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Mathematical simulation of gas bubble moving in central region of the short vortex chamber

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S u m m a r y. The research results of analytical investigations of insulated gas bubble moving in central region of the short vortex chamber in a dropping liquid are presented. The describing mathematical model of the trajectory of a bubble under influencing of forces, operating on it is compounded, in which one the change of radius of a bubble is taken into account. The ratio of magnitudes rendering the main effect on kinematic parameters of a bubble moving is obtained. The outcomes of researches can be applied at research of behavior of a bubble in a dropping liquid, rotated with a constant angular velocity.

K e y w o r d s: motion path, gas bubble, dropping liquid, angular velocity, vortex chamber.

INTRODUCTION

In engineering processes coal, oil producing, chemical industries, agriculture, pipeline transport, heat-and-power engineering etc the transportation of different fluid mediums is realized dynamic pumps, the overall performance which one by an essential image influences and a production efficiency. Thus, unfavorable operation conditions, such as the impact loads, chattering, chemical aggressiveness and heat of fluids, presence of abrasive fragments in transferred and enclosing mediums, reduce or restrict service performances of dynamic pumps [Rogovyi 2007].

Unite of superchargers virtues of centrifugal and ink-jet types has bringing in creation of a new type of pumps - with vortex working chamber called as us irrotational centrifugal pumps [Rogovyi 2007, Syomin 2005, 2007], which one have performances practically independent from the majority of the unfavorable factors reduced above. The transfer of power to a moving particle takes place in a field of an operation of centrifugal forces, that is a feature of a working process of a irrotational centrifugal pump. Thus, the particles having a density large, than density of an actuating medium, are displaced to peripherals of the vortex chamber to a plenum, and with a smaller density - to rotation axis [Rogovyi 2007, Syomin 2010. In many cases the presence of bubbles in a fluid can result in to essential change of transportation parameters, and to a collection them in axis zone [Syomin 2010, Chahine 1996, Van Nierop 2007]. Thereof, at transportation of dropping liquid the origin on an axis of the chamber of a gas rotational cord lowering expenditure of transferable liquid is possible. The cavitational conditions in such pump do not result in loss of its functionability, and only reduce its performance parameters.

Thus, the study of behavior of a gas bubble in central zone of the short vortex chamber of a irrotational centrifugal pump is an actual problem, the solution will allow which one to increase power performances irrotational centrifugal pumps at transportation dropping liquid.

RESEARCH OBJECT

To simulation of gas bubble moving there are two approaches: maiden is grounded on the share solution of three-dimensional flow equations of liquid-gas bubble mixture, that requires enough long time of calculation and considerable computational capabilities [Akhtar 2006, Hua 2007], owing to what the calculation more precise, but not always usables for an estimation of the factors effecting on behavior of a gas bubble. The second approach is grounded on a superposition method of moving of an insulated bubble on known flow of a fluid [Syomin 2010, Chahine 1996, Hsiao 2004, Latorre 1980, Maxey 1983, Raoufi 2006], with usage of a balance of power operating on a bubble.

Many researches [Syomin 2009, Akhtar 2006, Chahine 1996, Hsiao 2004, Hua 2007, Latorre 1980, Maxey 1983, Raoufi 2006] are dedicated to behavior and calculation of pathways of bubbles, but the majority of them consider behavior of a cavitational bubble [Chahine 1996, Hsiao 2004, Latorre 1980, Plesset 1948] or gas liquid streams which are transportation in tubes [Hua 2007, Maxey 1983, Raoufi 2006]. The operations concerning to behavior of a bubble in a vortex [Arndt 1995, Van Nierop 2007], will use pressure profiles intrinsic to a volume or diverse engineering device, that hampers their correct usage at calculations of bubble behavior in the short vortex chamber of irrotational centrifugal pump. Schematize it is possible conditionally to dissect flow in the vortex chamber, it on two zones: quasisolid rotation of a fluid and potential voretx in at an axial zone [Rogovyi 2007, Syomin 2002]. Both indicated zones have a different radial pressure gradient. The given operation is dedicated to motion study of a bubble in axis zone of the short vortex chamber, where is watched quasisolid rotation of a fluid, as it has a place in rotary vessel. In operation [Syomin 2010] the research of bubble motion in the short vortex chamber, without the registration of quasisolid rotation zone of a fluid was conducted, but because the bubbles at formation of a rotational cord are concentrated in axis zone of the chamber, the gas bubble behavior in this area requires further researches. Besides on performances of irrotational centrifugal pump, exerts influence presence of a gas cord, and thus, it is necessary to clarify, whether it is possible at definite ratio of kinematic parameters of bubble motion, escape by him of axis zone of the pump, and by that fading or not origin in general of gas cord.

