ORIGINAL ARTICLE

Economic optimization of the rotation age of stands

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Abstract

The central task of this research was to choose the age at which stands of growing timber should be harvested. The choice governs how long each stand must continue to earn interest, and also governs the size of the total inventory that must be maintained to sustain the annual harvest. It is a problem that calls for analysis of biological as well as economic relationships over time, and has intrigued economists for more than two centuries. The paper presents the theoretical background and results of economic optimization of the rotation age of a single stand. It analyses the timber production function depending on rotation age, growth, cost and other characteristics of forest, as well as the costs of land. The prerequisite for achieving the economic optimum of the rotation age of a stand is to balance the current increase in the total timber production value (growth and yield) and the value of opportunity costs from delaying cutting till the next year. This paper demonstrates how this economically optimum rotation age can be calculated, and how it varies according to the biological growth and economic parameters of the forest.

Introduction

One of the most critical economic questions in forestry is the age at which a single tree or forest stand should be harvested, or the crop rotation period.

Foresters have develop many criteria for selecting the age to cut forest stands, some of them do not take into consideration any of the economic factors involved. Some good examples of criteria are the age at which the trees reach a best size for making some products, the age at which the volume in stands is maximized, and the age at which the rate of growth in volume is maximized. These technical criteria have widely divergent rotation ages with major implications on the costs and benefits generated. This paper focuses on finding the age that will yield maximum economic returns, taking into account how the volume and technical characteristics of the forest change within age are reflected in its economic value.

Here, determining the most economically advantageous crop rotation period is regarded as an investment

problem. The goal is to find the age at which the net benefits from growing stock of timber are maximized. Since the costs and benefits associated with a forest stand accrue at different times, both must be discounted to their present values to be compared. Then the age, which shows the greatest difference between the present value of benefits and the present value of costs, can be identified. This paper demonstrates how this economically optimum rotation age can be calculated, and how it varies according to the biological growth and economic parameters of the forest.

To present the problem as clear as possible, it is necessary to make some initial simplifications. To begin with, let's assume that the only benefit of concern is commercial timber. Recreational benefits, wildlife, and the other non-wood forestry products and values will be set aside. Anyhow, it is necessary to note, that some of these other forest values may call for modification of the calculated rotation period.

It is also convenient to assume that the management regime involves clear cutting the entire crop when it reaches harvesting age. It means that determining the rotation period of even-aged forests, where rotation period is the number of years between complete harvests of all trees on the site. All costs and prices remain constant during the growing period.

The issue of the volume and value of timber production, which is the basis of economic optimization of the rotation age of forests, is covered in the regulation of forest utilization. According to Siekierski (1995), such a regulation requires, on the one hand, to reconcile the multiple functions of forest, each of which should be fulfilled to the highest degree, and to respect the principles of forest sustainability, on the other hand.

The traditional ways of regulating forest utilization put the greatest emphasis on the timber production function. Felling forest at the age considered optimal from the point of view of the set goal best fulfils this function. The ideal situation is when the productive and non-productive functions are positively correlated, which means that by maximizing the productive function of forests, their ability to perform the non-productive functions is also increased. According to Dawidziuk and Czuba (1993), the condition of the forest most desirable from the point of view of timber production best fulfils the other functions.

Nobel laureate in economics Samuelson (1976) spoke about the matters of regulation of timber harvest. Experts in the discussed issue were in the past, among others: Faustmann (1849), Pressler (1860), Ohlin (1921), Goundrey (1960), Klocek and Rutkowski (1986), Smykala (1993), Yin and Newman (1995), Olschewski and Benitez (2010), Asante at all (2011), Piekutin and Skręta (2012), and Klocek and Piekutin (2015).

Rotation age in the forest management practice

In Polish forest management practice, crop rotation period is considered as one of the elements of forest management planning. Cutting age means the age at which the single tree or forest stand is considered mature for felling, and felling maturity is the condition most desirable from the standpoint of forest management objective. This means that the rotation age should be considered as one of the factors ensuring the maximum achievement of the objective of the silvicultural practice. This objective is accepted in the forest management plan, in general, for a given forest management unit (1–3 per one forest management unit) and, in particular, for a forest stand or a group of forest stands. Additionally, it is suggested that the rotation age serving the management objective should result from the expertise, taking into account the specific conditions and using a reliable optimization account method.

