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Elastostatic problems in multicomponent, multilayered periodic composites

Key words: elastostatics, multicomponent composite, tolerance modelling, oscillating micro-shape function

Introduction

The object of the presented study is tolerance modelling for an elastostatic problem in a multicomponent, multilayered periodical structure. Tolerance modelling (tolerance averaging technique) proposed by Woźniak (1993, 1999) is well known in the literature and is applied to investigate various problems for two-component periodic and functionally graded (FGM) structures. Applications of this theory in thermomechanics and dynamics can be found in monographies by Woźniak and Wierzbicki (2000), Woźniak, Michalak and Jędrysiak (2008) as well as Woźniak (2010). Stationary elastic problems, using the asymptotic variant of tolerance modelling, were investigated in Wągrowska and Witkowska-Dobrev (2010), Witkowska-Dobrev (2014) as well as Witkowska-Dobrev and Wągrowska (2015). These publications also contained the analysis of the boundary layer effect for multilayered composites with transversal and longitudinal gradation of effective properties. In the above-mentioned papers the examples were narrowed to one-dimensional problems. Elastostatic problems for bifunctionally graded composites within the framework of tolerance modelling were studied by Czarnecka (2014). Thermal stresses in periodic two-component multilayered structures were considered by Bagdasaryan (2016). Perliński, Gajdzicki and Michalak (2014) analysed the stability problems for a thin annular two-component functionally graded plate interacting with elastic subsoil. Two models (tolerance and asymptotic) of thermo-elasticity problems for two-component transversally graded laminates were proposed by Jędrysiak (2011) as well as Pazera and Jędrysiak (2015). A similar problem was analysed by

Ostrowski (2014) for a two-component longitudinally graded hollow cylinder.

The tolerance modelling for structures which are composed of more than two materials was conducted for the heat conduction problems. The basic concept for modelling of heat conduction in multicomponent composites was presented by Woźniak (2012, 2013) and applied for periodic structures by Wągrowska and Woźniak (2014), Szlachetka and Wągrowska (2015), Wągrowska and Szlachetka (2016b) and for composites with transversal gradation by Szlachetka and Wągrowska (2016), Wągrowska and Szlachetka (2016a). The basic difference between the modelling of multicomponent composites and the modelling of two-component composites is the new form of the shape function which is called an oscillating micro-shape function.

The primary aim of this paper is, basing on Woźniak (2012, 2013), to propose the form of the oscillating micro-shape function for elastostatic problems for a multicomponent multilayered composite and present some examples of boundary value problems.

Object of analysis

Let the physical space be parameterized by an orthogonal Cartesian coordinate system $Ox_1x_2x_3$. The object of analysis is a periodic, multicomponent, multilayered, elastic composite which occupies a region $\Omega \equiv (0, L_1) \times (0, L_2) \times (0, L_3)$ in the physical space and consists of a large number $N\left(\frac{1}{N} << 1\right)$ of layers with constant thickness η, $\eta = \frac{L_1}{N}$.

Each layer is composed of P sublayers made of M homogeneous, orthotropic, perfectly combined linear elastic materials. The number of sublayers is at least equal to the number of materials. Let us assume that the axes of orthotropy of the components coincide with the axes of the coordinate system $Ox_1x_2x_3$. The scheme of the periodic layer of the considered composite is presented in Figure 1.

For the two-dimensional problem the elastic material properties in the p-th, $p = 1, ..., P$, orthotropic sublayer are described by the values of elastic modulus tensors:

$$C_p = \begin{bmatrix} C_p^{1111} & C_p^{1122} & 0 \\ & C_p^{2222} & 0 \\ & & C_p^{1212} \end{bmatrix}, \quad p = 1, .., P$$

Moreover, let $\varphi_p, p = 1, ..., P$, be positive constant values, such that $\varphi_1 + ... + \varphi_P = 1$. The thickness of the p-th ($p = 1, ..., P$) sublayer in each layer is equal to $\eta_p = \eta\varphi_p$.