As the bubble during the moving passes areas with various pressure, there is a change of bubble radius, it is necessary to allow for which one at calculation, and it is described by an Rayleigh-Plesset equation [Syomin 2010, Plesset 1948]. One of the purposes of the given operation is the justification of Rayleigh-Plesset equation applicability at calculations of gas bubble motion in the short vortex chamber of irrotational centrifugal pumps at transportation dropping liquid.

The purpose of a paper is deriving ratio of kinematic parameters which are exerting influence on gas bubble motion in a transportation dropping liquid in axis zone of irrotational centrifugal pumps. The definition of motion characteristics, at which one is possible evacuating a gas bubble from axis zone.

RESULTS OF RESEARCH

For definition of gas bubble motions in a stream of a transportation dropping liquid most frequently will use the theory of a single spherical cavitational bubble oscillations, i.e. arranged far from other bubbles and other microparticles, walls of a vessel and free surface of a fluid. To take into account change of bubble radius owing to change and effect on it of a gas pressure in a bubble, pressure in a fluid, viscosity and surface-tension, will use a general purpose dynamical equation of a cavitational bubble, or the Rayleigh-Plesset equation [Syomin 2010, Hsiao 2004, Plesset 1948], which one is updated by the optional component injection, bound with a variance of velocities of a bubble moving and fluid flow:

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2+ \\ +\frac{1}{\rho_L}\left[p_\infty-p_v+\frac{2\sigma}{R}+\frac{4\nu}{R}\frac{dR}{dt}-p_0\left(\frac{R_o}{R}\right)^{3k}\right]- \\ -\frac{\left(\overline{V}-\overline{V}_b\right)^2}{4}=0, \quad (1)$$

where: R - current radius of a bubble; p_∞ – the static pressure of a fluid, is received according to [Hsiao 2004], as middle pressure scaled on a surface of a bubble; p_0 – gas pressure in a bubble at $R = R_0$; p_v - saturated vapor pressure in a fluid; ρ_L – density of a fluid; ν – kinematic viscosity of a fluid; σ – surface-tension constant of a fluid; k – polytropic exponent; $\overline{V}$ - velocity of a fluid; $\overline{V}_b$ – velocity of a bubble. The pressure variation of gas inside a bubble owing to change of its radius is received under the adiabatic law of compression [Hsiao 2004, Latorre 1980].

The gas bubble motion equation in view of forces operating on a spherical bubble of radius R is recorded as follows [Maxey 1983, Raoufi 2006]:

$$m_b\frac{d\overline{V}_b}{dt}=\overline{F}_g+\overline{F}_p+\overline{F}_D+\overline{F}_{add}, \quad (2)$$

where: m_b – mass of gas in bubble; $\overline{F}_g$ – gravity; $\overline{F}_p$ – force of fluid pressure on a body surface, immersed in it; $\overline{F}_D$ – drag force [Syomin 2010, Hsiao 2004, Raoufi 2006], $\overline{F}_{add}$ – added mass of a fluid [Pozdeev 2003, Syomin 2010] (fig. 1). Sometimes at calculations of gas bubble motion will use lift force operating on a bubble rotation [Van Nierop 2007] and force, originating owing to change of a bubbles volume [Hsiao 2004, Latorre 19809, Raoufi 2006], which one in the given research were leave outed of account to their smallness, that is connected to features of flow field in fluid rotated with a constant angular velocity.