The rotation age of the main tree species accepted for a forest management unit in the forest management plan are regarded as:
a) optimal for natural–forest regions,
b) average for the dominant tree species,
c) long-term, that is for periods longer than 10 years.

However, it is not clear from the above that the rotation age established so far ensures the maximum achievement of the set management objectives.

In practice, it is recommended that the rotation age of a single stand be specified in the stand description, which may differ in plus or in minus from the rotation age adopted for the dominant species, depending on the current condition of the stand. Therefore, the concept of real felling age of the stand has been introduced, which is the minimum age that the main dominant tree species must reach. Although the real felling age is not specified in the forest management plan, it must be taken into consideration in all final cuts, ensuring that:

a) average cutting age of the main tree species in a given forest management unit is longer than the real felling age of this tree species by at least 10 years,

b) the age of felling maturity is not shorter than the real felling age of the stand,

c) the age of the dominant tree species in the forest is not shorter than the real felling age of the forest in the year of the final harvest.

The current rules for determining rotation age assume that:

1) the applied rotation age derives from the geographical diversification of the natural conditions of forest production, forest condition and the set production and non-production goals; the term 'production goal' should be understood as the production of the desired amount of sawmill timber, while minimizing losses in average volume increment;

2) forest management unit is the basic unit in which timber harvest is subject to regulation; the possibility of differentiating rotation ages in individual management units is foreseen; the rules of forest sustainability, continuity of use and fulfilment of non-productive functions constitute the basis for regulating the level of harvest;

3) rotation age characterizes the average maturity age of a forest stand earmarked for the final harvest; however, it is not synonymous with the felling age of a specific forest stand, as stand maturity for commercial use, is not the only criterion for determining the urgency of felling of a specific forest stand, as the deciding criteria are: forest functions and the commercial maturity, quality, the actual stand age and the spatial and temporal order, the need to rebuild forests, silvicultural requirements (e.g., initiation of natural regeneration) and economic considerations, including, in particular, the situation on the timber market;

4) the instructions, guidelines and ordinances concerning the regulation of timber utilization highlight the possibility of treating the rotation age of each stand individually, which is sometimes associated with a significant departure from the average rotation age set for a given forest management unit;

5) the boundary and target diameters at breast height play a role of the primary criterion of the commer-

cial maturity of forests, whereas technical quality may cause the acceleration of delay of harvest;

6) the need to preserve the spatial and temporal order may dramatically change the order of forest utilization resulting from the rotation;

7) the situation on the timber market is regarded as one of the factors relevant for the implementation of the principle of continuous maximization of the set goal. These are mainly changes in the plan of cuts forced by natural disasters.

A complex nature of the requirements regulating the amount of cuts results from a review of the key factors affecting and hierarchizing the sequence of selection of individual forests for harvest. At the same time, the application of model solutions in this area is still conditioned by the insufficient accuracy of determining stand volume and the lack of principles for economic analysis in this regard.

The timber production function and the general principles for determining the economic rotation age for single forest stand

Forest production, like any manufacturing process, is aimed to meet the set goals using the available resources, which, due to their role in the production process, are called production factors. With a certain amount of labour and land, and the adequate quantities of other production factors (raw materials, machinery and equipment), a certain amount of goods and services can be produced. The volume of such production depends, *inter alia*, on the adopted technology. With a novel technological process, the resources employed can generate more production. Always, however, there will be a maximum level of production that can be achieved with a given amount of production factors (Ohlin 1995). The relationship between the resources employed (input) and the results achieved (output) in the framework of a given technology is called „the production function".

The production function describes technical link (relationship) between the maximum production volume and the production factors employed that are essential for its achievement. Knowledge of the relationship between the factors and the size (scale, volume) of production is crucial for the management process. First of all, it allows answering the question – what results can be expected from the use of different factors, or how

many and what factors should be used to obtain the desired production volume.

The main objective of the study is to formulate the principles of correct economic analysis from the point of view of its suitability for the forest management practice where forest is treated only as a source of timber sold on the free market. This means that, at this stage, the externalities and the non-wood forestry products are not taken into consideration.

