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Synthesis of toothed gearing with reduced energy capacity

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S u m m a r y . Recommendations for synthesis of an original profile providing reduction of friction in toothing have been developed; geometrical parameters of the profile have been determined.
K e y w o r d s . Synthesis, original profile, energy capacity, slide velocity, toothed gearing.

PROBLEM

Operational performance and economic efficiency of modern machines in different industries mostly depend on performance quotient of gear transmissions. Designing a gear transmission with high performance quotient results in both perfection of the transmission and the machine itself, which is of vital importance in modern machine-building. One way of perfecting gear transmissions is development of toothed gearing with reduced energy capacity.

ANALYSIS OF RESEARCH LITERATURE

Energy capacity of toothed gearing depends on friction in gearing [1] which is determined mainly by the geometric parameters of the original profile used for shaping teeth of the gear wheel. Recently there has been an intensive research on designing non-involute gearing with high load capacity [1-6, 8-20]. Tooth geometry of these transmissions reduces, in some degree, the friction in gearing [1] of gearing wheels. Research on synthesis of toothed gearing whose teeth profiles are determined according to target values of friction in gearing, however, has not been carried out.

AIM AND OBJECTIVES

Development of a mathematical model of synthesis; synthesis of the original profile according to target values of friction in gearing which reduces energy capacity of the toothed gearing in cylindrical gear wheels.

MATHEMATICAL MODEL OF SYNTHESIS AND SYNTHESIS OF THE ORIGINAL PROFILE

Let us consider the order of geometric synthesis of the original profile according to the friction value in gearing. Friction value is:

$$F_{mp} = F_n f\ , \qquad (1)$$

where: F_n - the normal value in gearing;

f - friction coefficient in gearing.

We use friction coefficient value in sliding friction for the synthesis and comparative evaluation of friction [1]:

$$f = 0{,}09 q_n^{0,1}\left[10 + \lg\left(\frac{HB \cdot R_a \cdot \chi}{E_{np}}\right)\right]\chi^{0,25}\nu^{-0,07}V_{\Sigma}^{-0,1}V_{12}^{-0,35}$$

(2)

where: q_n –normal force acting on the unit of length of the contact line of teeth;

HB – hardness of the less hard of the contacting teeth;

R_a – roughness of the harder of the contacting teeth;

E_{np} – reduced elasticity module of gearing teeth materials;

ν – oil viscosity;

χ – relative curvature of teeth working surface;

V_{12} – sliding velocity in gearing;

V_{Σ} – rolling velocity sum in gearing.

For the assessment of forces of friction in gearing of the synthesized and involute transmissions we use the equation

$$\bar{f} = \frac{F_n f}{F_{nэ} f_э}, \tag{3}$$

where: $F_{nэ}$, $f_э$ – values for the involute transmission.

Analysis shows that $\frac{F_n}{F_{nэ}} \cong 1$ at similar parameters of the synthesized and involute transmissions and at similar loads on these transmissions. We can also assume in the first approximation that the ratio of values in the square brackets (2) for the synthesized and involute transmissions can equal one. Then equation (3) with the account of (2) will take the form :

$$\bar{f} = \frac{\bar{\chi}^{0,25} V_{\Sigma}^{-0,1} V_{12}^{-0,35}}{\bar{\chi}_э^{0,25} V_{\Sigma э}^{-0,1} V_{12Э}^{-0,35}}, \tag{4}$$

where: $\bar{\chi}$, $\bar{\chi}_э$ – relative curvatures for the synthesized and involute transmissions.

At large values of the radii of the pitch circles for the synthesized transmission (1) [1]:

$$V_{12} = \frac{(u+1)}{u}\frac{f}{\xi}; \quad V_{\Sigma} = 2R_1\sqrt{\frac{\xi}{\chi}}, \tag{5}$$

where: u – gear ratio;

R_1 – circle radius of the smaller wheel;

$\xi = \sin\alpha$ – (α - pressure angle of the original synthesized transmission, fig. 1)

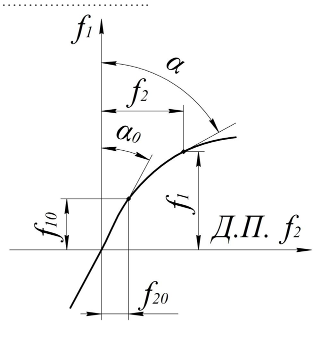

Fig. 1. Schematic diagram of the original profile (*ДП* – pitch line)