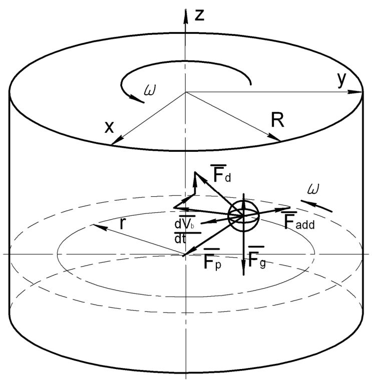


Fig. 1. Forces, operating on a bubble in central area of the vortex chamber

Thus, after a forces substitution and simplifications for steady flow is obtained a following bubble motion equation:

$$\frac{d\overline{V}_b}{dt}=2\frac{\rho_b}{\rho_L}\overline{g}+\frac{2}{\rho_L}\overline{\nabla}p+\frac{3}{4R}C_D\left(\overline{V}-\overline{V}_b\right)\left|\overline{V}-\overline{V}_b\right|. \quad (3)$$

Integrating equations (1) and (3) and recording in projections on an coordinates axes, for gas bubble moving in the fluid rotated field with a constant angular velocity is obtained a system of ordinary non-linear differential equations (4).

The calculations are conducted with the help of a license software Matlab on a Runge-Kutta method of the fourth and fifth orders. For each instant t the coordinates $x(t),\ y(t),\ z(t), r(t),\ r=\sqrt{x^2+y^2}$, velocity $\frac{dx_b}{dt},\ \frac{dy_b}{dt},\ \frac{dz_b}{dt},\ \frac{dR}{dt}$ were determined. The outcomes of calculation of radius bubble change R, in different points of the chamber are displayed in a fig. 2.

$$\begin{cases} R\dfrac{d^2R}{dt^2}+\dfrac{3}{2}\left(\dfrac{dR}{dt}\right)^2+\dfrac{1}{\rho_L}\left[p_\infty-p_v+\dfrac{2\sigma}{R}+\dfrac{4\nu}{R}\dfrac{dR}{dt}-p_0\left(\dfrac{R_o}{R}\right)^{3k}\right]- \\ -\dfrac{\left(\overline{V}-\overline{V}_b\right)^2}{4}=0; \\ \dfrac{dV_{bx}}{dt}=2\omega^2x+\dfrac{3}{4R}C_D\left(V_x-V_{bx}\right)\left|V_x-V_{bx}\right|; \\ \dfrac{dV_{by}}{dt}=2\omega^2y+\dfrac{3}{4R}C_D\left(V_y-V_{by}\right)\left|V_y-V_{by}\right|; \\ \dfrac{dV_{bz}}{dt}=-2\dfrac{\rho_b}{\rho_L}g+2g+\dfrac{3}{4R}C_D\left(V_z-V_{bz}\right)\left|V_z-V_{bz}\right|; \\ C_D=\dfrac{24}{\mathrm{Re}_b}\left(1+0{,}197\cdot\mathrm{Re}_b^{0{,}63}+2{,}6\cdot10^{-4}\,\mathrm{Re}_b^{1{,}38}\right); \\ \mathrm{Re}_b=\dfrac{2R\left|V-V_b\right|}{\nu}, \end{cases} \quad (4)$$

where: ω – constant angular velocity of fluid rotation; ρ_b - gas density in a bubble.