Let us introduce the decomposition of the i-th interval of periodicity into P subintervals Δ^i_p which are defined as:

$$\Delta^i_p \equiv \left(\eta(i-1) + \sum_{k=1}^{p-1} \varphi_k \eta_k, \left(\eta(i-1) + \sum_{k=1}^{p} \varphi_k \eta_k \right) \right)$$

$p = 1, 2, ..., P, \ i = 1, 2, ..., N$

The set which is occupied by the p-th sublayer in the discussed composite can be described as follows:

$$\Omega_p = \sum_{i=1}^{N} \Delta^i_p \times (0, L_2) \times (0, L_3),$$

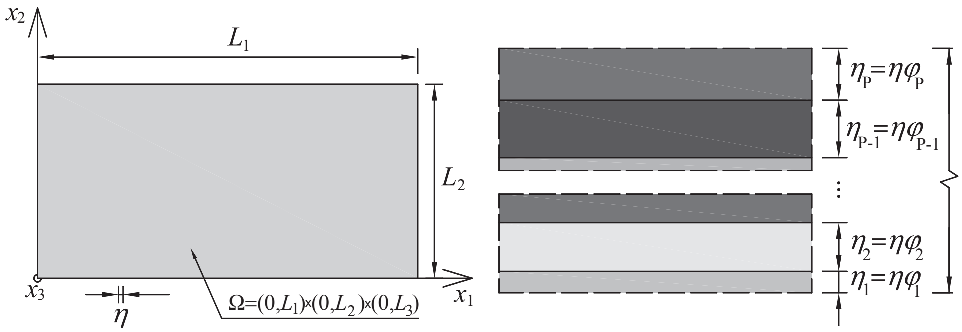

FIGURE 1. The scheme of a multicomponent multilayered periodic composite and its periodic layer

$p = 1, 2, ..., P$

For the two-dimensional elastostatic problems the displacements are not functions of time and they depend on two variables x_1 and x_2, where $x_1 \in (0, L_1)$, $x_2 \in (0, L_2)$.

Assuming that the body forces per unit volume are equal to zero, the equations of equilibrium for the two-dimensional problem take the form:

$$\sigma^{11}_{,1} + \sigma^{12}_{,2} = 0$$
$$\sigma^{12}_{,1} + \sigma^{22}_{,2} = 0$$

For orthotropic components of composite these equations in displacements take the form:

$$\begin{aligned}
&\left(C^{1111}u_{1,1} + C^{1122}u_{2,2}\right)_{,1} + \\
&+\left(C^{1212}\left(u_{1,2} + u_{2,1}\right)\right)_{,2} = 0 \\
&\left(C^{1212}\left(u_{1,2} + u_{2,1}\right)\right)_{,1} + \\
&+\left(C^{1122}u_{1,1} + C^{2222}u_{2,2}\right)_{,2} = 0
\end{aligned} \tag{1}$$

where $C^{ijkl} = C^{ijkl}_p$ when $(x_1, x_2) \in \Omega_p$,

$p = 1, 2, ..., P$

Equations (1) are a system of partial differential equations with discontinuous and highly oscillating coefficients. The solution of these equations is very difficult or even impossible. That is why there are so many methods of finding the approximated solution of these equations. Among them the following methods can be distinguished: asymptotic homogenization (Jikov, Kozlov & Oleinik, 1994), modelling with microlocal parameters (Matysiak, 1994) and tolerance modelling (Woźniak, 1999). This paper applies the tolerance modelling method.

Modelling concepts

In the process of tolerance modelling for periodic composites notions of slowly varying function and tolerance averaging approximation are needed. These notions will be cited as in Woźniak et al. (2010).

Slowly varying functions

Let stand for an arbitrary convex set in the space R^m, and $f \in C^1(\Pi)$ be an arbitrary real-valued function. Let us define the tolerance parameter $d \equiv (\eta, \delta_0, \delta_1)$ as a triplet of real positive numbers.

The notation $\partial_j \equiv \dfrac{\partial}{\partial x_j}$, $j = 1, ..., m$ will be used.

Function $f \in C^1(\Pi)$ is a weakly slowly varying function $\left(f \in \mathrm{WSV}_d^1(\Pi) \subset C^1(\Pi)\right)$ if the condition $\|\mathbf{x}-\mathbf{y}\| \le \eta$ implies the conditions $\left|f(\mathbf{x})-f(\mathbf{y})\right| \le \delta_0$ and $\left|\partial_j f(\mathbf{x})-\partial_j f(\mathbf{y})\right| \le \delta_1$ for $j = 1, ..., m$ and for all $(\mathbf{x},\mathbf{y}) \in \Pi^2$.