Of the several theoretical criteria and rules of economic optimization of the rotation age of forests (e.g., maximization of the volume of timber production, NPV (*net present value*) and IRR (*internal rate of return*), land rent is one to be considered in economic calculations. This need results from the requirements of economic profitability. Forest land, regarded as a permanent component of forest capital, should generate a profit corresponding to a standard interest rate, taking into account some elements of risk. The opinion that forestry has a unique position among other sectors of the economy and that its capital need not necessarily bring interest (direct consequence of this view is the principle of the highest output, which is maximization of timber production) is completely inconsistent with the requirements of profitability set for any economic activity.

THE METHOD OF DETERMINATION OF ECONOMIC ROTATION AGE

The determination of rotation that is most advantageous from the economic point of view may be considered as a typical investment problem. In order to maximize the net income from timber production, it is necessary to examine how the earned income and the production costs differ depending on the age of the stand. Due to the fact that both the costs and the revenues associated with timber production refer to different periods of time, they must therefore be discounted to be comparable, that is converted into their equivalent present value. Only then one can determine the age at which there is the greatest difference between the current value of income and the current value of costs. The subject of this study is the method of determining this difference, which is the search for an economically optimal rotation and its dependence on the natural conditions of growth

and economic characteristics of stands under the growing threat to forest environment.

To simplify the problem, it is assumed that revenues generated from timber sales are the only type of income taken into account in the calculations aimed at determining the optimal rotation age. The value of other benefits resulting from the protection, recreation and other functions performed by forests will be skipped at this stage of the analysis. In addition, it is assumed that the principles of management provide for the harvest of the whole timber volume by clear-cutting upon reaching the rotation age by the forest stand. This, in turn, narrows the analysis of determining the rotation age to even-aged stands, where rotation is the difference in the number of years (interval) between two consecutive cuts of forest stands growing in the same area.

### The value of standing timber *versus* stand age

The value of timber in the form of growing stock is called as its stumpage value. In the market economy, it is the maximum price that buyers would be willing to pay for timber standing in the forest. Hence, the value of standing timber, that is the value of growing stock (S) equals the gross income (D), which the producer can expect from harvesting timber and selling it on the market, minus the expected costs of timber harvest (C), or

$$S = D - C \qquad (1)$$

The costs in formula (1) should cover both: the operating costs and capital costs, as well as the standard share of profit for the producer. The value of standing timber embodies all possible income and all necessary outlays that forest owner must take into consideration when planning timber harvest. A forest stand, growing on a specific area, increases in stumpage value as its age increases, following the general pattern indicated by the curve $S_{(t)}$ displayed in field *a*, Figure 1, where *t* refers to the age of forest.

This multiplication of the value of standing timber with age is due to at least three reasons. Firstly, the volume of utility timber per unit area of the forest increases with forest growth. The age-related change in stand volume is shown by the broken line $V_{(t)}$, and is also shown in field *a* in Figure 1. It follows a sigmoid curve S, its slope increasing up to an inflection point du-

ring the lifetime of the forest, then decreasing, a growth pattern frequently observed in biology. In the case of a forest stand, the volume continues to increase as long as the (diminishing) annual increment of growth exceeds the (increasing) losses caused by the process of natural mortality of trees, insects or diseases.
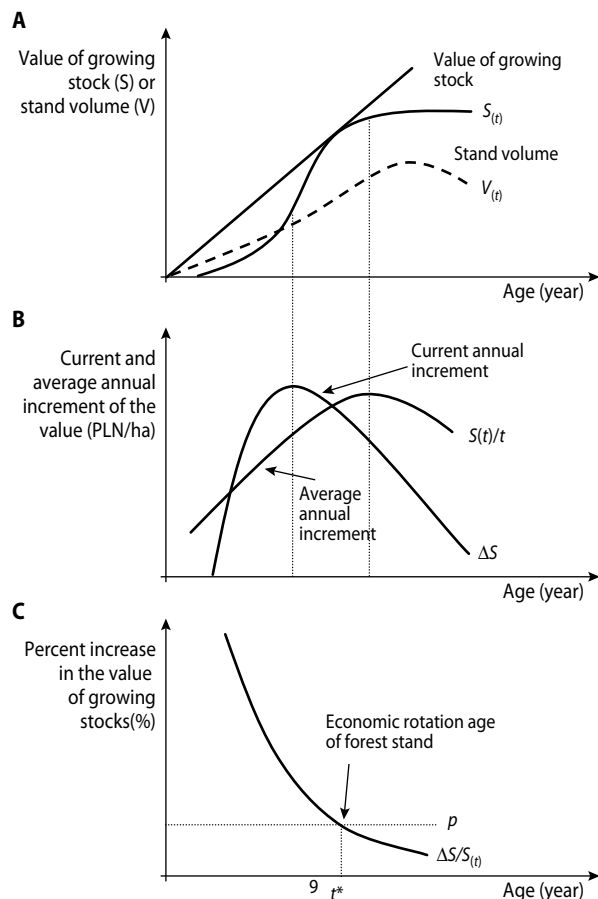
Figure 1. Field A and B: Relationship between the age, value and volume of the growing stock and its current and average incremental growth; Field C: Graphical determination of the economic rotation age a of a single forest stand