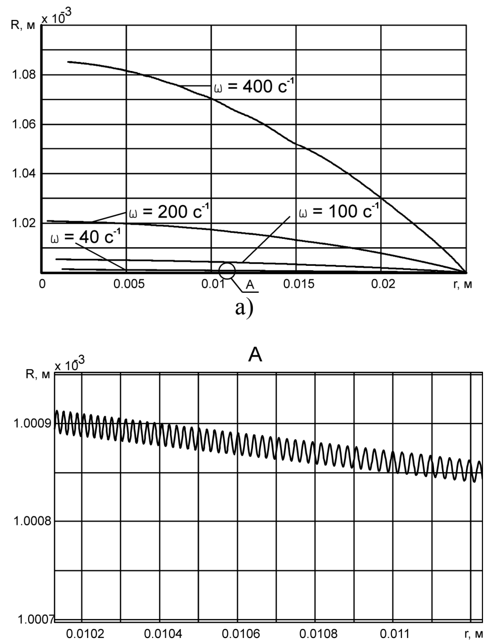
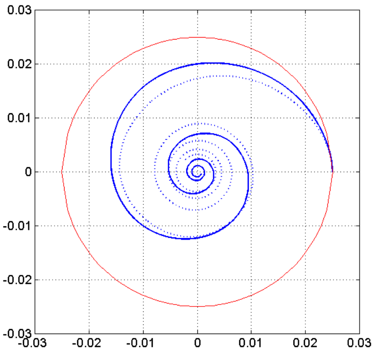
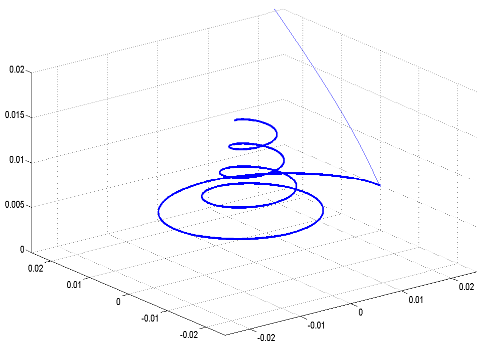


Fig. 2. Dependence of bubble radius to radius of the vortex chamber

As it is visible from a fig. 2, greater angular fluid rotation velocity, there corresponds the greater change of a radius bubble value. For $\omega \le 200$ s^{-1} change of bubble radius on exceeds of one percent. Besides as usage of an Rayleigh-Plesset equation is visible from a fig. 2.b, to take into account influencing change of bubble radius on its motion characteristics, results that time of calculation owing to a considerable variability of radius change is augmented sharply. Thus, at $\omega \le 100$ s^{-1}, there is no necessity to apply to calculation of bubble radius of an Rayleigh-Plesset equation, and simplistically in further calculations it is possible to count, that the motion bubble in the a fluid rotated field with a constant angular velocity, has constant radius, besides the time expended on calculation is diminished.



a)



b)

Fig. 3. Results of a trajectory calculation of a gas bubble in a fluid rotated with a constant angular velocity (a - in meridian plane, b - in space)

In a fig. 3 typical trajectories of gas bubble motion in the vortex chamber are displayed. They have the good agreement with theoretical and experimental researches, cited in operation [Van Nierop 2007]. At change of main specifications influential in motion characteristics of moving (ω, R_b, ν), aspect of trajectories invariable, that is visible from a fig. 3.a, where the solid and dash lines two trajectories with a various set ω, R_b, ν. The main difference between trajectories - time, during which one a gas bubble reaches zone of an axial output channel of the vortex chamber. Thus, at an identical aspect of trajectories, two versions of bubble moving in the vortex chamber are possible: if the bubble reaches an axis, it there and remains, if the bubble reaches the upper end wall of the vortex chamber (light line in a fig. 3.b) earlier, than axis of the vortex chamber, it can be evacuated by flow of a fluid in a boundary layer on end walls escaping the vortex chamber.

By main specifications influential in time, for which one the bubble can reach radius of an output axial channel, are ω, R_b, ν, that is visible from a set of equations (4). As at bubble motion in a fluid flow through a definite instant, velocity of a bubble and the fluids are completed, it is possible to consider, that the flow steamlining of a bubble takes place to small numbers of the Reynolds Re_b, then it is possible to suppose, that $C_D = \dfrac{24}{\mathrm{Re}_b}$ or $C_D = \dfrac{12\nu}{R\omega r}$. Then, by transformations of a system (4) it is possible to receive, that the basic complex which is exerting influence on a trajectory looks like this - $\dfrac{2}{9}\dfrac{\omega R}{\nu}$. It is confirmed by outcomes of timing of reaching by a gas bubble of radius of an output axial channel of the short vortex chamber $\left(\overline{R}_0 = \dfrac{R_0}{R_k} 0,2\right)$, displayed on a fig. 4.