Function $f \in \mathrm{WSV}_d^1(\Pi)$ is a slowly varying function $\left(f \in \mathrm{SV}_d^1(\Pi)\right)$ if conditions $\eta\left|\partial_j f(\mathbf{x})\right| \le \delta_0$ hold for $j = 1, ..., m$ for every $\mathbf{x} \in \Pi$.

Obviously, $\mathrm{WSV}_d^1(\Pi) \supset \mathrm{SV}_d^1(\Pi)$.

Tolerance averaging approximation

Define interval $\Delta \equiv \left(-\dfrac{\eta}{2}, \dfrac{\eta}{2}\right)$ and a local interval $\Delta(x) \equiv \left(x-\dfrac{\eta}{2}, x+\dfrac{\eta}{2}\right)$ for every $x \in [0, L]$.

Let $f_x \in L^2((0, L))$. The averaging of function f in point x over interval $\Delta(x)$ is equal to: $\langle f\rangle(x) \equiv \dfrac{1}{\eta}\int\limits_{\Delta(x)} f_x(z)dz$.

Let $f_x \in L^2(\Delta(x))$ and $F \in \mathrm{WSV}_d^1((0,L))$. The tolerance averaging approximation of functions $\langle fF\rangle_T(x)$, $\langle f\partial_1 F\rangle_T(x)$ is given by functions $\langle f\rangle(x)F(x)$ and $\langle f\rangle(x)\partial_1 F(x)$, respectively.

$$\langle fF\rangle_T(x) \equiv \langle f\rangle(x)F(x),$$

$$\langle f\partial_1 F\rangle_T(x) \equiv \langle f\rangle(x)\partial_1 F(x)$$

Oscillating micro-shape function

Function $\gamma(\cdot)$ is an oscillating micro-shape function (for linear elastostatic problems) if $\gamma(\cdot)$ is piecewise linear, with values on the interfaces between sublayers of a periodicity layer given by

$$\gamma_p = \gamma_{p-1} + \eta\varphi_p\left(\frac{C_0^{1111}}{C_p^{1111}} - 1\right), p = 1, 2, ..., P$$

where $C_0^{1111} \equiv \left(\sum\limits_{i=1}^{P}\dfrac{\varphi_i}{C_i^{1111}}\right)^{-1}$ and $\langle\gamma\rangle = 0$.

An example of an oscillating micro-shape function for a three-component structure is presented in Figure 2.

Modelling procedure

The process of tolerance modelling is based on two assumptions. The first assumption says that the displacement field $\boldsymbol{u}(\cdot)$ is approximated by $\tilde{\boldsymbol{u}}(\cdot)$ in the form, Woźniak et al. (2010):

$$\boldsymbol{u}(\boldsymbol{x}) \approx \tilde{\boldsymbol{u}}(\boldsymbol{x}) = \boldsymbol{w}(\boldsymbol{x}) + \gamma(x_1)\boldsymbol{v}(\boldsymbol{x}) \qquad (2)$$

Fields $\boldsymbol{w}(\cdot, x_2)$, $\boldsymbol{v}(\cdot, x_2) \in SV_d^1((0,L_1))$ are unknown vectors, which are called macro-displacement and the amplitude of fluctuation of displacement, $\gamma(\cdot)$ is the oscillating micro-shape scalar function, which is given *a priori*.

Before formulating the second assumption let us define the residual field of $\tilde{\boldsymbol{u}}(\cdot)$ in the region $\boldsymbol{\Omega}$, Woźniak et al. (2015):

$$r_i(x_1) \equiv \left(C^{ijkl}(x_1)\frac{1}{2}\left(\tilde{u}_k(x_1)_{,l} + \tilde{u}_l(x_1)_{,k}\right)\right)_{,j}$$