The second reason for increase in the value of standing timber during forest aging is that individual trees not only increase their volume, but by becoming thicker they at the same time achieve the dimensional and qualitative characteristics qualifying certain sections of the stem into assortments of higher commercial value. The described qualitative changes are reflected in the age-related increase in the unit price of standing timber volume (1 m³). A third reason is that larger timber can

usually be harvested at lower cost per cubic meter, reflecting economies of log size.

As a result, changes in the stumpage value (standing timber) are similar to changes in its volume but at a faster rate and over a longer period of time, as shown in field *a* in Figure 1. With information about the stumpage value of growing stock, illustrated by curve $S_{(t)}$, the average rate of growth in the stumpage value of the stand to any selected age can be easily calculated. It is simply the value of the stand at a particular age divided by the number of years to that age, namely, $S_{(t)}/t$. Geometrically, this value is determined by the tangent of the angle that forms the radius vector of curve $S_{(t)}$ with the positive direction of the abscissa axis. Such calculated increase in the average value of the forest depending on its age is shown in field *b* in Figure 1.

The current incremental growth in the value of forest stand is an increase in the value of the growing stock observed in subsequent years of the stand's lifetime. In other words, this increment shows how much the value of forest stand will increase, if the harvest is delayed an additional year, whereas the value of this increment also varies with the age of the stand. Geometrically, this annual incremental growth in value is represented by the slope of the stumpage value curve $S_{(t)}$ from the upper field in Figure 1. It increases up to the inflection point on the stumpage value curve, and then falls, as shown in the middle field B of Figure 1.

The interrelationship between the current growth curve and the average growth curve corresponds to the relation between the marginal cost curve and the average cost curve in the economic theory of the production. As long as the increment in the value growth from one year to the next is greater than the average growth in value to that age, the average curve must continue to rise. At its maximum the average and incremental value growth are equal and it declines over the range where the incremental growth is less than average, as shown in Figure 1 (field B).

The economic optimum of rotation

The incremental growth in the value of a stand, expressed as a percent of its current value, is illustrated in Figure 1 (field C). The value of such age-depended percent growth continues to decline, as in the given formula, the value of the denominator increases and the increment in value growth declines over a wide range. Attemp-

ting to determine the most favourable felling age of a stand, one must take into account the marginal values of income and costs in the case of postponing felling to the next year. Specifically, it is necessary to compare, for each year separately, the growth in the forest capital value by growing the crop from one year to another ($\Delta S_{(t)}/S_{(t)}$) against the costs arising from this decision.

Leaving aside for the moment the cost of land, the interest that might be earned on capital in the form of the growing stock, if it was cashed and then invested at the current interest rate p, would be the expense incurred in connection with delaying harvest. So, while seeking to maximize income (profits), one should suspend cutting only as long as the rate of return, resulting from the increase in the value of the stand, exceeds the amount of the required interest rate.

It can be said that the stand will be held uncut as long as the marginal income achieved through the same exceeds the marginal costs of holding the stand uncut, and no year longer. The age at which the rate of return on forest capital (growing stock) equals the amount of interest rate, is the first attempt to approximately determine the optimal rotation age. This age described with the symbol t* is shown schematically in Figure 1 (field C).