For the involute transmission [1]

$$V_{12э} = \frac{(u+1)f_1}{u\sin\alpha_э}; \quad V_{\Sigma э} = 2R_1\sin\alpha_э; \quad \bar{\chi}_э = \frac{1}{\sin\alpha_э}. \tag{6}$$

With the account of (5), (6) we receive from (4)

$$\bar{f} = \bar{\chi}^{0,3}\xi^{0,3}. \tag{7}$$

Relative value $\bar{\chi}$ for the synthesized transmission equals to [Shishov V.P. 2006]

$$\bar{\chi} = \frac{(\xi - f_1\xi')^2}{\xi^3}, \tag{8}$$

where: ξ' – derived function ξ for f_1;

f_1 – profile coordinate of the original profile (fig.1)

From equations (7) and (8) follows:

$$\xi' f_1 = \left(1 - \bar{f}^{1,67}\right)\xi. \tag{9}$$

Solution of differential equation (9) determines the actual angle of the original profile of the synthesized transmission. $\bar{f}$ <1 shows in which degree the friction of gearing in the synthesized transmission is less than that in the involute transmission. It should be noted that $\bar{f}$ can be both constant and variable in the area of gearing of gear wheels. When $\bar{f} = const$ equation (9) can be solved as follows

$$\xi = Cf_1^{\lambda}., \tag{10}$$

where: C – constant value. This value can be determined giving $\alpha = \alpha_0$ at $f_1 = f_{10}$. The n from equation (10) follows

$$\xi = \xi_0 f_1^{\lambda}, \tag{11}$$

where: $\xi_0 = \alpha_0$ (α_0 – is the angle of the original profile at $f_1 = f_{10}$)

At the preset value $\bar{f}$ equation (11) determines the angle of the original profile at a certain point. The value of this angle permits to determine the curvature equation which describes the original profile. This can be done as follows:

-take the constant value $\bar{f} < 1$ (at $\bar{f} = 1$ we have $\xi = \xi_0 = const$ - the original profile for involute teeth);

-take $\alpha_{\max}$ and $f_{10} = f_{1\max}$ ($f_{1\max}$ can be $f_{1\max} = 1$ at module $m = 1$ mm). Then $\xi_0 = \sin\alpha_{\max}$, and value $\alpha_{\max}$ must provide overlapping coefficient $\varepsilon_\alpha \geq 1{,}2$ and teeth thickness at knot point $S_a = (0{,}2...0{,}4)m)$ [11];

-determine from (11) and (10) values ξ .and ξ' within value limits $f_{\min} \leq f_1 \leq f_{\max}$ and determine the derived function f_2 by equation [1]:

$$f_2'' = \frac{\xi'}{\left(1-\xi^2\right)^{1,5}}, \tag{12}$$

where: f_2'' – constant coefficients;

$$f_2'' = \sum_{i=1}^{k} a_i f_1^i , \tag{13}$$

where: a_i – constant coefficients;

-integrating (13) we receive [7]:

$$f_2'' = \sum_{i=1}^{k} \frac{a_i}{(i+1)} f_1^{i+1} + C_1;$$
$$f_2'' = \sum_{i=1}^{k} \frac{a_i}{(i+1)(i+2)} f_1^{i+2} + C_1 f_1 + C_2, \tag{14}$$

where: C_1, C_2 – arbitrary constants.

Constants C_1, C_2 equal to:

$$C_1 = tg\alpha_{\max} - \sum_{i=1}^{k} \frac{a_k}{(i+1)} f_{10}^{i+1};$$
$$C_2 = f_{20} - \sum_{i=1}^{k} \frac{a_k}{(i+1)(i+2)} f_{12}^{i+2} - C_1 f_{12}, \tag{15}$$

where: f_{20} – value of function f_2 at $f_1 = f_{12}$.

According to anticipatory data f_{12} should be set within limits $f_{12} = 0{,}025...0{,}1$.