for $i, j, k, l = 1, 2$

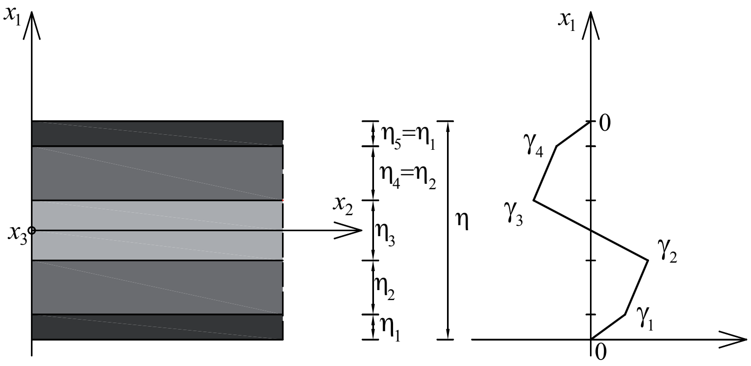

FIGURE 2. An example of an oscillating micro-shape function for a three-component periodic composite

The second assumption can be written with formulas:

$$\langle r_i \rangle_T(\cdot) = 0, \ \langle \gamma r_i \rangle_T(\cdot) = 0, \ \ i = 1, \ 2$$

Modelling equations

Taking into account both of the mentioned assumptions and that $\boldsymbol{w}(\cdot, x_2)$, $\boldsymbol{v}(\cdot, x_2) \in SV_d^1((0, L_1))$, the system of equations for unknown functions $\boldsymbol{w}(\cdot)$ and $\boldsymbol{v}(\cdot)$ takes the form (Woźniak, Michalak & Jędrysiak, 2008):

$$\begin{aligned}
&\langle C^{1111}\rangle w_{1,11} + \langle C^{1111}\gamma_{,1}\rangle v_{1,1} + \\
&+\langle C^{1122}\rangle w_{2,21} + \langle C^{1212}\rangle w_{2,12} + \\
&+\langle C^{1212}\gamma_{,1}\rangle v_{2,2} + \langle C^{1212}\rangle w_{1,22} = 0 \\
&-\langle C^{1111}\gamma_{,1}\rangle w_{1,1} - \langle C^{1111}(\gamma_{,1})^2\rangle v_1 + \\
&+\underline{\langle C^{1111}(\gamma)^2\rangle} v_{1,11} - \langle C^{1122}\gamma_{,1}\rangle w_{2,2} + \\
&+\underline{\langle C^{1122}(\gamma)^2\rangle} v_{2,21} + \underline{\langle C^{1212}(\gamma)^2\rangle} v_{2,12} + \\
&+\underline{\langle C^{1212}(\gamma)^2\rangle} v_{1,22} = 0 \qquad (3)
\end{aligned}$$

$$\begin{aligned}
&\langle C^{1212}\rangle w_{2,11} + \langle C^{1212}\gamma_{,1}\rangle v_{2,1} + \\
&+\langle C^{1212}\rangle w_{1,21} + \langle C^{1122}\rangle w_{1,12} + \\
&+\langle C^{1122}\gamma_{,1}\rangle v_{1,2} + \langle C^{2222}\rangle w_{2,22} = 0 \\
&-\langle C^{1212}\gamma_{,1}\rangle w_{2,1} - \langle C^{1212}(\gamma_{,1})^2\rangle v_2 + \\
&+\underline{\langle C^{1212}(\gamma)^2\rangle} v_{2,11} - \langle C^{1212}\gamma_{,1}\rangle w_{1,2} + \\
&+\underline{\langle C^{1212}(\gamma)^2\rangle} v_{1,21} + \underline{\langle C^{1122}(\gamma)^2\rangle} v_{1,12} + \\
&+\underline{\langle C^{2222}(\gamma)^2\rangle} v_{2,22} = 0
\end{aligned}$$

The above system of partial differential equations and formula (2) represent what will be called the standard tolerance model.

It should be emphasized that the system of Eqs. (3) obtained in the process of tolerance modelling has constant coefficients in contrast to the Eqs. (1).