Starting with the simplest possible assumptions, an attempt can be made to formulate a rule for the determination of the optimal rotation age. This age falls on such time in the forest's lifetime in which marginal revenues, earned as a result of holding the growing stock uncut for one more year (i.e., a percent characterizing the increase in the value of the growing stock) equal the opportunity cost of the capital represented by this stock. Using the above symbols, this condition can be presented in the form of the following equation:

$$\Delta S_{(t)} = p \cdot S_{(t)} \qquad (2)$$

Equation (2) shows that the crop should be carried until the incremental rate of return from another year's growth falls to the opportunity cost of capital. On this basis, it can be concluded that the optimal rotation age is the longer, if the higher and more delayed is the percent increase in the value of the forest, and the lower is the interest rate, as illustrated in Figure 1 (field C).

This equation enables maximization of the current value of timber to be harvested in line with the assumptions to the analysis of forest investment assessment.

The current value of timber to be harvested from the forest at its rotation age, discounted at the beginning of the rotation is:

$$W_0 = \frac{S_{(t)}}{\left(1+p\right)^t} \qquad (3)$$

The optimal rotation age is the age at which this value is greatest. Having information about the rate (percent) of growth in stumpage value of the stand and the opportunity cost of capital, this optimum age can be calculated as the age at which the annual increase in present value of the stand drops to zero, that is:

$$\Delta W_0 = 0$$

or

$$\frac{S_{(t+1)}}{\left(1+p\right)^{t+1}-1} - \frac{S_{(t)}}{\left(1+p\right)^t} = 0$$

This equation is equivalent to:

$$S_{(t+1)} = \left(1+p\right) \cdot S_t$$

which is the same as the rule in equation 2:

$$\Delta S_{(t+1)} = p \cdot S_{(t)}$$

In the presented formula, only one type of cost was taken into account, which resulted from the fact that the capital value was kept ('frozen') in the form of growing stock. Conducting production in a forest stand requires, in addition to human labour, the involvement of at least two more basic production resources, namely, forest land and capital. As a result, in economic calculations the costs of use of these two types of resources should be taken into account. Hence, also the analyses aimed at determining the economically optimal rotation age need to include the cost of use of forest land as a production factor.

Assuming that forest land can be used only for timber production, or more precisely, that this kind of management activity ensures its most productive use, and in addition, each next forest stand in this place will generate the same revenues at the same level of costs. The current value of all future revenues incoming regularly every rotation of t years, can be expressed as follows:

$$W_0 = \frac{S_{(t)}}{\left(1+p\right)^t} + \frac{S_{(t)}}{\left(1+p\right)^{2t}} + \frac{S_{(t)}}{\left(1+p\right)^{3t}} + \ldots + \frac{S_{(t)}}{\left(1+p\right)^{+\infty}}$$

where each element appearing successively on the right side shows the current value of the forest, determined for each additional rotation of *t* years. The above formula can be reduced to the following form:

$$W_0 = \frac{S_{(t)}}{(1+p)^t - 1}$$

This formula enables the calculation of the current value of an infinite series of future revenues from felling one stand after another less the cost of generating them. Such value is sometimes referred to as 'the expected value of forest land' or 'land rent' or the value of land not covered by forest. It seems that, in this case, the term 'habitat value' (W_s) is the most appropriate. Excluding the stand establishment costs, the formula for calculating the forest habitat value will take the following form:

$$W_s = \frac{S_{(t)}}{(1+p)^t - 1} \qquad (4)$$

The habitat value is the value of land designated for forest management in accordance with the principle of continuity of forest production. This value is related to the land which is not covered by forest (at the beginning of the rotation period). In the case of a continuous succession of one stand after another, the economically optimal rotation will be that which generates the highest forest habitat value. Hence, it is the age at which the net present value cannot be increased by extending the rotation age by one more year, i.e.