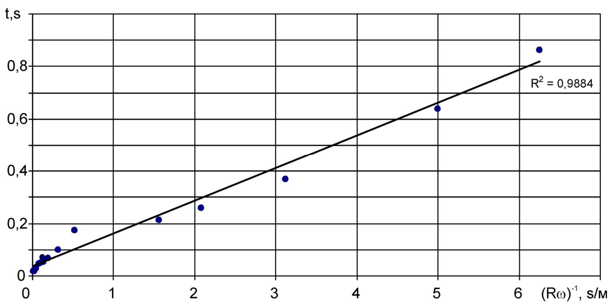


Fig. 4. Dependence of bubble moving time in the chamber from a complex of parameters $R\omega$

As the requires time for a gas bubble is visible from a fig. 4, to reach an axial channel in an end wall of the vortex chamber, is directly proportional to a complex of magnitudes $(R\omega)^{-1}$, that is confirmed by quadrate of a mixed correlation, equal 0,988. The relation of time to kinematic viscosity of a fluid is not showed, as the researches were conducted at constant viscosity, equal water viscosity. Generally speaking, time of moving, as well as the motion characteristics of bubble moving depend on a complex of values - $\dfrac{\nu}{\omega R}$. Thus, than more angular velocity of a fluid rotation, the more initial gas bubble radius, that the smaller time is necessary for it to reach an axis of the vortex chamber, and on the contrary, the smaller angular velocity of rotation and radius, the greater is necessary time.

CONCLUSIONS

1. The more angular velocity of a fluid rotation, the greater change of bubble radius at moving it to an axis of the vortex chamber. For $\omega \le 100$ s^{-1} change of bubble radius on exceeds of one percent.

2. At $\omega \le 100$ s^{-1}, there is no necessity to apply to calculation of a bubble radius an Rayleigh-Plesset equation, and it is possible to count, that the bubble at moving in the field of a fluid rotated with a constant angular velocity, has constant radius.

3. Two versions of a bubble moving in the vortex chamber are possible, that it will reach - axis of the chamber or end wall earlier. If the bubble reaches an axis, it there and remains, if the bubble reaches an end wall, it can be evacuated by flow of a fluid in a boundary layer on end walls escaping the vortex chamber.

4. The time for a gas bubble to reach an axial channel in an end wall of the vortex chamber, is directly proportional to a complex of magnitudes $(R\omega)^{-1}$. Therefore, than more angular velocity of a fluid rotation, and, the more initial radius of a gas bubble, that the smaller time is necessary for it to reach an axis of the vortex chamber, and on the contrary, the smaller angular velocity of rotation and radius, the greater is necessary time.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДВИЖЕНИЯ ГАЗОВОГО ПУЗЫРЬКА В ЦЕНТРАЛЬНОЙ ОБЛАСТИ КОРОТКОЙ ВИХРЕВОЙ КАМЕРЫ

Сёмин Дмитрий, Роговой Андрей

Аннотация. Приведены результаты теоретических исследований движения изолированного газового пузырька в центральной области короткой вихревой камере в капельной несущей среде. Составлена математическая модель, описывающая траекторию движения пузырька под влиянием сил, действующих на него, в которой учтено изменение радиуса пузырька. Получено соотношение величин, оказывающих основное воздействие на кинематические параметры движения пузырька. Результаты исследований могут быть применены при исследовании поведения пузырька во вращающейся с постоянной угловой скоростью капельной жидкости.

Ключевые слова: траектория движения, газовый пузырек, капельная жидкость, угловая скорость, вихревая камера.