The underlined components in the Eqs. (3) depend on the length parameter η. If $\eta \rightarrow 0$, then the asymptotic model is obtained and it is possible to determine the amplitudes of fluctuation by displacements:

$$v_1 = -\frac{\left\langle C^{1111}\gamma_{,1}\right\rangle w_{1,1} + \left\langle C^{1122}\gamma_{,1}\right\rangle w_{2,2}}{\left\langle C^{1111}\left(\gamma_{,1}\right)^2\right\rangle}$$

$$v_2 = -\frac{\left\langle C^{1212}\gamma_{,1}\right\rangle \left(w_{1,2} + w_{2,1}\right)}{\left\langle C^{1212}\left(\gamma_{,1}\right)^2\right\rangle}$$

Then the equations for unknown displacement fields $w_1(\cdot)$ and $w_2(\cdot)$ take the form:

$$\left(C_0^{1111} w_{1,1} + \tilde{C}^{1122} w_{2,2}\right)_{,1} + \\ + \left(C_0^{1212}\left(w_{1,2} + w_{2,2}\right)\right)_{,2} = 0$$

$$\left(C_0^{1212}\left(w_{1,2} + w_{2,2}\right)\right)_{,1} + \\ + \left(\tilde{C}^{1122} w_{1,1} + \tilde{C}^{2222} w_{2,2}\right)_{,2} = 0$$

where:

$$C_0^{1111} = \left\langle C^{1111}\right\rangle - \frac{\left(\left\langle C^{1111}\gamma_{,1}\right\rangle\right)^2}{\left\langle C^{1111}\left(\gamma_{,1}\right)^2\right\rangle}$$

$$\tilde{C}^{1122} = \left\langle C^{1122}\right\rangle - \frac{\left\langle C^{1111}\gamma_{,1}\right\rangle \left\langle C^{1122}\gamma_{,1}\right\rangle}{\left\langle C^{1111}\left(\gamma_{,1}\right)^2\right\rangle}$$

$$C_0^{1212} = \left\langle C^{1212}\right\rangle - \frac{\left(\left\langle C^{1212}\gamma_{,1}\right\rangle\right)^2}{\left\langle C^{1212}\left(\gamma_{,1}\right)^2\right\rangle}$$

$$\tilde{C}^{2222} = \left\langle C^{2222}\right\rangle - \frac{\left(\left\langle C^{1122}\gamma_{,1}\right\rangle\right)^2}{\left\langle C^{1111}\left(\gamma_{,1}\right)^2\right\rangle}$$

Examples

This section presents the distribution of an approximate displacement field for two specific cases of multicomponent multilayered periodic composites. It is assumed that all materials of the discussed composites are homogeneous and isotropic, so the values of elastic modules are reduced to: $C^{1111} = C^{2222} = 2\mu + \lambda$, $C^{1122} = \lambda$, $C^{1212} = \mu$ where λ, μ are Lamé parameters.

Let us assume that the composite, which occupies the region $\Omega \equiv (0, L_1) \times (0, L_2)$ where $L_1 = 1.2$ m, $L_2 = 1$ m, is composed of $P = 12$ layers with constant thicknesses $\eta = 10$ cm. It means that the thickness of the periodicity layer is equal to 10 cm. The periodicity layer consists of five sublayers made of three different materials. Thicknesses of sublayers “1”, “5”, are equal to 0.1η ($\eta_1 = \eta_5 = 1$ cm), thicknesses of sublayers “2”, “4” are equal to 0.2η ($\eta_2 = \eta_4 = 2$ cm) and the thickness of sublayer “3” is equal to 0.4η ($\eta_3 = 4$ cm). The sublayers made of the same material are distributed symmetrically with respect to the midplane of the periodicity layer.

The Lamé parameters related to the corresponding sublayers in considered cases are shown in the Table.

The graphs of the oscillating micro-shape functions $\gamma(\cdot)$ for considered examples are shown in Figure 3.

It should be noted that if sublayers made of the same material are symmetrically distributed with respect to the midplane of periodicity layer, the graph of the oscillating micro-shape function is antisymmetric with respect to this midplane and that the oscillating micro-shape function is equal to 0 on the edges of the periodicity layer.

