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# Assessment of strength parameters of beech plywood in terms of failure criteria

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**Abstract:** *Assessment of strength parameters of beech plywood in terms of failure criteria.* The failure state in a composite layer being in the complex state of stress is defined using the so-called material failure theories. They describe the yield surface of the material, for which the limit of linear elasticity is determined. Failure theories formulated for anisotropic materials are more complex. The paper presents the results of research on strength and deformation of the beech veneer during tension loading in relation to slope of grain. Results of the performed investigations were presented in the form of graphs end figures.

*Keywords:* failure criterion, plywood strength, wood composites.

## INTRODUCTION

The failure state in a composite layer being in the complex state of stress is defined using the so-called material failure theories. They describe the yield surface of the material, for which the limit of linear elasticity is determined. Failure theories formulated for anisotropic materials are more complex. The sources of hypotheses for anisotropic materials frequently include modifications of failure criteria concerning isotropic materials. It turns out that at the uniaxial tension of the orthotropic material for certain angles failure occurs at a specific load, while at others it does not. The number of failure criteria for composite materials is much greater, estimated at several hundred and practically impossible to determine, since new ones are constantly being presented in literature (Nahas 1986; German 1996; Muc 2003). Development of these criteria for these materials is much more difficult than for isotropic materials. In view of the diversity of composite materials and their configuration a universal effort criterion providing precise results may hardly be established. It is assumed that a pre-condition for the safe state of a layer is for the stresses in the principal material directions to be below boundary values. When the material of an individual layer(lamina) is treated as homogeneous in the macroscopic scale it does not include the mechanisms of failure in the microscopic scale. Moreover, it is assumed that damage to even one layer causes destruction of the whole composite.

The basic failure hypotheses concerning orthotropic materials comprise the maximum stress hypothesis, the maximum strain hypothesis and interactive hypotheses (Noris 1962, Azzi,V.D., Tsai, S.W 1965,Tsai and Wu 1971).

## MATERIAL AND METHODS

Material constants were adopted after literature data (Keylwerth 1951, Neuhaus 1994), as mean values amounting to density of 690 kg/m$^3$, 12% moisture content. For beech wood the limit failure parameters amount to: $X_t$= 135.0 MPa, $Y_t$= 7.0 MPa, $X_c$ = 53.0 MPa, and S = 10.0 MPa (Kollmann 1951).

The *maximum stress* hypothesis originates from failure analyses of isotropic materials according to the Galileo hypothesis, i.e. maximum principal stresses (German 1996). In the hypothesis of maximum stresses - implemented for composite materials - a layer is in the state of limit failure when stresses in the directions of material orthotropic are lower than failure ones: $X_c \leq \sigma_{11} \leq X_t$ ; $-Y_c \leq \sigma_{22} \leq Y_t$ ; $|\tau_{12}| \leq S$. For a unidirectional fibrous composite lamina, the following strength date can be obtained from simple laboratory test: $X_t$, $X_c$– longitudinal tensile and compressive strength in 1 direction , $Y_t$, $Y_c$ –transverse tensile and compressive strength in 2 direction, S- shear strength. In the calculation of maximum stress $\sigma_{max}$ we first determine stress in the principal axis system of a layer:

$$\sigma_{11}=\sigma_{xx}\cos^2\theta, \qquad \sigma_{22}=\sigma_{xx}\sin^2\theta\,, \qquad \tau_{12}=-\sigma_{xx}\cos\theta\sin\theta\,, \tag{1}$$

where: $\sigma_{11}$ = stress in principal material 1-direction, $\sigma_{22}$ = stress in principal material 2-direction, $\tau_{12}$ = shear stress in principal material 1-2 direction, thus modified failure conditions take the form:

$$\frac{X_c}{\cos^2\theta} \le \sigma_{xx} \le \frac{X_t}{\cos^2\theta}\,, \qquad \frac{Y_c}{\sin^2\theta} \le \sigma_{xx} \le \frac{Y_t}{\sin^2\theta}\,, \qquad |\sigma_{xx}| \le \frac{S}{|\cos\theta\sin\theta|} \tag{2}$$