The original profile at $0 \leq f_1 \leq f_{12}$ can be outlined by the radial arc with the radius (fig.2):

$$\rho = \frac{\left(\sqrt{1+(f_{20}')^2}\right)^3}{f_{20}''}, \tag{16}$$

where: f_{20}', f_{20}'' – values (13) and (14) at $f_1 = f_{12}$. Function f_{20} is thus determined applying equation (fig.2):

$$f_{20} = \rho(\cos\alpha_n - \cos\alpha_{20});$$
$$\sin\alpha_n = \sin\alpha_{20} - \frac{f_{12}}{\rho}; \tag{17}$$
$$\cos\alpha_{20} = \frac{1}{\sqrt{1+(f_{20}')^2}},$$

where: α_n – profile angle of the original profile on the pitch line;

α_{20} - profile angle of the original profile at $f_1 = f_{12}$.

Radius ρ_Γ (fig.2) is determined on the assumption of complete rounding of the base of the original profile.

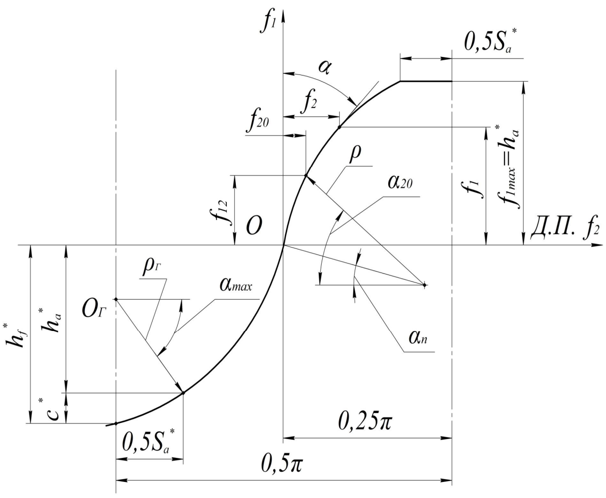

Fig. 2. Original profile ($m = 1$ mm, ДП – pitch line)

It equals :

$$\rho_\Gamma = 0{,}5S_a^*\sqrt{1-(f_{2\max}')^2}, \tag{18}$$

where: $f_{2\max}'$ – value of the first derivative (14) at $f_1 = f_{1\max}$;

$$S_a^* = 0{,}5\pi - 2f_{2\max}, \tag{19}$$

$f_{2\max}$ – value of function f_2 from (14) at $f_1 = f_{1\max}$.

Radial clearance in gearing of gear wheels is determined by value C^* which equals:

$$C^* = \rho_\Gamma\left[1 - \frac{f_{2\max}'}{\sqrt{1+(f_{2\max}')^2}}\right]. \tag{20}$$

CONCLUSIONS

1. Recommendations for synthesizing the original profile reducing friction in teeth gearing have been developed.
2. Geometrical parameters of the original profile which permits to reduce energy capacity of the toothed gearing have been determined.