TABLE. Lamé parameters

Case	Parameter [Pa]	Sublayer				
		1	2	3	4	5
1	$\lambda\ (\cdot 10^{10})$	4.583	5.108	9.515	5.108	4.583
	$\mu\ (\cdot 10^{10})$	0.625	2.632	4.478	2.632	0.625
2	$\lambda\ (\cdot 10^{10})$	5.108	4.583	9.515	4.583	5.108
	$\mu\ (\cdot 10^{10})$	2.632	0.625	4.478	0.625	2.632

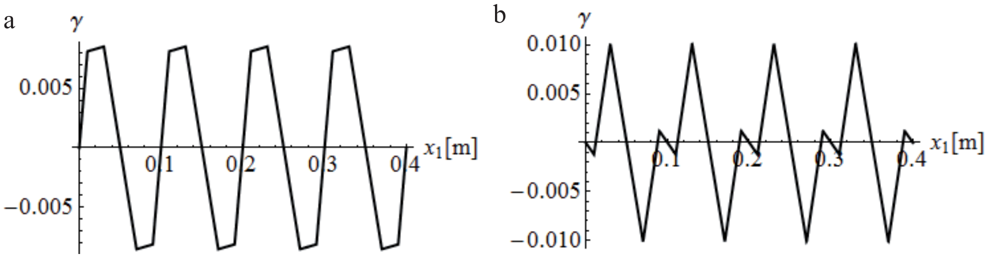

FIGURE 3. Graphs of the oscillating micro-shape function $\gamma(\cdot)$ in periodicity layer in Case 1 (a) and in Case 2 (b)

The boundary conditions for the displacements in both cases are: $w_1(0, x_2) = f(x_2)$, $w_1(L_1, x_2) = 0$, $w_1(x_1, 0) = 0$, $w_1(x_1, L_2) = 0$ and $w_2(0, x_2) = 0$, $w_2(L_1, x_2) = 0$, $w_2(x_1, 0) = 0$, $w_2(x_1, L_2) = 0$ where $f(x_2) = w_0 \sin\left(\frac{\pi x_2}{L_2}\right)$, $w_0 = 0.1$ m. The distributions of the macro-displacements w_1 and w_2, as well as displacements $\tilde{u}_1$ and $\tilde{u}_2$ for Case 1 are shown in Figures 4 and 5, respectively. Figure 6 presents the cross-sections of the macro-displacements w_1 and w_2, as well as displacements $\tilde{u}_1$ and $\tilde{u}_2$ for $x_2 = 0.1L_2, x_2 = 0.25L_2$ $x_2 = 0.5L_2$ in Case 1.

For Case 2 the cross-sections of the approximated displacements $\tilde{u}_1$ and $\tilde{u}_2$ for $x_2 = 0.1L_2, x_2 = 0.25L_2, x_2 = 0.5L_2$ are presented in Figure 7.

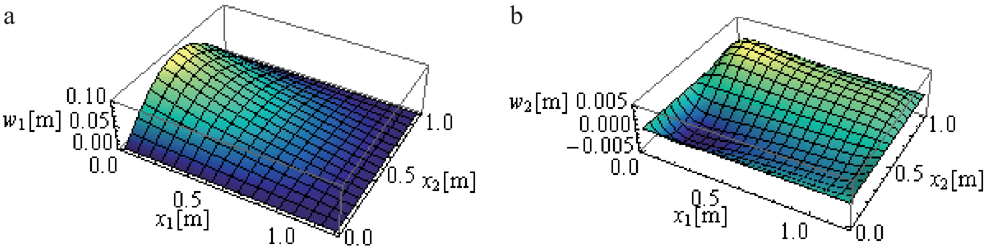

FIGURE 4. The distributions of macro-displacements: a – w_1, b – w_2 (Case 1)

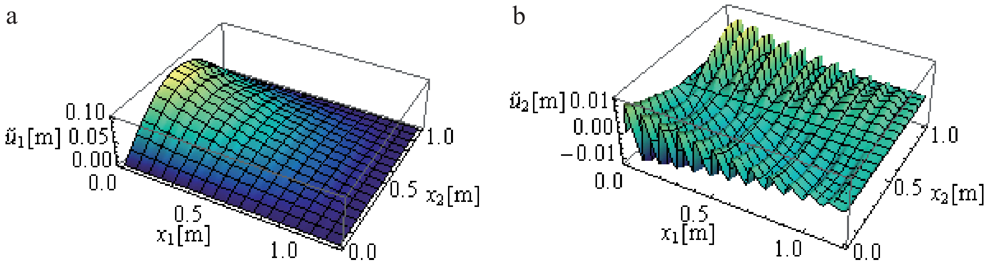

FIGURE 5. The distributions of the approximated displacements: a – $\tilde{u}_1$, b – $\tilde{u}_2$ (Case 1)