$$\Delta W_s = 0$$

which in turn means that:

$$\frac{S_{(t)}}{(1+p)^t - 1} = \frac{S_{(t+1)}}{(1+p)^{t+1} - 1}$$

And after its simplification:

$$\frac{\Delta S_{(t)}}{S_{(t)}} = \frac{p}{1 - (1+p)^{-t}} \qquad (5)$$

This equation will be satisfied at the optimal rotation age. This again suggests equating the marginal income, expressed as a percent increase in the value of

standing timber as a result of holding the stand uncut for the next year (the left side of equation 5) with marginal costs, also including the annual cost of land (the right side of equation 5). For each shorter age, it is more beneficial to suspend cutting, because the annual increase in forest value exceeds the annual cost and *vice versa*: for each age exceeding t^{**}, the cost increase exceeds the anticipated changes in the value of growing stock, as shown in Figure 2, which also illustrates the relationship between both sides of equation (5) and the optimal rotation t^{**}, when they are equal.
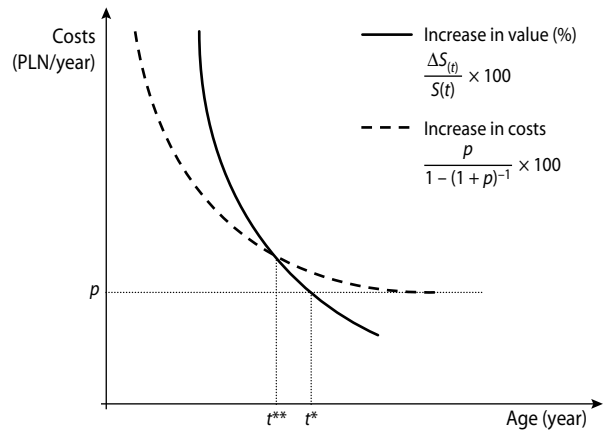
Figure 2. The effect of increase in the stand value and costs on the optimal rotation age

As shown in Figure 2, the optimal rotation in the case of continuity of forest management is shorter in comparison with the rotation t^* set for a single forest stand. The algebraic growth of costs under the conditions of establishing another next new forest stand: p: $1 - (1 + p)^{-t}$ is higher than the increase in costs p for a single forest stand (because the denominator in the first case is less than 1). Graphically, this means that the cost growth curve in continuous forest management is in a higher position and therefore, intersects the value growth curve at the earlier rotation age. As shown in Figure 2, the increase of costs p: $1 - (1 + p)^{-t}$ increases the rate of interest p.

The shorter rotation in the conditions of continuous management can be explained by additional costs of involvement of the second forest production factor that is land, which inevitably increases the costs related to delaying harvest over time, as a result of which the intersection of cost growth curve with the value growth

curve occurs at an earlier age. The opportunity cost of forest land, reflecting the value that might be generated, if the stand was cut and a new stand established, accelerates the urgency of harvest. Holding the current stand uncut for each subsequent year means, that the value of future stands represented by habitat value will be delayed by another year, and that it is most preferable to do the cutting at an earlier age of the stand.

The increase of costs related to holding stands uncut varies depending on the interest rate, both in the case of a single stand and sustained-yield management unit. If the rate is higher, an optimal rotation age is shorter.

RESULTS AND CONCLUSIONS OF CALCULATING THE ECONOMIC ROTATION AGE

The use of different economic categories for determining the optimal rotation age is illustrated in the Table. The data characterizes changes in volume and value of

a forest stand over time. In order to verify the methodical assumptions, the following data on stands with a share of pine of at least 80% of the total stand volume were taken from the Information System of the State Forests (SILP): stand area, growing stock, mean diameter at breast height, mean height of trees and volume of the harvested wood in pine stands. The volume of forest resources is as of 1 January 2014 r., however the data on harvested volume (sold timber) is presented as of 31 December 2014 r. The forest area was expressed in hectares with accuracy to 1 are (100 sq meters), but the volume is presented in m³ rounded to integer values. They serve as an example of calculating an economic rotation age of the pine stand (site index – III, interest rate – 1%).

The knowledge of the value of total timber production, given in column 7 of the Table, allows calculation of the habitat (forest land) value according to formula (4). Assuming a discount rate of 1%, the maximum habitat value amounting to 30,151 PLN/ha is

Table. Results of calculating the optimal economic rotation age of the pine stand (site index – III, interest rate – 1%)