The second more popular is failure criterion according to Azzi-Tsai. This criterion originates from the generalization of the yield condition of the Huber-Mises-Hencky energy hypothesis for orthotropic materials presented by Hill in 1950 (Hill 1950, Azzi and Tsai 1965; Tsai 1968, Tsai and Hahn 1980). For the plane stress in an orthotropic layer the strength condition, with identical tensile and compressive strength in the directions of its principal axes, criterion may be presented as:

$$\frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\tau_{12}^2}{S} = 1 \tag{3}$$

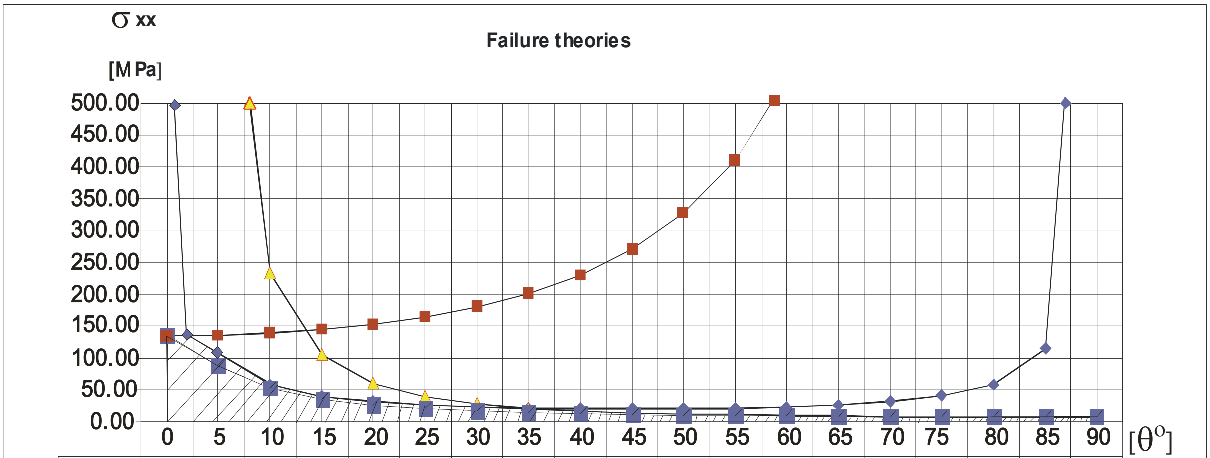
Using formula (1) and substituting in relationship (3) the failure criterion for uniaxial tension $\sigma_{xx}$ depending on angle $\theta$ takes the form:

$$\sigma_{xx} = \frac{1}{\sqrt{\frac{\cos^4\theta}{X^2} + \left(\frac{1}{S^2} - \frac{1}{X^2}\right)\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{Y^2}}}\,, \tag{4}$$

here: $X$,$Y$, are failure values of tensile strength in the directions of principal axes of material orthotropic, $S$- shear strength in the plane of principal material axes.

## RESULTS

According to the hypothesis of maximum stresses the picture of limit stresses $\sigma_{xx}$ at uniaxial tension of the layer comprises boundary curves presented in the graph in figure 1.




| | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| τ 12 | 1145.9 | 115.18 | 58.48 | 40.00 | 31.11 | 26.11 | 23.09 | 21.28 | 20.31 | 20.00 | 20.31 | 21.28 | 23.09 | 26.11 | 31.11 | 40.74 | 58.48 | 115.18 | 1145.9 |
| σ11 | 135.00 | 136.03 | 139.20 | 144.69 | 152.88 | 164.35 | 180.00 | 201.19 | 230.05 | 270.00 | 326.74 | 410.35 | 540.00 | 755.85 | 1154.0 | 2096.5 | 4477,0 | 17772. | |
| σ22 | | 921.52 | 232.14 | 104.50 | 59.84 | 39.19 | 28.00 | 21.28 | 16.94 | 14.00 | 11.93 | 10.43 | 9.33 | 8.52 | 7.93 | 7.48 | 7.22 | 7.05 | 7.00 |
| A–T | 135.00 | 87.64 | 52.63 | 36.25 | 27.22 | 21.58 | 17.76 | 15.03 | 13.00 | 11.47 | 10.29 | 9.37 | 8.66 | 8.10 | 7.68 | 7.36 | 7.16 | 7.04 | 7.00 |

Figure 1. A comparisons of limit curves of layer strength according to the criterion of maximum stress ($\sigma_{11}$, $\sigma_{22,}$ $\tau_{12}$) and according to the interactive Azzi-Tsai (A-T) criterion

The area of failure stresses (loads) in the layer of beech material in the graph is represented by the hatched area below the boundary curves.