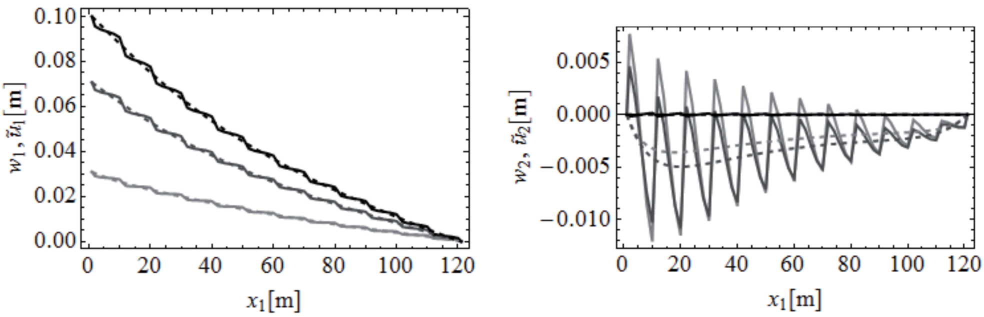

FIGURE 6. The distributions of the approximated displacements $\tilde{u}_1$ and $\tilde{u}_2$ (the continuous line) and macro-displacements w_1 and w_2 (the dashed line) for $x_2 = 0.1L_2$ – the light grey line, $x_2 = 0.25L_2$ – the dark grey line, $x_2 = 0.5L_2$ – the black line (Case 1)

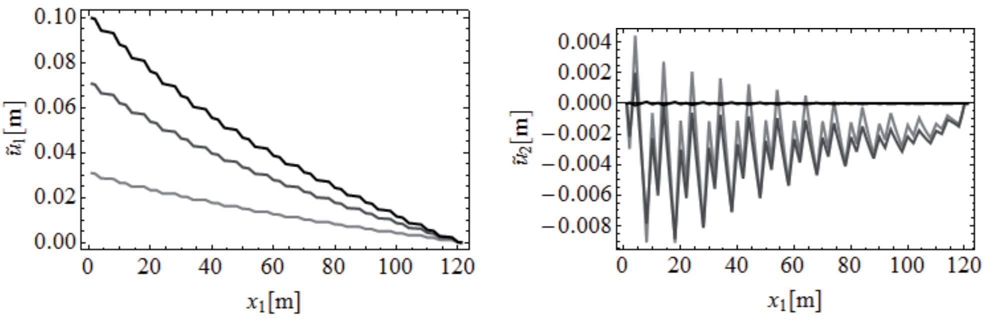

FIGURE 7. The distributions of the approximated displacements $\tilde{u}_1$ and $\tilde{u}_2$ for $x_2 = 0.1L_2$ – the light grey line, $x_2 = 0.25L_2$ – the dark grey line, $x_2 = 0.5L_2$ – the black line (Case 2)

Conclusions

Previous papers concerning the tolerance modelling analysed only two-component composites. The shape function formulated there had the form which was proper only for the structures of this type. A limit pass to a one-component body within the framework of this model was not possible (Wągrowska & Szlachetka, 2016b). The term of an oscillating micro-shape function for heat conduction problems, introduced by Woźniak (2012, 2013), makes it possible to describe the periodic multilayered composites made of many components. The form of this function gives the possibility of the limit pass from a multicomponent to a one--component body. Based on the definition of an oscillating micro-shape function for heat conduction problems, in this paper the form of an oscillating micro--shape function for elastostatic problems was proposed. This function has the same properties as an oscillating micro-shape function for heat conduction problems.

The influence of the structure of the composites is visible only on the approximated displacements distribution (compare Fig. 4 and Fig. 5 or lines of the same colour on Fig. 6). The presented examples show that approximated displacements distribution strongly depends on the distribution of sublayers made of the given material. The calculations were made with the use of Mathematica 8.0 software.

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Summary

Elastostatic problems in multicomponent, multilayered periodic composites. The object of the analysis is a two-dimensional elastostatic problem for multicomponent, multilayered periodic composites. The equations of equilibrium for this composite are obtained within the framework of tolerance modelling procedure. The paper presents two examples of solutions of boundary value problems.

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