Age of the stand (year)	Volume of the growing stock (m³·ha⁻¹)	Volume of the intermediate cutting (m³·ha⁻¹·year⁻¹)	Price of the intermediate cutting (PLN·(m³)⁻¹)	Value of the growing stock (PLN·ha⁻¹)	Value of the intermediate cutting (PLN·ha⁻¹)	Value of the total timber production (PLN·ha⁻¹)	Current annual increment of the stand value (PLN·ha⁻¹)	Value of the forest land (PLN·ha⁻¹)	Interest on the value of the growing stock (PLN·ha⁻¹)	Value of the forest land rent (PLN·ha⁻¹)	Annual increase of opportu-nity costs (PLN·ha⁻¹)
1	2	3	4	5	6	7	8	9	10	11	12
25	45	1	96	4 645	748	5 393	-	16 447	46	164	348
35	96	1	104	10 581	2 174	12 755	736	25 399	106	254	407
45	140	2	111	16 420	4 201	20 621	787	29 071	164	291	466
55	179	2	117	21 966	6 719	28 685	806	30 151	220	302	521
65	212	2	123	27 047	9 583	36 630	795	29 742	270	297	572
75	238	2	128	31 512	12 623	44 135	751	28 411	315	284	617
85	259	2	132	35 232	15 647	50 880	674	26 495	352	265	654
86	261	2	133	36338	16162	52500	664	26855	363	269	665
87	263	2	133	37312	17090	54402	653	27104	373	271	675
95	274	2	136	38 101	18 446	56 547	567	24 214	381	242	683
105	282	2	139	40 033	20 800	60 832	429	21 724	400	217	702
115	285	1	142	40 964	22 481	63 444	261	19 140	410	191	711
125	281	1	144	40 853	23 259	64 112	67	16 548	409	165	710
135	271	0	145	39 454	23 259	62 713	-140	13 933	395	139	696

reached when the rotation age is 55 years. The annual equivalent value of the habitat represents the annual opportunity costs of forest land, or land rent (*r*), in the form of:

$$r = 0.01 \cdot 30{,}151 = 302 \text{ PLN/ha}$$

and is the highest value shown in column 11 of the Table. The maximum habitat value and its equivalent in the form of an annual forest land rent determine the optimal, from the economic point of view, rotation age. When deciding about rotation, both should be considered – the annual growth in the value of stand (column 8) and the annual costs of forest land (column 11). The opportunity costs of land and forest capital associated with delaying cutting of a stand from year to year reflect the value that could be obtained when selling the timber. By cutting down a stand, capital in the form of timber ($S_{(t)}$) is obtained, on which the annual interest on the value of the growing stock ($p \cdot S_{(t)}$) is calculated.

At the same time it allows the sale of forest land, which at this point has a value equal to habitat value W_s, and whose annual equivalent is land rent *r*. The aforementioned two types of cost increase related to holding the forest uncut should be countered by an increase in the value of the stand (growing stock ΔS) taking place at that time. The optimal rotation age t^{**} should meet the following condition:

$$\Delta S_{(t)} = p \cdot S_{(t)} + r$$

From the quoted formula, delaying the cutting of standing timber makes sense only till the time at which the increase in its value exceeds the increases in costs, but no longer. Using the table data, the discussed relationships are shown in Figure 3. The figure shows that the increase in the value $\Delta S_{(t)}$ exceeds the increases in the opportunity costs only till the age of 86 years, which at the same time indicates the economic optimum of rotation.

Let us repeat once again, the term 'economically optimal rotation' means – according to equation (5), the age at which the annual increase in the value of growing stock is equal to the increase in the costs of maintaining the stand. The cost increase includes not only the interest on capital in the form of growing stock (forest stand), but also the interest on the value of forest land not covered with forest, that is an annual rent that land

is capable of generating in the case of maintaining the continuity of forest production.
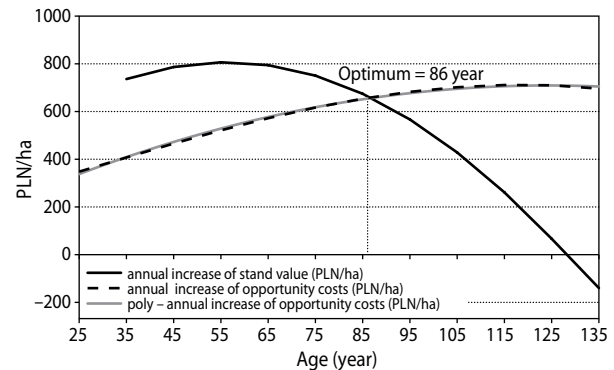

Figure 3. Economic optimum of the rotation age

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