A picture of the interactive Azzi-Tsai criterion is given by the curve presented too in the graph figure 1.

An example of experimental studies on tensile strength of beech material depending on the fiber inclination angle $\theta = (0^o, 10^o, 30^o, 90^o)$ are presented in figure 2, while the picture of failure of beech veneer is given in figure 3.

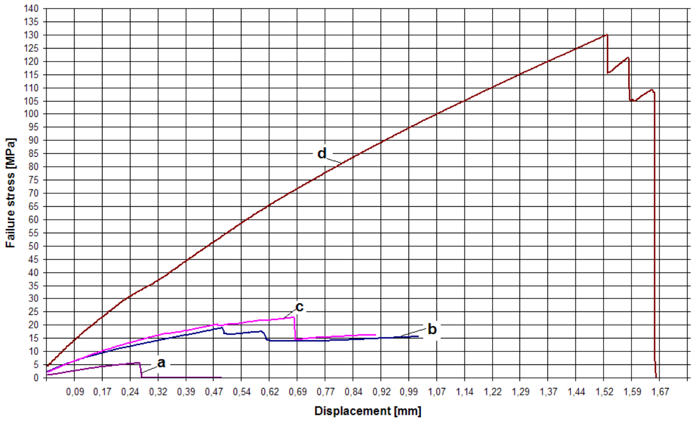



Figure 2. Limit stresses and strains of beech veneer in the function of fibre inclination angle: a) $\theta = 90^O$ specimen, b) $\theta = 30^O$ specimen, c) $\theta = 10^O$ specimen, d) $\theta = 0^O$ specimen

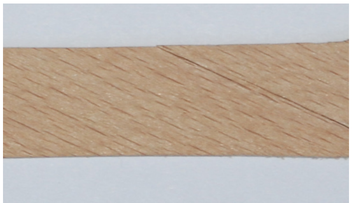
a)  b) 

Figure 3. The picture of failure for beech material samples in the function of fiber inclination angle: a) a $\theta = 0^O$ specimen, b) a $\theta = 30^O$ specimen.

CONCLUSIONS

Strength of layer laminates is dependent mainly on the structure and mechanical parameters of a given material. The primary difference in the determination of strength between isotropic and orthotropic materials is connected with the fact that the latter generally have other stresses or strains in the so-called principal directions of the material. Moreover, at the general complex state of stress in composite structures frequently an additional problem results from the occurring misalignment of stress and strain tensors.

It results from the calculations that for small angles where $\theta = (0^o \approx 1.5^o)$, failure of the composite layer occurs as a result of exceeding its tensile strength along the fibers figure 1. For angles $\theta = (1.5^o \div 35^o)$ failure is connected with exceeding failure shear stresses. When angle $\theta$ is greater than $35^o$ ($\theta \geq 35^o$), failure is determined by tensile strength perpendicular to fibers in the layer. While the above criterion presents the mechanism of failure, it does not consider conjugation between individual stresses, which is its drawback.

The conducted analytical and numerical calculations of a multilayer wood-based composite under complex load conditions provided information on how strength of laminates changes depending on the configuration arrangement of the layer and to what extent strength hypotheses are suitable for the determination of material failure of the laminate.

It results from analyses of calculated stress values in individual layers of laminas at the imposed loading, the limit value of failure stresses defined according to the failure hypotheses was not exceeded. Experimental studies confirmed that the strength and the shape of layers failure - and also whole laminates - are determined by the fiber orientation directions in relation to the loading direction.

**Streszczenie:** *Oszacowanie wytrzymałości warstwy sklejki bukowej w aspekcie wybranych hipotez wytrzymałosciowych.* Stan wytężenia warstwy kompozytu znajdującej się w złożonym stanie naprężenia określa się korzystając z tzw. hipotez wytężeniowych. Opisują one powierzchnię graniczną materiału, do której określony jest koniec liniowej sprężystości. Formułowane dla materiałów anizotropowych hipotezy wytężeniowe, są bardziej złożone. Badania obejmowały określenie wytrzymałości forniru sklejki bukowej przy statycznym rozciąganiu o różnej konfiguracji włókien.

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