REFERENCES

1. **Shishov V.P. Nosko P.L., Fil P.V., 2006.:** Theoretical foundation for synthesis of gear transmission: Monography. – Luhansk: Publishing House of East-Ukrainian Volodymyr Dahl National University. – 408 pp
2. **Shabanov I.R., 1967.:** On toothed gear with conchoidal line of gearing Durability and quality of toothed gearing. – NII TYAZHMASH. 18-67-106. –p. 1-8
3. **Anikyn Y.V.. 1975.:** Sinusoidal gearing – Voronezh: Publishing House of Voronezh State University. – 56 pp
4. **Bashansky M., Tokoli P., Vanya F., Kozhuh I., 2011.:** Possibility of use of non-involute gearing in the gearbox of agricultural machines Bulletin of NTU 'KhPI' - Kharkiv: NTU 'KhPI' . – 2011. - №29. – p.21-30
5. **Shishov V.P. Nosko P.L., Muhovatiy A.A., 2011.:** Highly loaded toothed gearing Bulletin of NTU 'KhPI' . – 2011. - №28. p. 180-186
6. **Protasov R.V., Ustinenko A.V., 2011.:** Research of overlapping coefficient of non-involute transmission Bulletin of NTU 'KhPI' . – №29. – p. 154-165
7. **Vinberg E.V., 1980.:** Polynomial algebra – M.: Education, 1980. - 176 pp
8. **Shishov V.P., Tkach P.N., Revyakina O.A., 2000.:** Cutting of teethsof cylindrical cogwheels with the generalized geometry. Lugansk, Visnik East Ukrainian national University. - №9(31). – p. 227-229
9. **Shishov V.P., Pilipec D.A., Revyakina O.A., Muhovaty A.A., 2001.:** About factors of overlapping of cylindrical tooth gearings with corcet and arch teeths. Lugansk, Visnik East Ukrainian national University. - №2(36). – p. 144-147
10. **Shishov V.P., Fil P.V., Revyakina O.A., Muhovaty A.A., 2001.:** The geounderground – kinematic indicators of cylindrical tooth gearing with arch teeths. Lugansk, Printing House of EUNU. The collection of proceedings: Resursosberegajushchie technologies manufactures and processings pressure of materials in mechanical engineering – p. 193-200
11. **Shishov V.P., Fil P.V., Revyakina O.A., Muhovaty A.A., 2001.:** The loaded tooth gearings with arch teeths. Lugansk, Visnik East Ukrainian national University. - №6(40). – p. 217-222
12. **Shishov V.P., Revyakina O.A., 2001.:** Definition of losses a friction in gearing of cylindrical wheels with arch teeths. Kharkov, Visnik National Technical university «KhPI». The thematic collection of scientific works «Technologies in mechanical engineering». - №12. – p. 41-48
13. **Shishov V.P., Pilipec D.A., Revyakina O.A., Muhovaty A.A. 2001.:** Cylindrical tooth gearings with extreme geometrical kinematic indicators. Lugansk, Visnik East Ukrainian national University. - №11(45). – p. 143-147
14. **Shishov V.P., Tkach P.N., Sklyar U.A., Tkach I.G., 2009.:** Geometrokinematichesky criterion for synthesis of the loaded cylindrical transfers with direct teeths. Odessa, The Odessa national polytechnical university. The podemno-transport technics №3 (31), - p.109-114
15. **Shishov V.P., Nosko P.L., Tkach P.N., Sklyar U.A., 2009.:** Bases of syntesis of a target contour of cylindrical tooth gearings with the raised wear resistence of wheels. . Kharkov, Visnik National Technical university «KhPI». The thematic collection of scientific works «Technologies in mechanical engineering». – p. 171-176
16. **Sklyar U.A., 2010.:** Geometrical criteria conhoidal transfers with displacement of an initial contour. . Kharkov, Visnik National Technical university «KhPI». The thematic collection of scientific works «Technologies in mechanical engineering». - №27. – p. 162-167
17. **Shishov V.P., Revyakina O.A., Tkach P.N., Sklyar U.A., 2010.:** Criteria of working capacity rod transfers with displacement rod. Kharkov, Visnik National Technical university «KhPI». The thematic collection of scientific works «Technologies in mechanical engineering». - №26. – p. 187-194
18. **Shishov V.P., Tkach P.N., Muhovaty A.A., Sklyar U.A., 2010.:** Geometrical kinematic criteria of cylindrical transfers with direct teeths at displacement of an initial contour. Odessa, The Odessa national polytechnical university. The podemno-transport technics №2 (34), - p.31-37
19. **Shishov V., Nosko P., Revyakina O., Muhovaty A., Karpov A., 2010.:** Teeth geometry of arched transmissions within initial contour shift TEKA Commission of Motorization and Power Industry in Agriculture, p. 279-286
20. **Shishov V., Nosko P., Revyakina O., Muhovaty A., Fil P., 2010.:** Geometrical and kinematic criteria of arched transmissions within initial contour shift TEKA Commission of Motorization and Power Industry in Agriculture, p. 287-293

СИНТЕЗ ЗУБЧАТЫХ ПЕРЕДАЧ С ПОНИЖЕННОЙ ЭНЕРГОЕМКОСТЬЮ

Александр Муховатый

А н н о т а ц и я . Разработаны рекомендации по синтезу исходного контура, обеспечивающего снижение сил трения в зацеплении зубчатых колес, определены геометрические параметры такого исходного контура.

К л ю ч е в ы е с л о в а . Синтез, исходный контур, энергоемкость, скорость скольжения, зубчатая передача.