{xy}}{\sqrt{1+(k^l_{xy})^2}}v^{*l}_{ik}+\frac{\mu}{\sqrt{1+(k^l_{xy})^2}}q^l_{xyik}+\frac{k^l_{xy}}{\sqrt{1+(k^l_{xy})^2}}v^{*l}_{cik}=0,\\&-\frac{\mu}{\sqrt{1+(k^l_{xz})^2}}\tau^l_{xzik}-\frac{k^l_{xz}}{\sqrt{1+(k^l_{xz})^2}}w^{*l}_{ik}+\frac{\mu}{\sqrt{1+(k^l_{xz})^2}}q^l_{xzik}+\frac{k^l_{xz}}{\sqrt{1+(k^l_{xz})^2}}w^{*l}_{cik}=0.\end{aligned} \quad (40)$$

The next step – the problem numerically is simulated using the method of discrete orthogonalization by S.K.Hodunov [8]. Differential equations in partial derivatives are solved using the method of Runge-Kutta-Merson. This problem is programmed by the Fortran programming language. Depending on the geometry and initial-boundary conditions; temperature, displacement and stress are determined at certain points of the construction.

CONCLUSIONS

The suggested modification of the method of lines significantly increases the accuracy of calculation. The problem of setting boundary function is solved and this allows the solution of problem of dynamics and thermoelasticity.

REFERENCES

1. **Vekua I.N., 1982.** Some common methods of constructing different versions of the theory of shells. M .: Science. Home Editorial physical and mathematical literature, 288 p. (in Russian).
2. **Vinokurov L.P., 1956.** Direct methods for solving spatial and contact problems for arrays and foundations. Kharkiv. Publ Kharkov. Univ, 279 p. (in Russian).
3. **Shkelёv L.T., Stankevich A.N., Poshivach D.V., 2004.** Application of the method of lines for the determination of the stress and strain states of the spatial and plate structural elements: Monograph. – K .: KNUCA, 2004. -136 p. (in Russian).
4. **Mikhlin S.M., 1957.** Variational methods in mathematical physics // State-ing because of technical and theoretical l-ry: M – 476 p. (in Russian).
5. **Kovalenko A.D., 1970.** Fundamentals of thermoelasticity, Naukova Dumka, – 308. (in Russian).
6. **Kovalenko A.D.,1975.** Thermoelasticity. Textbook. To .: Vishcha School, – 215 p. (in Russian).
7. **Stankevich A.M., Chibiryakov V.K., Skele L.T., 2010.** The method of lines in prostoroviy zadachi teoriï pruzhnosti // Naukova-tehnichny zbirnik “Opir materialiv i teoriya sporud” – Preview Issue №87, – p.109 – 117. (in Ukrainian).
8. **Godunov S.K**., **1961**. On the numerical solution of boundary value problems for systems of linear ordinary differential equations // Advances Mathematical Sciences. – v.16 – No.3. – P.171 – 174. (in Russian).

ПРИМЕНЕНИЕ ОБОБЩЕННОГО МЕТОДА ПРЯМЫХ К ЗАДАЧАМ ТЕРМОУПРУГОСТИ ТОЛСТЫХ ПЛИТ. СООБЩЕНИЕ 1. ПОСТРОЕНИЕ РАЗРЕШАЮЩИХ УРАВНЕНИЙ

Аннотация. Авторами данной работы предложен новый вариант понижения размерности методом прямых, что существенно расширило его возможности. Обобщенный метод прямых применим для плит переменной толщины, а также в задачах динамики. Основная идея состоит в понижении размерности по пространственной координате с помощью проекционного метода (к проекционному методу относится метод Бубнова – Галеркина, обобщенный Г. И. Петровым [4]). В работе методика понижения размерности используется для редукции уравнений термоупругости.

Ключевые слова: метод прямых, метод Бубнова-Галёркина-Петрова, термоупругость, толстые пластины, строительная механика.